Announcements

- Midterm: Friday next week
  - Open everything
  - ... but cheat sheet recommended instead
  - Who can not stay till 5pm?

- Optional homework next week
  - Will give you midterm and practice questions

- How’s lab I going?

How do you know this guy is insane?

Spectrum not symmetric, so cat must be imaginary

http://xkcd.com/26/

Last Time

- Frequency Analysis with DFT
- Windowing
- Zero-Padding

- Today:
  - Time-Dependent Fourier Transform
  - Heisenberg Boxes

Discrete Transforms (Finite)

- DFT is only one out of a LARGE class of transforms
- Used for:
  - Analysis
  - Compression
  - Denoising
  - Detection
  - Recognition
  - Approximation (Sparse)

Spectrum of a bird chirping

Interesting,.... but...
- Does not tell the whole story
- No temporal information!

Example of spectral analysis

Sparse representation has been one of the hottest research topics in the last 15 years in sp
To get temporal information, use part of the signal around every time point

\[ X[n, \omega] = \sum_{m = -\infty}^{\infty} x[n + m] w[m] e^{-j\omega m} \]

*Also called Short-time Fourier Transform (STFT)

• Mapping from 1D \( \rightarrow \) 2D, \( n \) discrete, \( w \) cont.
• Simply slide a window and compute DTFT

\[ X_r[k] = \sum_{m=0}^{L-1} x[rR + m] w[m] e^{-j2\pi km/N} \]

• \( L \) - Window length
• \( R \) - Jump of samples
• \( N \) - DFT length
• Tradeoff between time and frequency resolution

\[ X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \]

\[ \Delta \omega = \frac{2\pi}{N} \]
\[ \Delta t = \frac{N}{2\pi} \]
\[ \Delta \omega \cdot \Delta t = 2\pi \]
**DFT**

\[
X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}
\]

\[\Delta \omega = \frac{2\pi}{N}\]
\[\Delta t = N\]
\[\Delta \omega \cdot \Delta t = 2\pi\]

Question: What is the effect of zero-padding?
Answer: Overlapped Tiling!

**Discrete STFT**

\[
X[r, k] = \sum_{m=0}^{L-1} x[r R + m] w[m] e^{-j2\pi km/N}
\]

\[\Delta \omega = \frac{2\pi}{L}\]
\[\Delta t = L\]

Question: What is the effect of R on tiling? what effect of N?
Answer: Overlapping in time of frequency or both!

**Applications**

- Time Frequency Analysis

![Spectrogram of Orca whale](image)

**Spectrogram**

- What is the difference between the spectrograms?
  a) Window size B<A
  b) Window size B>A
  c) Window type is different
  d) (A) uses overlapping window

**Sidelobes of Hann vs rectangular window**

![DTFT of Hamming Window](image)
![DTFT of Rectangular Window](image)
• What is the difference between the spectrograms?
  a) Window size B < A
  b) Window size B > A
  c) Window type is different
  d) (A) uses overlapping window

\[ y_c(t) = A \cos \left( 2\pi f_c t + 2\pi f_p \int_0^t x(\tau) d\tau \right) \]
\[ y[n] = y(nT) = A \exp \left( j2\pi f_p \Delta \int_0^n x(\tau) d\tau \right) \]

Spectrogram of FM

\[ x(t) = (L + R) + 0.1 \cos(2\pi f_p t) + (L - R) \cos(2\pi (3f_p) t) + 0.05 \text{ RBDS}(t) \cos(2\pi (3f_p) t) \]

Spectrogram of FM radio Baseband

Subcarrier FM radio (Hidden Radio Stations)

Applications

• Time Frequency Analysis
STFT Reconstruction

\[ x[rR + m]w_L[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[n, k] e^{j2\pi km/N} \]

- For non-overlapping windows, \( R=L \):

\[ x[n] = \frac{x[n - rL]}{w_L[n - rL]} \quad rL \leq n \leq (r + 1)R - 1 \]

- What is the problem?

STFT Reconstruction

\[ x[rR + m]w_L[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[n, k] e^{j2\pi km/N} \]

- For non-overlapping windows, \( R=L \):

\[ x[n] = \frac{x[n - rL]}{w_L[n - rL]} \quad rL \leq n \leq (r + 1)R - 1 \]

- For stable reconstruction must overlap window 50% (at least)

Applications

- Noise removal

  - Recall bird chirp

  - Example: Bird Chirp

    Play Sound!

  - Spectrum of a bird chirp

  - Time, s

  - Frequency, Hz

Application

- Denoising of Sparse spectrograms

  - Spectrum is sparse! can implement adaptive filter, or just threshold!

Limitations of Discrete STFT

- Need overlapping ⇒ Not orthogonal

  - Computationally intensive \( O(MN \log N) \)

  - Same size Heisenberg boxes
From STFT to Wavelets

• Basic Idea:
  – low-freq changes slowly - fast tracking unimportant
  – Fast tracking of high-freq is important in many apps.
  – Must adapt Heisenberg box to frequency

• Back to continuous time for a bit.....