EE123
Digital Signal Processing

Lecture 16
Lab II - Time-Frequency

- compute spectrograms with w/o windowing
Lab II - Time-Frequency

• Compute with overlapping window
Lab II - Time-Frequency

• Look at temporal/frequency resolution tradeoffs:
FM Broadcast Radio - KPFA 94.1MHz
Spectrogram of Broadcast FM

Non-demodulated

FM demodulated
Spectrogram of Broadcast FM

FM demodulated

Broadcast FM baseband signal

L+R (mono) pilot signal

L-R (stereo)

RBDS

Subcarriers

19KHz 38KHz 57KHz 67.65KHz 92KHz

M. Lustig, EECS UC Berkeley
Filter Mono and down

- To play we need to filter the right signal
- Downsample to 48KHz so we can play on the computer

120KHz after filtering

24KHz after decimation
Demodulate subcarriers: Example 92KHz

demodulate by 92KHz
Demodulate subcarriers: Example 92KHz

- Filter and decimate

- FM demodulate and filter
Topics

• Last time
  – Ideal reconstruction D/C
  – D.T processing of C.T signals
  – C.T processing of D.T signals (ha?????)

• Today
  – Changing Sampling Rate via DSP
  – Downsampling
  – Upsampling
Example:

Non-integer delay: \[ H(e^{j\omega}) = e^{-j\omega\Delta} \]

• What is the time-domain operation when \( \Delta \) is not an integer (\( \Delta = 1/2 \))?

Let: \( H_c(j\Omega) = e^{-j\Omega\Delta T} \) delay of \( \Delta T \) in time

\[ x[n] \rightarrow \text{D/C} \rightarrow x_c(t) \rightarrow H_c(j\Omega) \rightarrow y_c(t) \rightarrow \text{C/D} \rightarrow y[n] \]

C.T recon delay \( \Delta T \) sampling
Example: Non Integer Delay

- The block diagram is only for interpretation!

\[ y_c(t) = x_c(t - \Delta) \]

\[ y[n] = y_c(nT) = x_c(nT - T\Delta) \]

\[ = \sum_k x[k]\text{sinc} \left( \frac{t - kT - T\Delta}{T} \right) \bigg|_{t=nT} \]

T’s cancel!

\[ = \sum_k x[k]\text{sinc}(n - k - \Delta) \]
Example: Non Integer Delay

\[ h[n] = \text{sinc}(n - \Delta) \]

Example: a discrete delta is a representation of a sampled sinc

shifted by partial samples results in many coefficients!
DownSampling

• Much like C/D conversion
• Expect similar effects:
  – Aliasing
  – mitigate by antialiasing filter

• Finely sampled signal $\Rightarrow$ almost continuous
  – Downsample in that case is like sampling!
Changing Sampling-rate via D.T Processing

Downsampling:

\[ x[n] \rightarrow \downarrow M \rightarrow x_d[n] = x[nM] \]

\[ = x_c(nT) \]

\[ = x_c(n MT) \]

The DTFT:

\[ X(e^{j\omega}) = \frac{1}{T} \sum_k X_c \left( j \left( \frac{\omega}{T} - \frac{2\pi}{T} k \right) \right) \]

\[ X_d(e^{j\omega}) = \frac{1}{MT} \sum_k X_c \left( j \left( \frac{\omega}{MT} - \frac{2\pi}{MT} k \right) \right) \]
Changing Sampling-rate via D.T Processing

The DTFT:

\[ X(e^{j\omega}) = \frac{1}{T} \sum_{k} X_c \left( j \left( \frac{\omega}{T} - \frac{2\pi}{T} k \right) \right) \]

\[ X_d(e^{j\omega}) = \frac{1}{MT} \sum_{k} X_c \left( j \left( \frac{\omega}{MT} - \frac{2\pi}{MT} k \right) \right) \]

we would like to bypass \( X_c \) and go from \( X(e^{j\omega}) \Rightarrow X_d(e^{j\omega}) \)

substitute \( r = kM + i \quad i=0,1,\ldots,M-1 \)
\( k=-\infty,\ldots,\infty \)

two counters

e.g., k: hours, i: minutes
Changing Sampling-rate via D.T Processing

\[
X_d(e^{j\omega}) = \frac{1}{MT} \sum_k X_c \left( j \left( \frac{\omega}{MT} - \frac{2\pi}{MT} k \right) \right)
\]

\[
= \frac{1}{M} \sum_{i=0}^{M-1} \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c \left( j \left( \frac{\omega}{MT} - \frac{2\pi}{MT} i - \frac{2\pi}{T} k \right) \right)
\]

\[
X \left( e^{j \left( \frac{\omega}{M} - \frac{2\pi}{M} i \right)} \right)
\]

\[
X_d(e^{j\omega}) = \frac{1}{M} \sum_{i=0}^{M-1} X \left( e^{j \left( \frac{\omega}{M} - \frac{2\pi}{M} i \right)} \right)
\]

stretch by M

replicate
Changing Sampling-rate via D.T Processing

\[
X_d(e^{j\omega}) = \frac{1}{M} \sum_{i=0}^{M} X \left( e^{j \left( \frac{\omega}{M} - \frac{2\pi i}{M} \right)} \right)
\]
Changing Sampling-rate via D.T Processing

\[ X_d(e^{j\omega}) = \frac{1}{M} \sum_{i=0}^{M} X \left( e^{j(\omega/M - 2\pi i/M)} \right) \]
Anti-Aliasing

\[ x[n] \xrightarrow{\text{LPF } \pi/M} \tilde{x}[n] \xrightarrow{\downarrow M} \tilde{x}_d[n] = \tilde{x}[nM] \]