EE 123 DIGITAL SIGNAL PROCESSING, Spring 2009
Homework # 4, Due February 19, Thursday

1. Show that a causal LTI system with impulse response $h[n]$ is BIBO stable if and only if all poles of $H(z)$ are inside the unit circle in the complex plane.

   Hint: Note that the stability condition $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$ we studied earlier is equivalent to the condition that $\sum_{n=-\infty}^{\infty} |h[n]z^{-n}| < \infty$ for $|z| = 1$. What does this imply for the ROC and, thus, for the poles of $H(z)$?

2. Problem 6.44, Oppenheim and Schafer, 2nd ed.

3. Calculate explicitly the realizable pole positions in Figure 6.42(a) of Oppenheim and Schafer, 2nd ed.

4. Consider the sequence

$$x[n] = \sin\left(\frac{2\pi}{N}n + \Theta\right)$$

   where N is an integer and Θ is a random variable, uniformly distributed over the interval $[-\pi, \pi]$.

   a) Calculate the mean m_x, autocorrelation sequence $\phi_{xx}[m]$, and the autocovariance sequence $\gamma_{xx}[m]$. Make a sketch of $\phi_{xx}[m]$.

   b) Let $N = 10$, and use MATLAB to generate and plot a sequence $x[n]$ as above for $n = 0, 1, 2, \cdots, 99$. (You can use the `rand` command to generate a Θ for each n, but note that this command assumes a uniform distribution in the interval $[0, 1]$; therefore you need to do a subtraction and scaling to stretch this interval to $[-\pi, \pi]$.)

5. Problem 6.40, Oppenheim and Schafer, 2nd ed.