

Name Solution SID _____

1. (3 points) Given the system difference equation $y[n] + 0.5y[n-1] = x[n]$, with $x[n] = (-1)^n$, the maximum value of the output $y[n]$ is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+0.5z^{-1}}$$

$$(-1)^n \text{ input} \Rightarrow H(-1) \cdot (-1)^n \text{ output } \{ \text{Eigen function} \}$$

$$= 2 \cdot (-1)^n \Rightarrow \max \{ \text{output} \} = 2$$

2. (4 points) Let the transfer function of an LTI system be given by,

$$H(z) = \frac{z^8 - 2}{1 - z^{-8}}, \quad |z| > 1, |z| < \infty.$$

Then,

$$h[-4] = \underline{0} \quad h[0] = \underline{-1} \quad \text{Is the system causal? } \underline{\text{No}} \quad \text{Is the system stable? } \underline{\text{No}}$$

$$H(z) = z^8 - \frac{1}{z^{-8}} = z^8 - \sum_{k=0}^{\infty} (z^{-8})^k$$

$$h[n] = S[n+8] - \sum_{k=0}^{\infty} S[n-8k]; \quad \sum_{n=-\infty}^N |h[n]| \rightarrow \infty \text{ as } N \rightarrow \infty$$

↳ non-causal part Unstable.

3. (3 points) Let $x[n]$ be given by,

$$x[n] = \frac{1}{8} \sum_{k=0}^7 e^{\frac{j2\pi nk}{8}}, \quad 0 \leq n \leq \infty.$$

$$\text{Then, } x[1] = \underline{\underline{0}} \quad x[8] = \underline{\underline{1}} \quad x[15] = \underline{\underline{0}}$$

$x[n] = s[(n)_8]$. (See DFT and inverse DFT).

$$\Rightarrow x[1] = 0 ; \quad x[15] = 0 ; \quad x[8] = 1 .$$