EE 123 University of California, Berkeley
Anant Sahai February 15, 2007

Digital Signal Processing

Midterm 1

Name: _____ **SID:** _____

Instructions

- Total time allowed for the exam is 80 minutes
- Some useful formulas:
  - Discrete Time Fourier Transform (DTFT)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

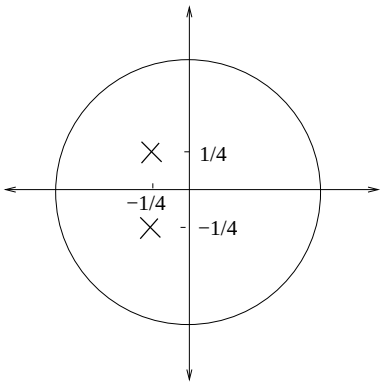
  - Inverse Fourier Transform

$$x[n] = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

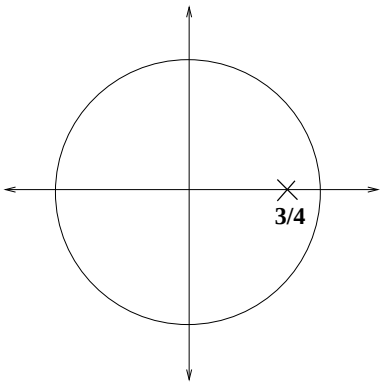
  - Z Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

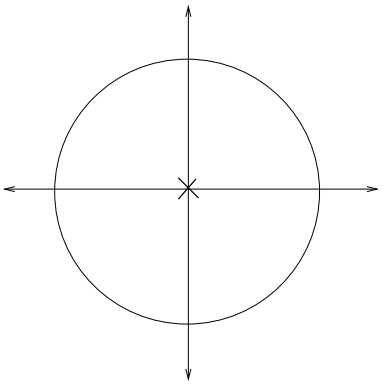
1. (*20 points*) Match each of the pole-zero plots to the corresponding magnitude and phase plots given below. Write your answer in the space provided below each magnitude and phase plot.



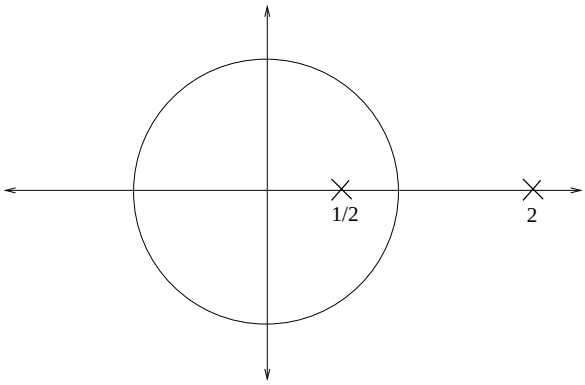
(a)



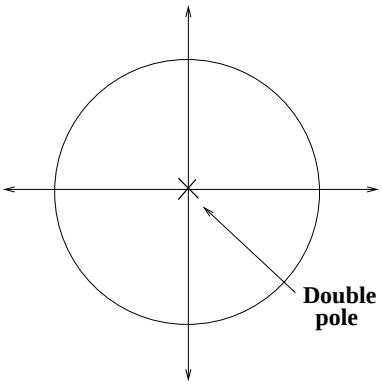
(b)



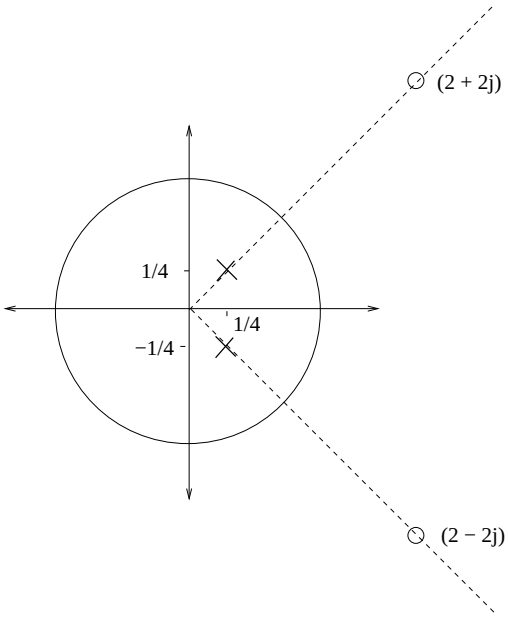
(c)



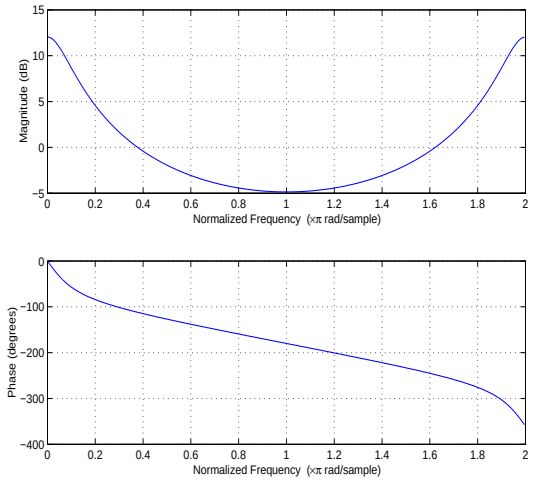
(d)



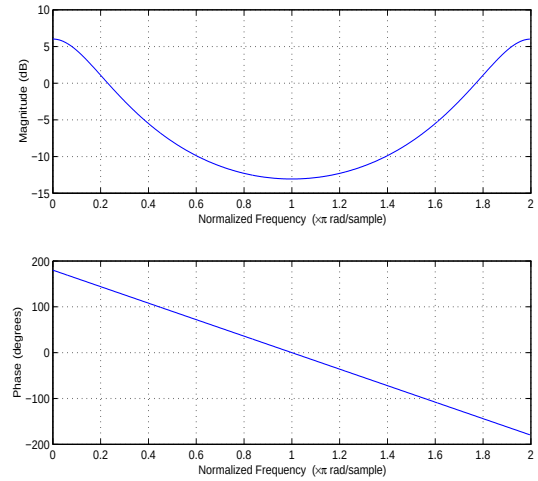
(e)



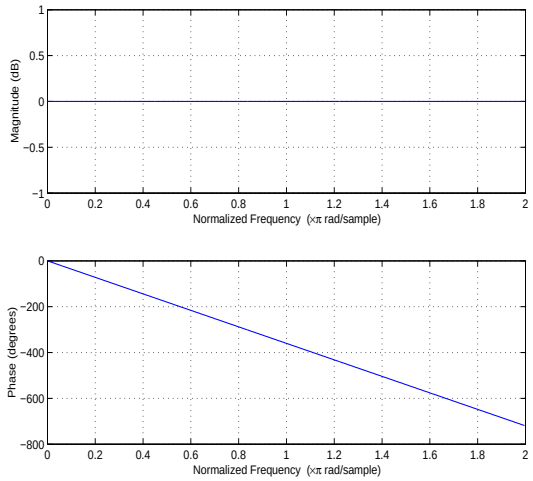
(f)


Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase (degrees)
Normalized Frequency (×π rad/sample)

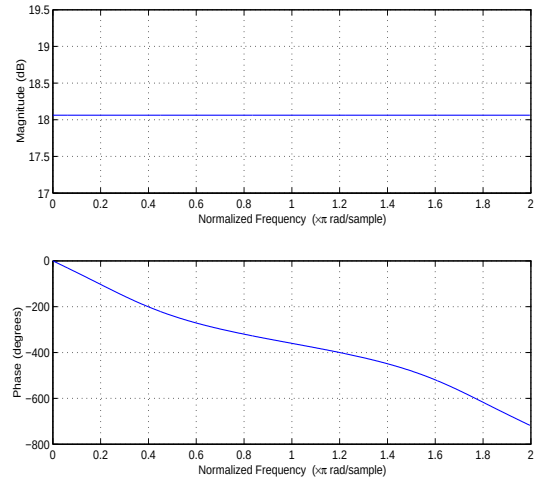
(1)-( )


Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase (degrees)
Normalized Frequency (×π rad/sample)

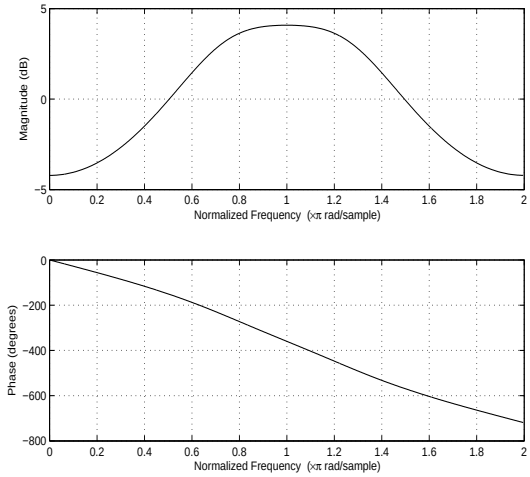
(2)-( )


Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase (degrees)
Normalized Frequency (×π rad/sample)

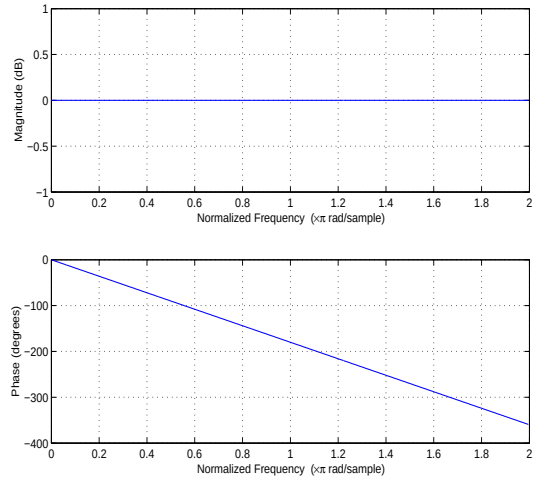
(3)-( )


Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase (degrees)
Normalized Frequency (×π rad/sample)

(4)-( )


Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase (degrees)
Normalized Frequency (×π rad/sample)

(5)-( )

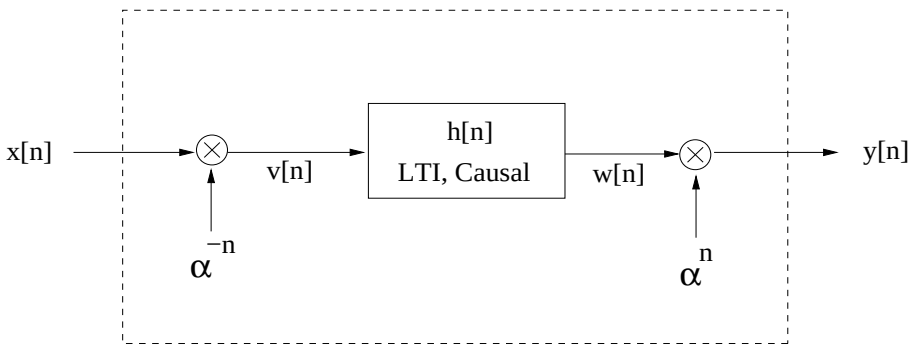

Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase (degrees)
Normalized Frequency (×π rad/sample)

(6)-( )

2. (*70 points*) Consider a causal linear time-invariant system with impulse response $h[n]$. The z-transform of $h[n]$ is

$$H(z) = \frac{1 - z^{-1}}{1 - 0.5z^{-1}}$$

Consider the cascade configuration given in the figure below. Here we assume that α is a real number.



(**a**) Draw the pole-zero plot of $H(z)$. Also, sketch the region of convergence. Is this system stable?

(**b**) Assume that $\alpha = \frac{1}{3}$ and $x[n] = \left(\frac{1}{4}\right)^n u[n]$. Compute the output $y[n]$ for the above choice of input $x[n]$.

(**c**) Is the overall system enclosed by the dashed box linear? Explain your reasoning.

(**d**) Is the overall system enclosed by the dashed box time-invariant? Explain your reasoning.

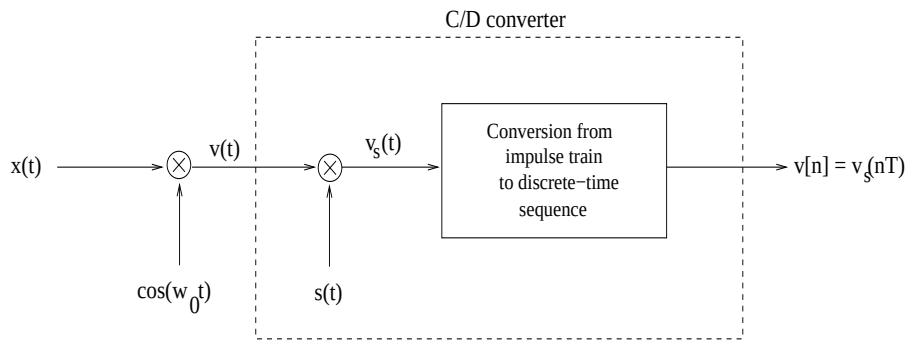
(**e**) Find the response $g[n]$ of the overall system within the enclosed box to the input $x[n] = \delta[n]$.

(**f**) Plot the pole-zero plot of $G(z)$, the z-transform of $g[n]$. Also, sketch the region of convergence.

(**g**) What happens to the pole-zero plot in part (**f**) if $\alpha = e^{-j\omega_0}$, for some real number ω_0.

3. (*40 points*) A real-valued signal $x(t)$ is known to be band-limited, i.e., $X(j\omega) = 0$, for $|\omega| > W$. $x(t)$ is first mixed by multiplying by $\cos(\omega_0 t)$. The resulting signal $v(t)$ is sampled uniformly (sampling time T) using the architecture given in the figure below. The periodic impulse train used for sampling is

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



Assume that $x(t) = \frac{\sin Wt}{Wt}$ and $\omega_0 = 2W$.

(**a**) Compute and draw $X(j\omega)$, the Fourier transform of $x(t)$.

(**b**) Compute and draw $V(j\omega)$, the Fourier transform of $v(t)$.

(**c**) Compute and draw $V_s(j\omega)$, the Fourier transform of $v_s(t)$.

(**d**) What is the minimum sampling rate $\Omega = \frac{2\pi}{T}$ required to reconstruct the original signal $x(t)$ from the discrete time signal $v_s[n]$.