EE123
Digital Signal Processing

Lecture 10
Announcements

• Midterm: Friday next week
  – Open everything
  – ... but cheat sheet recommended instead
  – Who can not stay till 5pm?

• Optional homework next week
  – Will give you midterm and practice questions

• How’s lab I going?
How do you know this guy is insane?

Hi, Dr. Elizabeth?
Yeah, uh... I accidentally took the Fourier transform of my cat...

Spectrum not symmetric, so cat must be imaginary

http://xkcd.com/26/
Last Time

- Frequency Analysis with DFT
- Windowing
- Zero-Padding

Today:
- Time-Dependent Fourier Transform
- Heisenberg Boxes
Discrete Transforms (Finite)

- DFT is only one out of a LARGE class of transforms
- Used for:
  - Analysis
  - Compression
  - Denoising
  - Detection
  - Recognition
  - Approximation (Sparse)

Sparse representation has been one of the hottest research topics in the last 15 years in sp
Example of spectral analysis

- Spectrum of a bird chirping
  - Interesting,... but...
  - Does not tell the whole story
  - No temporal information!

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Time Dependent Fourier Transform

• To get temporal information, use part of the signal around every time point

\[ X[n, \omega) = \sum_{m=-\infty}^{\infty} x[n + m] w[m] e^{-j\omega m} \]

*Also called Short-time Fourier Transform (STFT)

• Mapping from 1D \( \Rightarrow \) 2D, \( n \) discrete, \( w \) cont.

• Simply slide a window and compute DTFT
Time Dependent Fourier Transform

- To get temporal information, use part of the signal around every time point

\[ X[n, \omega] = \sum_{m=-\infty}^{\infty} x[n + m] w[m] e^{-j\omega m} \]

*Also called Short-time Fourier Transform (STFT)*
Discrete Time Dependent FT

\[ X_r[k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N} \]

• L - Window length
• R - Jump of samples
• N - DFT length

• Tradeoff between time and frequency resolution
Heisenberg Boxes

- Time-Frequency uncertainty principle

\[ \sigma_t \cdot \sigma_\omega \geq \frac{1}{2} \]

\[ \omega \]

\[ \sigma_t \]

\[ \sigma_\omega \]

\[ t \]
DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$

$$\Delta \omega = \frac{2\pi}{N}$$

$$\Delta t = N$$

$$\Delta \omega \cdot \Delta t = 2\pi$$

one DFT coefficient
DFT

\[ X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \]

\[ \Delta \omega = \frac{2\pi}{N} \]
\[ \Delta t = N \]
\[ \Delta \omega \cdot \Delta t = 2\pi \]

Question: What is the effect of zero-padding?
Answer: Overlapped Tiling!
Discrete STFT

\[ X[r, k] = \sum_{m=0}^{L-1} x[rR + m] \omega[m] e^{-j2\pi km/N} \]

\[ \Delta \omega = \frac{2\pi}{L} \]

\[ \Delta t = L \]

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Discrete STFT

\[
X[r, k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j2\pi km/N}
\]

\[
\Delta \omega = \frac{2\pi}{L}
\]

\[
\Delta t = L
\]

Question: What is the effect of R on tiling? what effect of N?
Answer: Overlapping in time of frequency or both!

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Applications

• Time Frequency Analysis

Spectrogram of Orca whale
• What is the difference between the spectrograms?

a) Window size B<A  
b) Window size B>A  
c) Window type is different  
d) (A) uses overlapping window
Sidelobes of Hann vs rectangular window

```
Here we consider several examples. As before, the sampling rate is $\frac{s}{2\pi} = \frac{1}{T} = 20$ Hz.

Rectangular Window, $L = 32$

Hamming Window, $L = 32$
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Miki Lustig UCB. Based on Course Notes by J.M Kahn
Fall 2011, EE123 Digital Signal Processing
• What is the difference between the spectrograms?

a) Window size $B<A$

b) Window size $B>A$

c) Window type is different

d) (A) uses overlapping window

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Spectrogram

**Hamming Window, \( L = 32 \)**

![Hamming Window, \( L = 32 \)](image)

**Hamming Window, \( L = 64 \)**

![Hamming Window, \( L = 64 \)](image)
Spectrogram of FM

\[ y_c(t) = A \cos \left( 2\pi f_c t + 2\pi \Delta f \int_0^t x(\tau) d\tau \right) \]

\[ y[n] = y(nT) = A \exp \left( j2\pi \Delta f \int_0^{nT} x(\tau) d\tau \right) \]

Spectrogram of FM radio
Spectrogram of FM radio Baseband

\[ y[n] = y(nT) = A \exp \left( j2\pi \Delta f \int_0^{nT} x(\tau) d\tau \right) \]

\[ x(t) = (L + R) + 0.1 \cdot \cos(2\pi f_p t) + (L - R) \cos(2\pi (2f_p) t) + 0.05 \cdot \text{RBDS}(t) \cos(2\pi (3f_p) t). \]

Broadcast FM baseband signal

Spectrogram of \textbf{Demodulated} FM radio (Adele on 96.5 MHz)
Subcarrier FM radio (Hidden Radio Stations)
Applications

- Time Frequency Analysis

Spectrogram of digital communications - Frequency Shift Keying

$t=0$  
$t=1\text{sec}$
STFT Reconstruction

\[ x[rR + m]w_L[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[n, k]e^{j2\pi km/N} \]

• For non-overlapping windows, \( R=L \):

\[ x[n] = \frac{x[n - rL]}{w_L[n - rL]} \]

\[ rL \leq n \leq (r + 1)R - 1 \]

• What is the problem?
STFT Reconstruction

$$x[rR + m]w_L[m] = \frac{1}{N} \sum_{k=0}^{N-1} X[n, k] e^{j2\pi km/N}$$

- For non-overlapping windows, $R=L$:

$$x[n] = \frac{x[n - rL]}{w_L[n - rL]}$$

$$rL \leq n \leq (r+1)R - 1$$

- For stable reconstruction must overlap window 50% (at least)
STFT Reconstruction

• For stable reconstruction must overlap window 50% (at least)

• For Hann, Bartlett reconstruct with overlap and add. No division!
Applications

- Noise removal
- Recall bird chirp

Example: Bird Chirp

Play Sound!

Spectrum of a bird chirp

$X[n]$
Application

- Denoising of Sparse spectrograms

- Spectrum is sparse! can implement adaptive filter, or just threshold!
Limitations of Discrete STFT

• Need overlapping $\Rightarrow$ Not orthogonal

• Computationally intensive $O(MN \log N)$

• Same size Heisenberg boxes
From STFT to Wavelets

**Basic Idea:**
- Low-freq changes slowly - fast tracking unimportant
- Fast tracking of high-freq is important in many apps.
- Must adapt Heisenberg box to frequency

**Back to continuous time for a bit.....**