1. This problem sketches a method used for prevention of data-loss due to scratches on CD (compact discs). The same method is used in satellite communication links.

Consider a real signal $x[n]$ of length 12. Let $\bar{x}[n]$ be a 16 length signal obtained from $x[n]$ by padding four zeros. Let $\{X[k]\}_{k=0}^{15}$ be the 16-pt DFT of $\bar{x}[n]$. In the following questions, $X[k]$ is corrupted by additive noise $W[k]$. Answer the following:

(a) Let $Y_1[k] = X[k] + W_1[k], k = 0, \ldots, 15$. Assuming that $W_1[k]$ is non-zero for four known values of $k$, describe a procedure to obtain $x[n]$ from $Y_1[k]$. Assume that $0 \leq i_1 < i_2 < i_3 < i_4 \leq 15$ are the four locations where $W_1[k]$ is non-zero.

(b) Let $Y_2[k] = X[k] + W_2[k], k = 0, \ldots, 15$. You are told that $W_2[k]$ is non-zero only at two places $k = i_1$ and $k = i_2$, where $i_1, i_2$ are unknown. Describe (in detail) a procedure to obtain $x[n]$ from $Y_2[k]$.

(c) In this part, you will design the zero-padding to tolerate more errors. You are told that $W[k]$ will have $t$ non-zero values at unknown locations. How much zero-padding should be added in $x[n]$ to successfully reconstruct $x[n]$ from $X[k] + W[k]$? Describe in detail.

(d) (MATLAB) Apply the technique in (a) to $Y1.mat$ and the technique in (b) to $Y2.mat$ to obtain $\bar{x}[n]$. If you did it right, you should see the same $\bar{x}[n]$ as the solution.

The locations where $W_1[k]$ is non-zero are $k = 1, k = 5, k = 7, \text{and } k = 15$. (This noise affects $Y1.mat$).

The CD connection: $x[n]$ is the data to be stored. $X[k]$ is the data that is actually stored. The scratches in CD can be thought of as successive errors in $W[k]$. And finally, the algebra (computations) happen in a Galois field (instead of the real number field as in this problem).

2. Consider a ZOH waveform shown in the following figure:
By differentiation, the ZOH output is converted into a sequence of delta functions. In this problem, the constants \( \{c_1, c_2, c_3\} \) and \( \{t_1, t_2, t_3\} \) are unknown. The finite stream of delta functions is observed after passing through a filter with impulse response \( g(t) = e^{-\frac{t^2}{\tau}} \). Let \( y(t) \) be the output of the filter.

Using the samples in \( y.mat \), which contains \( y(0), y(1), \ldots, y(5) \), find \( \{c_1, c_2, c_3\} \) and \( \{t_1, t_2, t_3\} \) using the annihilation filter method. (Use \( zplane() \) for factorization of the annihilation filter).

3. Problem 4.46 from Oppenheim, Schafer, and Buck.

4. Consider the system

\[
\begin{array}{c|c|c|c|c}
  x[n] & 25 \cdot H_d(e^{j\omega}) & e[n] & f[n] & y[n] \\
\end{array}
\]

where \( H_d(e^{j\omega}) \) is an (approximate) FIR LPF \( \{h[n]\}_{n=0}^{N-1} \) with cutoff frequency at \( \frac{\pi}{25} \) and \( X_d(e^{j\omega}) = \pi - |\omega|, \ |\omega| \leq \pi \). For sketching spectrum, assume that \( H_d(e^{j\omega}) \) is ideal.

(a) Sketch \( E_d(e^{j\omega}), F_d(e^{j\omega}), G_d(e^{j\omega}), \) and \( Y_d(e^{j\omega}) \).

(b) What does this overall system implement?

(c) For accurate low-pass filtering, \( N \) must be very large, say \( N = 200 \), to accurately realize the narrowband filter \( H_d(e^{j\omega}) \). Why might the above implementation be preferred over a more direct implementation?