Back to Discrete

- Early 80’s, theoretical work by Morlett, Grossman and Meyer (math, geophysics)
- Late 80’s link to DSP by Daubechies and Mallat.

- From CWT to DWT not so trivial!
- Must take care to maintain properties

Discrete Wavelet Transform

$$d_{s,u} = \sum_{n=0}^{N-1} x[n] \Psi_{s,u}[n]$$

$$a_{s,u} = \sum_{n=0}^{N-1} x[n] \Phi_{s,u}[n]$$

Discrete Orthogonal Haar Wavelet

Haar for n=8

scaling $\Phi_0$ 

$\Psi_0$

$\Psi_10$

$\Psi_11$

Equivalent to DFT2!
Fast DWT with Filter Banks (more Later!)

Fast DWT with Filter Banks

Fast DWT with Filter Banks

Decomposition

Reconstruction

Example, Haar DWT - Level 0
Haar DWT Example

Approximation from 25/256 coefficients

Example: Denoising Noisy Signals

Example: Denoising by Thresholding

Compression - JPEG2000 vs JPEG

@ 66 fold compression ratio
Approximation/Compression

M. Lustig, EECS UC Berkeley
Robust 4D Flow Denoising using Divergence-free Wavelet Transform

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Running head: 4D Flow Denoising with Divergence-free Wavelet Transform

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Divergence Free Wavelets

Divergence-Free Wavelet Denoising