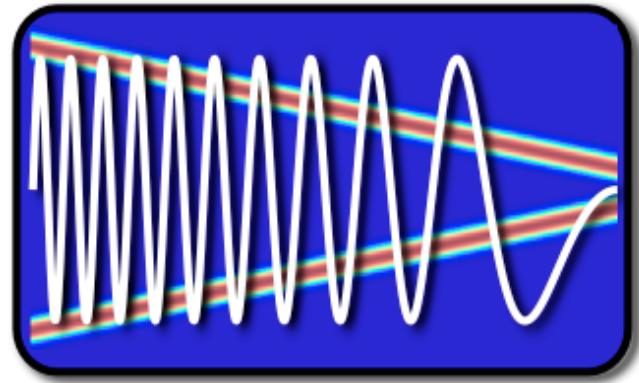


EE123



Digital Signal Processing

Lecture 14 Sampling

Announcements

- Ham exam Th 3/12 7-10+, The Woz Soda hall
- Lab:
 - Who is having trouble?

What is this Phenomena?

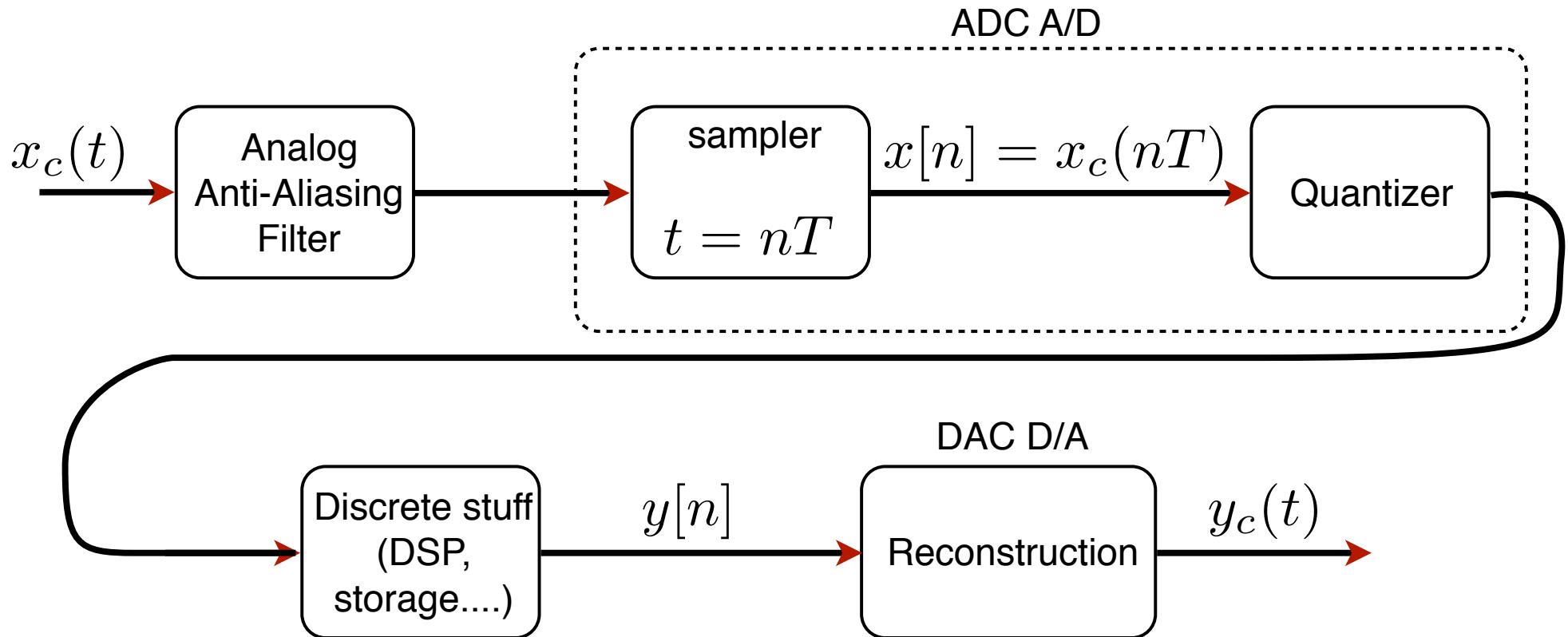


Sampling of Continuous Time Signals (Ch.4)

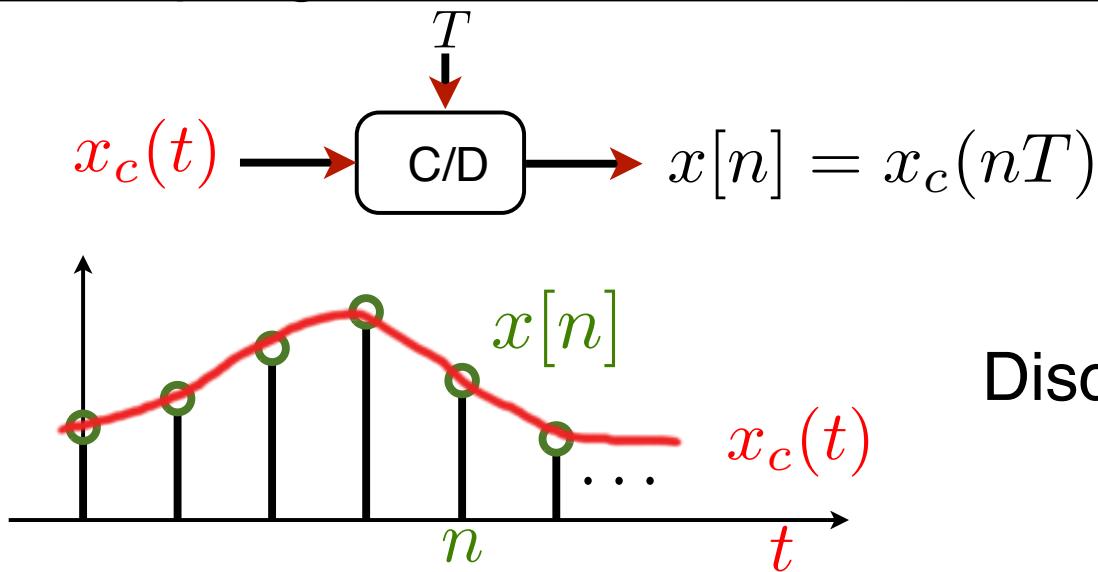
- Sampling:
 - Conversion from C.T (not quantized) into D.T (usually quantized)
- Reconstruction
 - D.T (quantized) to C.T
- Why?
 - Digital storage (audio, images, videos)
 - Digital communications (fiber optics, cellular...)
 - DSP (compression, correction, restoration)
 - Digital synthesis (speech, graphics)
 - Learning

Sampling of C.T. Signals

- Typical System:

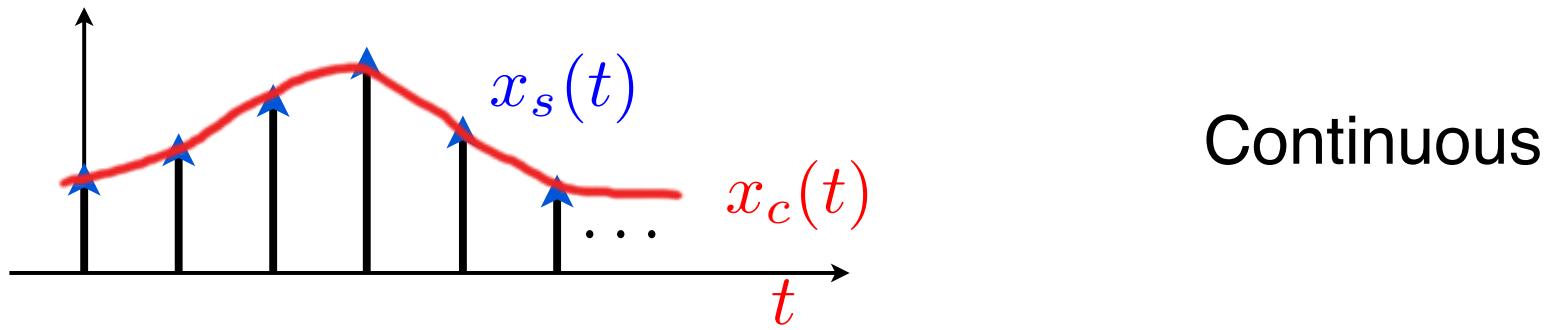


Ideal Sampling Model



Discrete and Continuous

define impulsive sampling:



Continuous

$$x_s(t) = \dots + x_c(0)\delta(t) + x_c(T)\delta(t - T) + \dots$$

$$x_s(t) = x_c \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

Ideal Sampling Model

$$x_s(t) = x_c \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

- Not physical: used for modeling & derivations

$$x[n] \leftrightarrow x_s(t) \leftrightarrow x_c(t)$$

- How is $x[n]$ related to $x_s(t)$ in freq. domain?

Frequency Domain Analysis

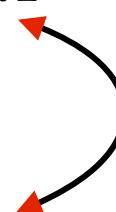
- How is $x[n]$ related to $x_s(t)$ in the Freq. Domain?

$$x_s(t) \text{ :C.T}$$

$$X_s(j\Omega) = \sum_n x_c(nT) e^{-j\Omega nT}$$

$$x[n] \text{ :D.T}$$

$$X(e^{j\omega}) = \sum_n x[n] e^{-j\omega n}$$



$$\omega = \Omega T$$

$$X(e^{j\omega}) = X_s(j\Omega)|_{\Omega=\omega/T}$$

$$X_s(j\Omega) = X(e^{j\omega})|_{\omega=\Omega T}$$

Frequency Domain Analysis

- How is $x_s(t)$ related to $x_c(t)$?

$$x_s(t) = x_c(t) \underbrace{\sum_n \delta(t - nT)}_{\triangleq s(t)}$$

Frequency Domain Analysis

- How is $x_s(t)$ related to $x_c(t)$?

$$x_s(t) = x_c(t) \underbrace{\sum_n \delta(t - nT)}_{\triangleq s(t)}$$

recall $\mathbb{U}(t) = \sum_n \delta(t-n)$ | notation break

$$\begin{aligned} s(t) &= \sum_n \delta(t-nT) = \sum_n \delta(T(\frac{t}{T}-n)) = \\ &\quad \text{recall } S(\omega) = \frac{1}{|a|} \delta(\omega) \\ &= \frac{1}{T} \sum_n \delta(\frac{t}{T}-n) = \frac{1}{T} \mathbb{U}(\frac{t}{T}) \end{aligned}$$

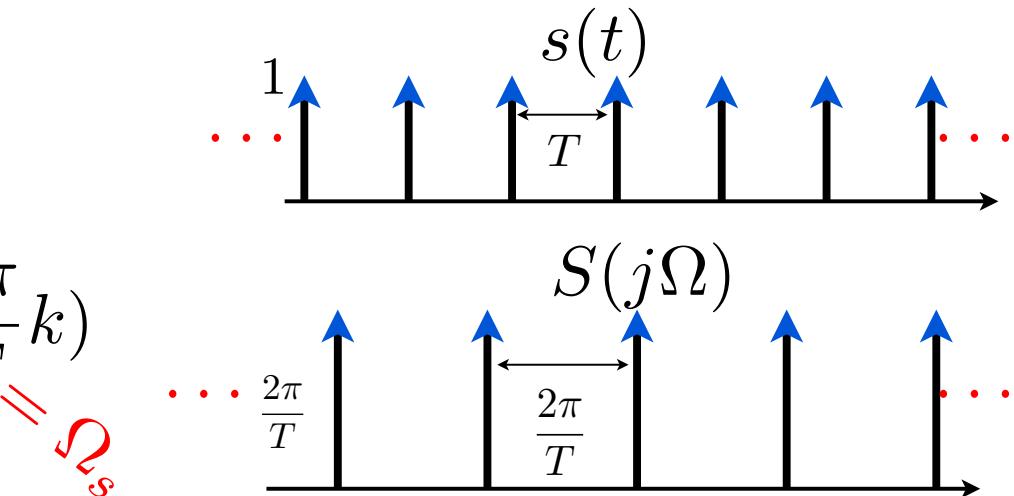
Frequency Domain Analysis

- How is $x_s(t)$ related to $x_c(t)$?

$$x_s(t) = x_c(t) \underbrace{\sum_n \delta(t - nT)}_{\triangleq s(t)}$$

$$s(t) \leftrightarrow S(j\Omega)$$

$$S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - \frac{2\pi}{T}k)$$

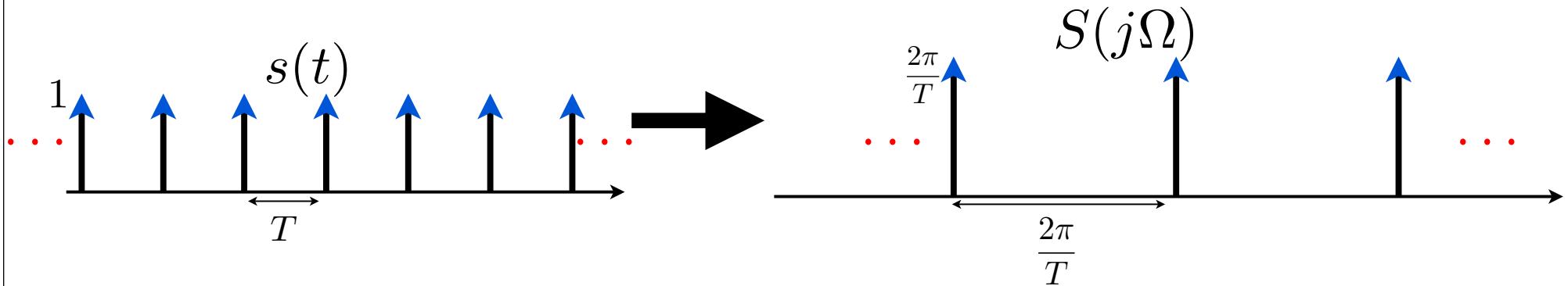


Frequency Domain Analysis

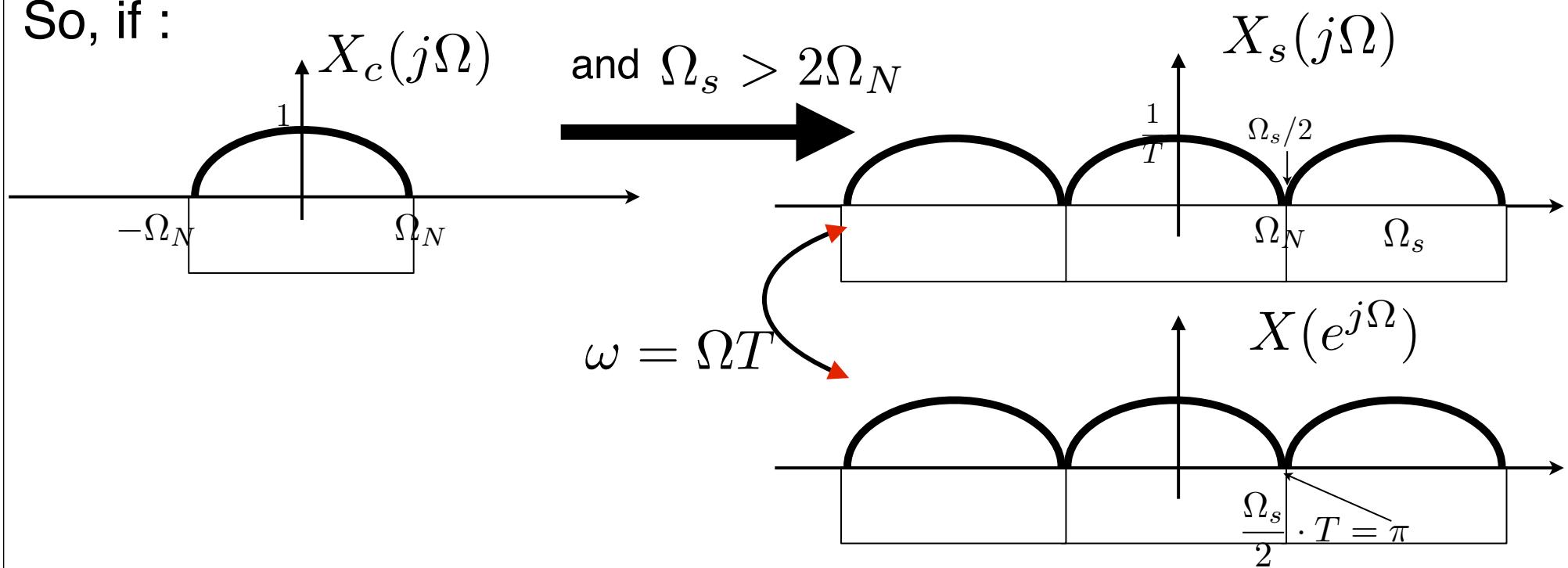
$$\begin{aligned} X_s(j\Omega) &= \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega) \\ &= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - \Omega_s)) \quad | \quad \Omega_s = \frac{2\pi}{T} \end{aligned}$$

- X_s is replication of X_c !

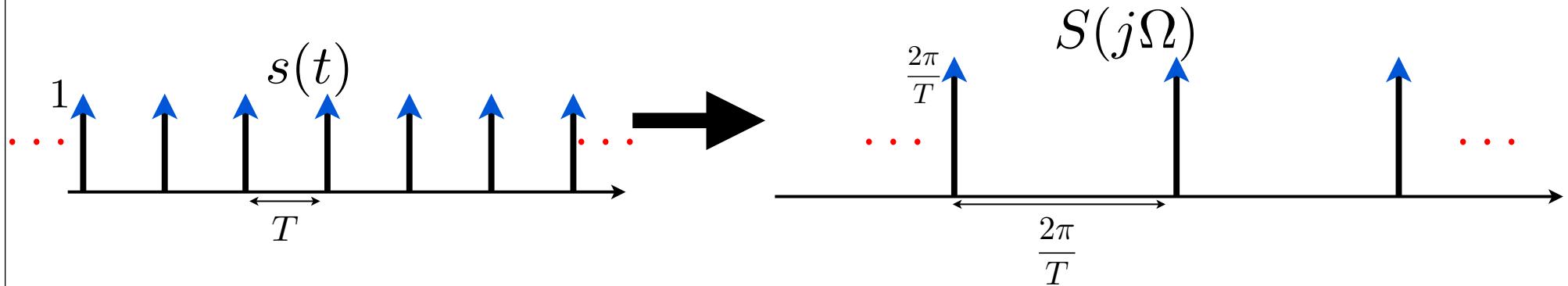
Frequency Domain Analysis



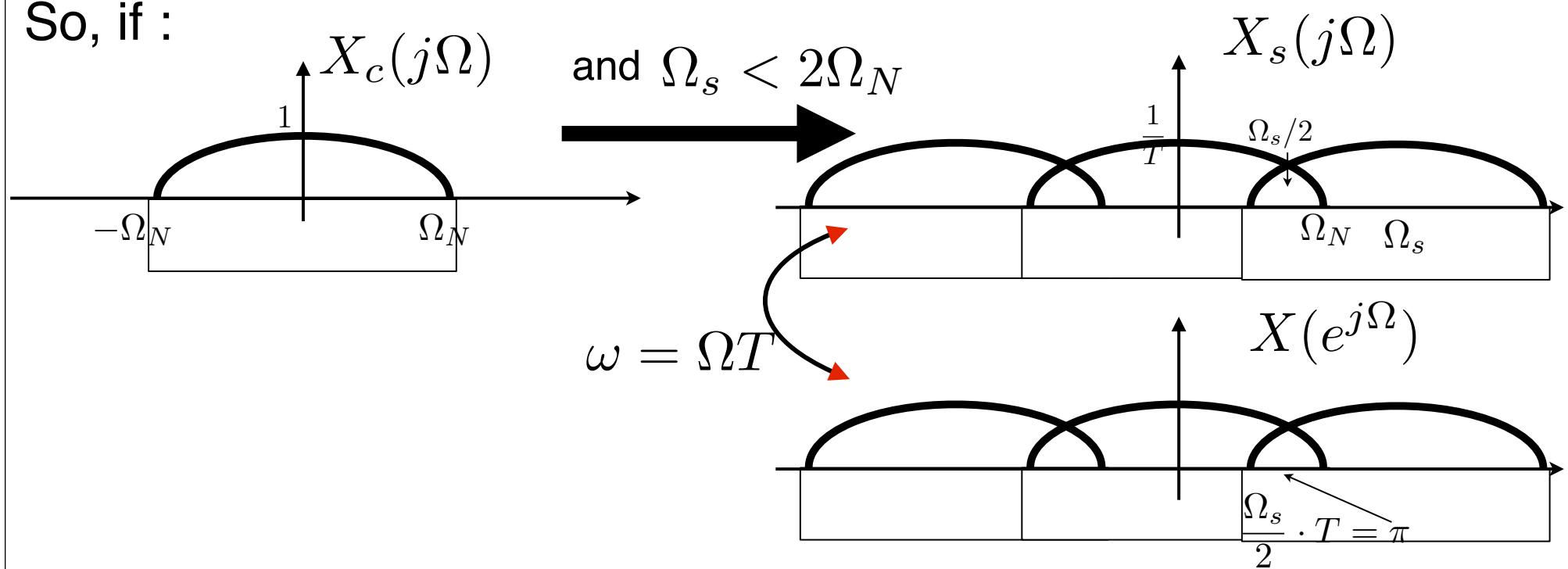
So, if :



Aliasing

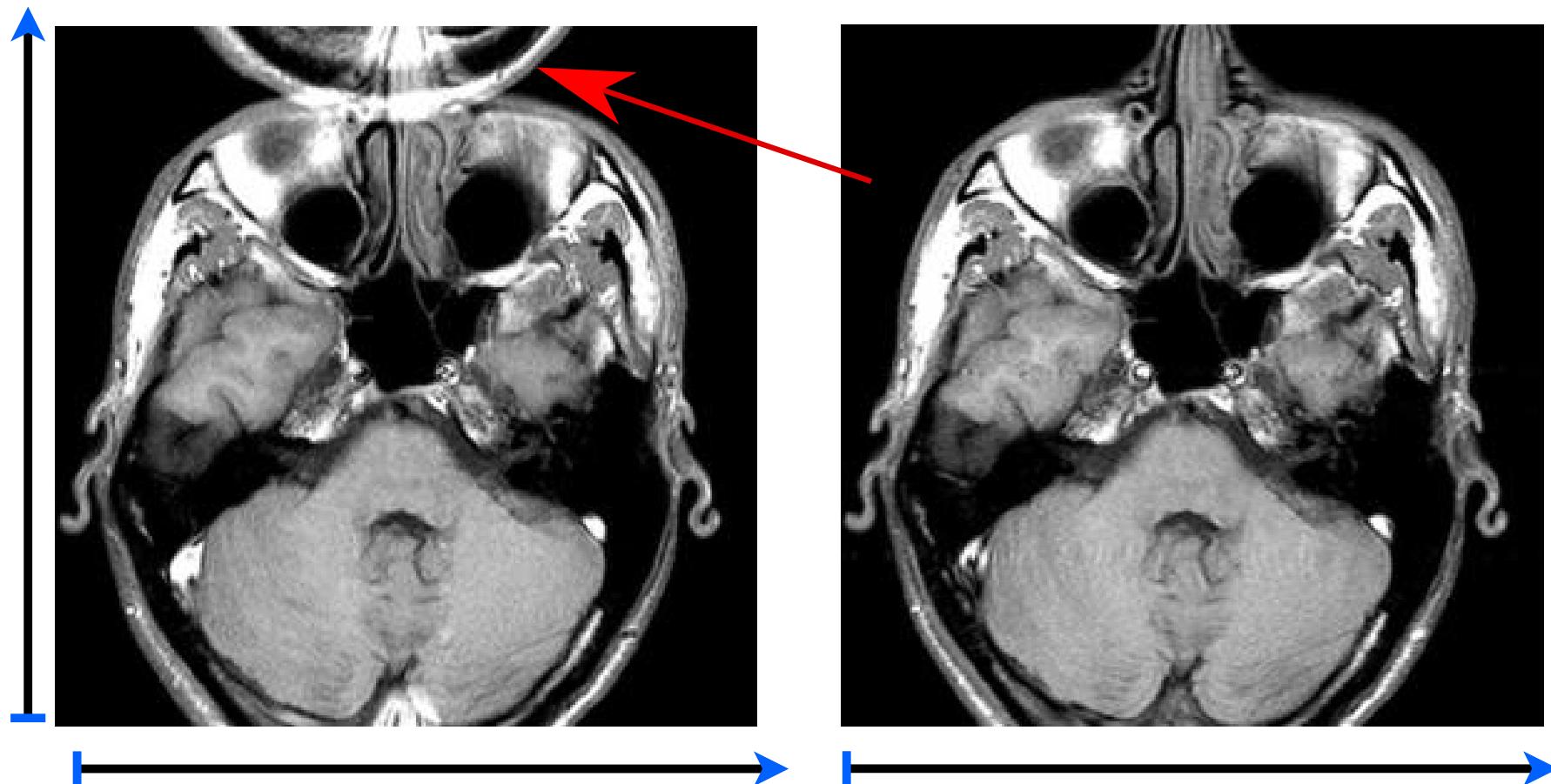


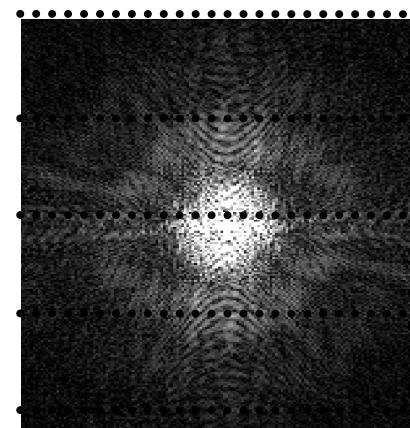
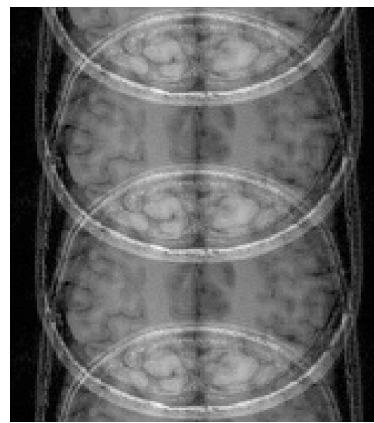
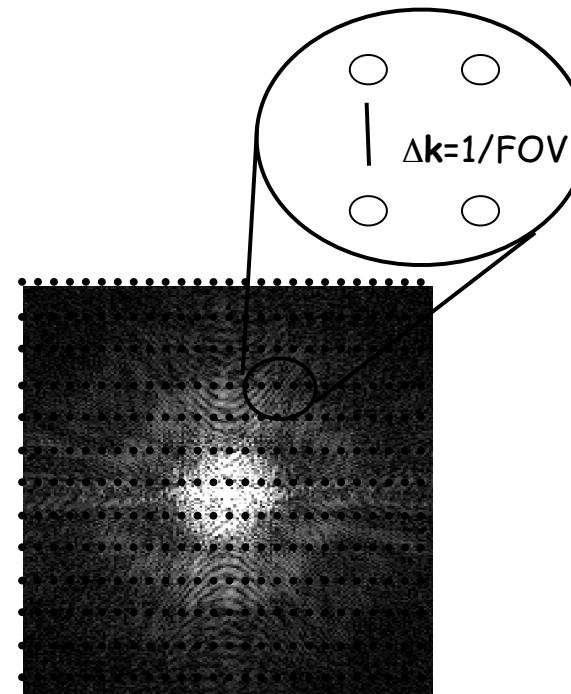
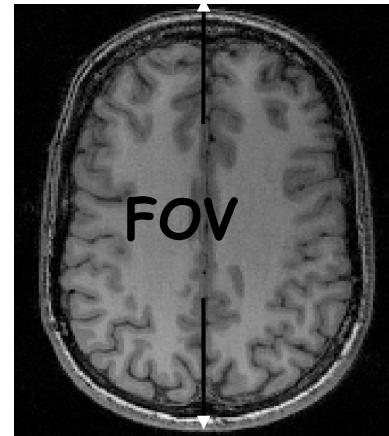
So, if :

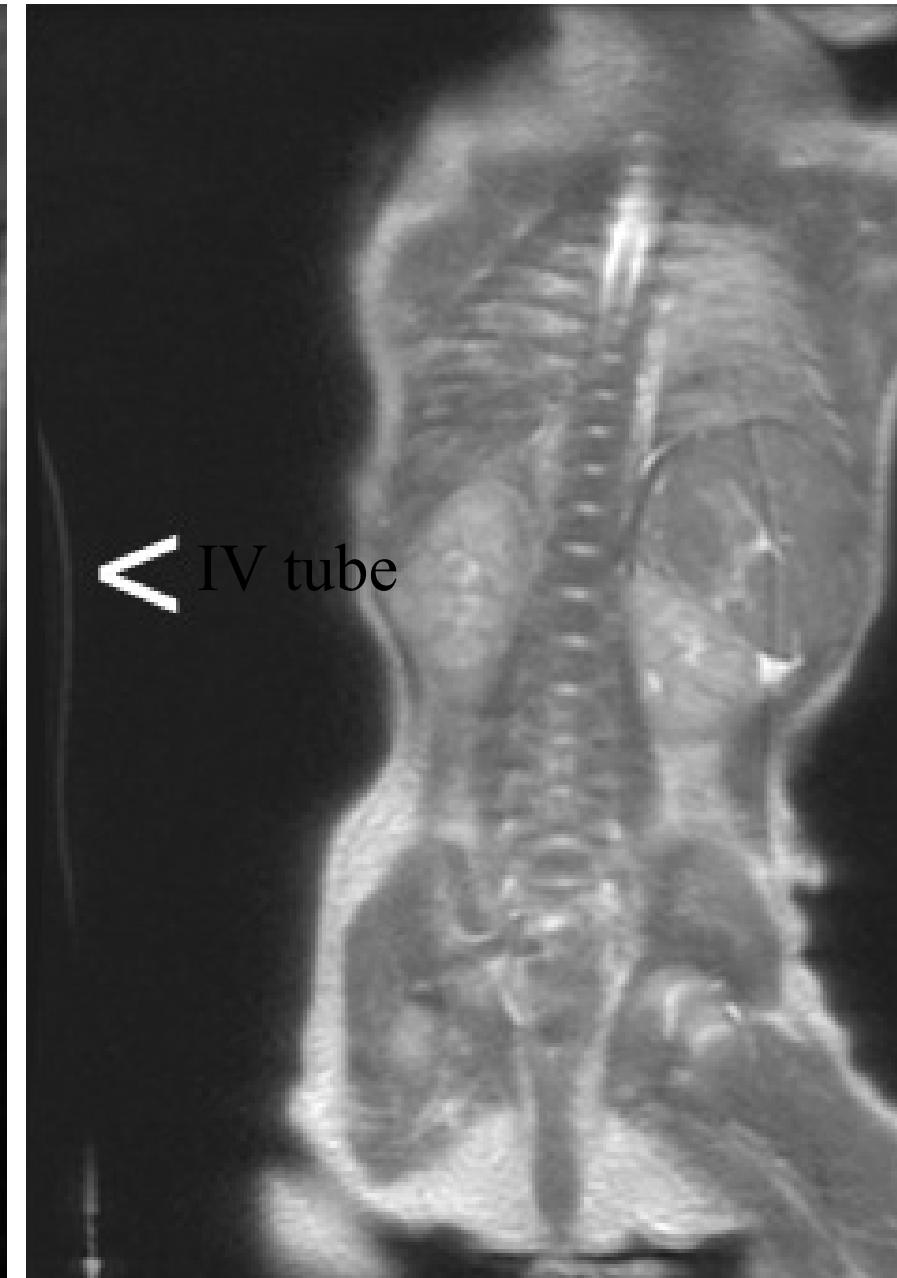


Aliasing

Q: What is the difference in acquisition between the two images ?







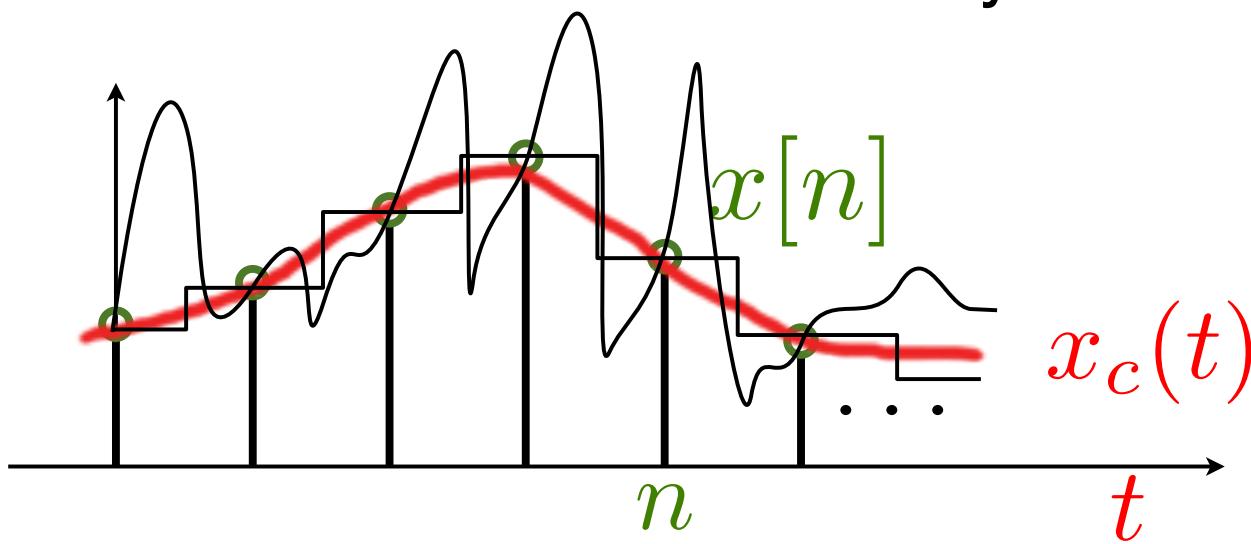
Reconstruction of Bandlimited Signals

- Nyquist Sampling Thm: suppose $x_c(t)$ is bandlimited

$$X_c(j\Omega) = 0 \quad \forall \quad |\Omega| \geq \Omega_N$$

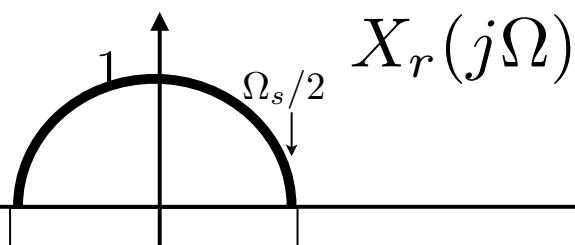
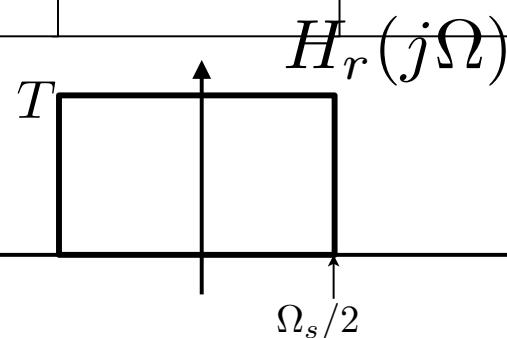
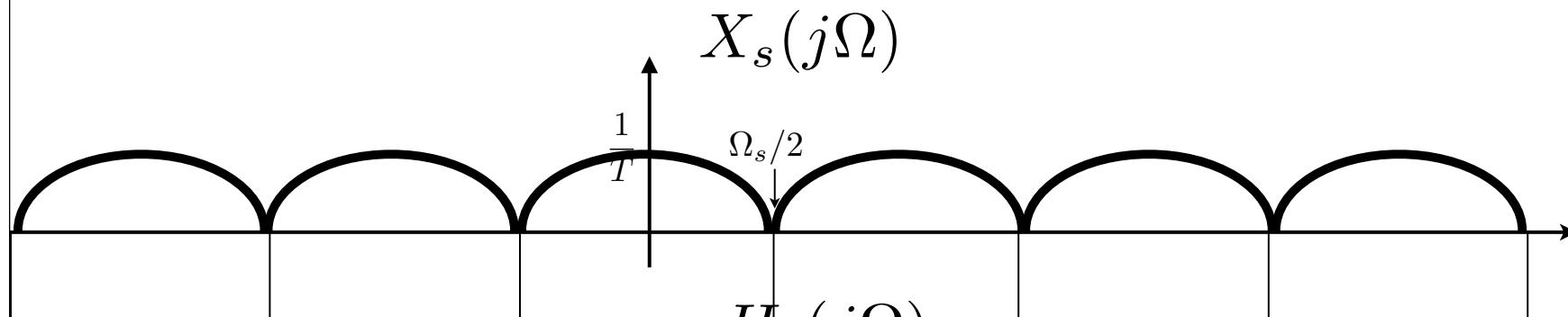
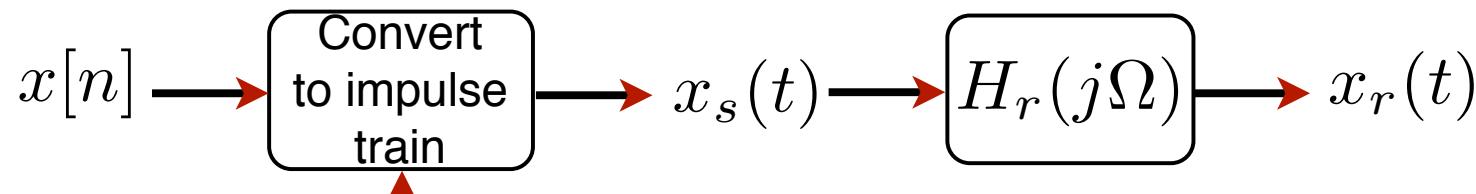
if $\Omega_s \geq 2\Omega_N$, then $x_c(t)$ can be uniquely determined from its samples $x[n] = x_c(nT)$

- Bandlimitedness is the key to uniqueness



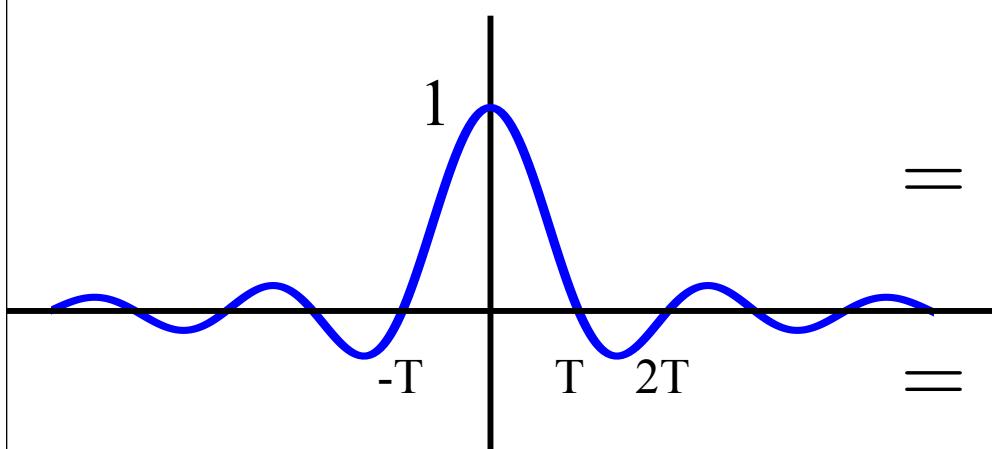
multiple signals go through the samples, but only one is bandlimited!

Reconstruction in Frequency Domain



Reconstruction in Time Domain

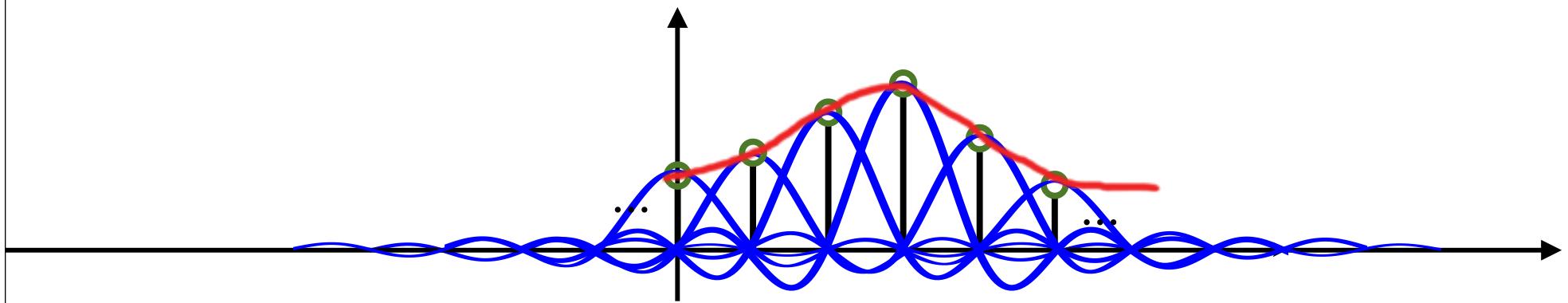
$$\begin{aligned} h_r(t) &= \frac{1}{2\pi} \int_{-\Omega_s/2}^{\Omega_s/2} T e^{j\Omega t} d\Omega \\ &= \frac{T}{2\pi} \frac{1}{jt} s^{j\Omega t} \Big|_{-\Omega_s/2}^{\Omega_s/2} \\ &= \frac{T}{\pi t} \frac{e^{j\frac{\Omega_s}{2}t} - e^{-j\frac{\Omega_s}{2}t}}{2j} \\ &= \frac{T}{\pi t} \sin\left(\frac{\Omega_s}{2}t\right) = \frac{T}{\pi t} \sin\left(\frac{\pi}{T}t\right) \\ &= \text{sinc}\left(\frac{t}{T}\right) \end{aligned}$$



Reconstruction in Time Domain

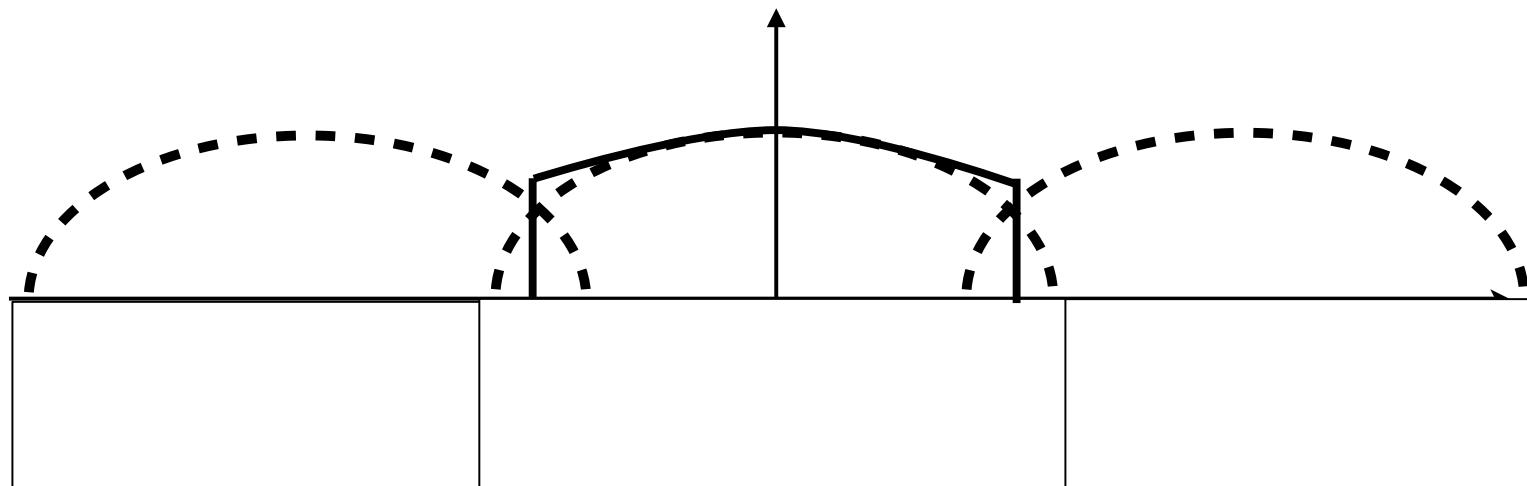
$$\begin{aligned}x_r(t) = x_s(t) * h_r(t) &= \left(\sum_n x[n] \delta(t - nT) \right) * h_r(t) \\&= \sum_n x[n] h_r(t - nT)\end{aligned}$$

The sum of “sincs gives $x_r(t)$ \Rightarrow Unique signal
bandlimited by Ω_s



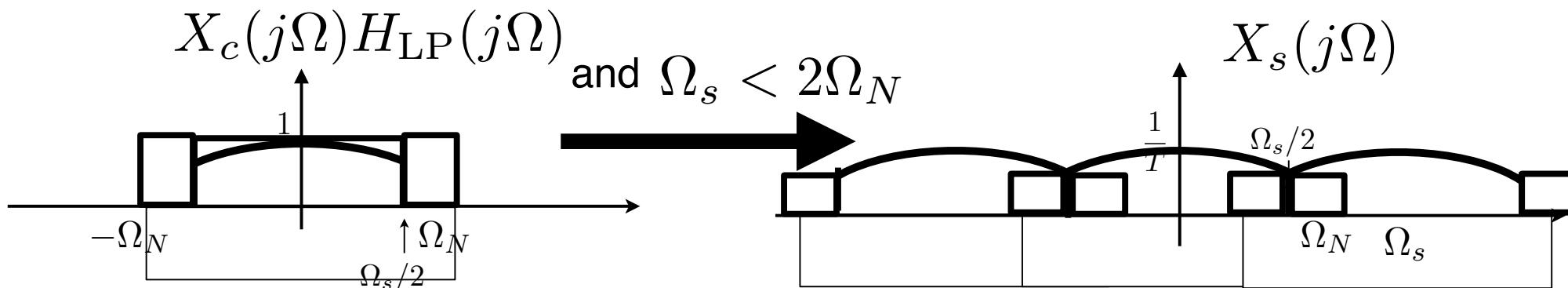
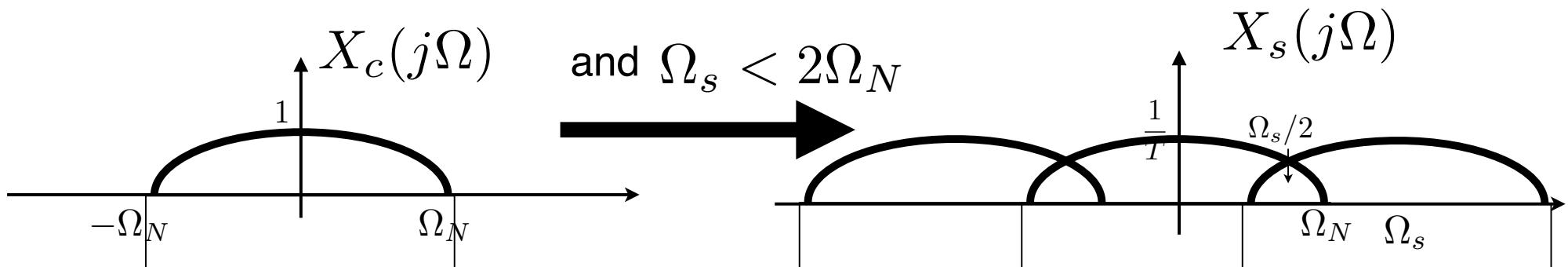
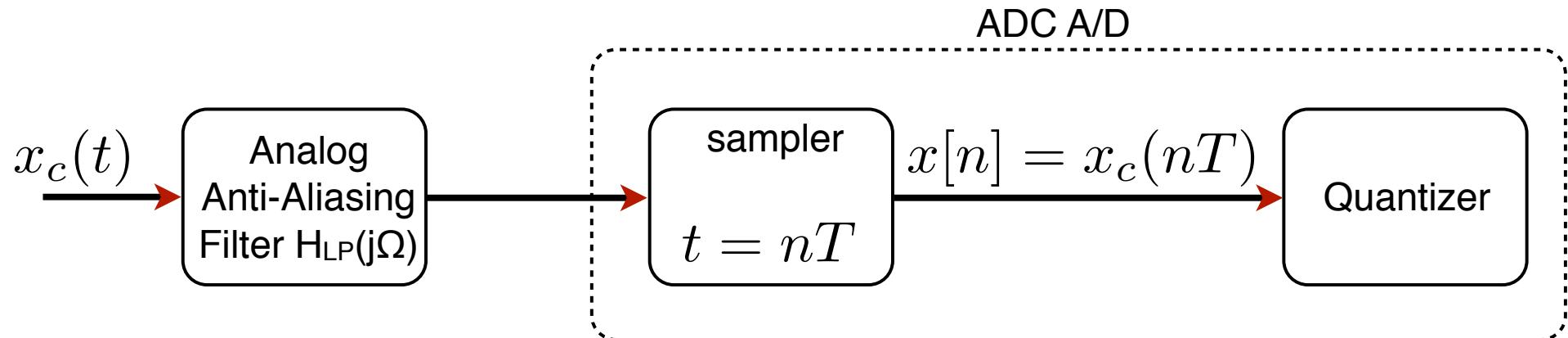
Aliasing

- If $\Omega_N > \Omega_s/2$, $x_r(t)$ an aliased version of $x_c(t)$



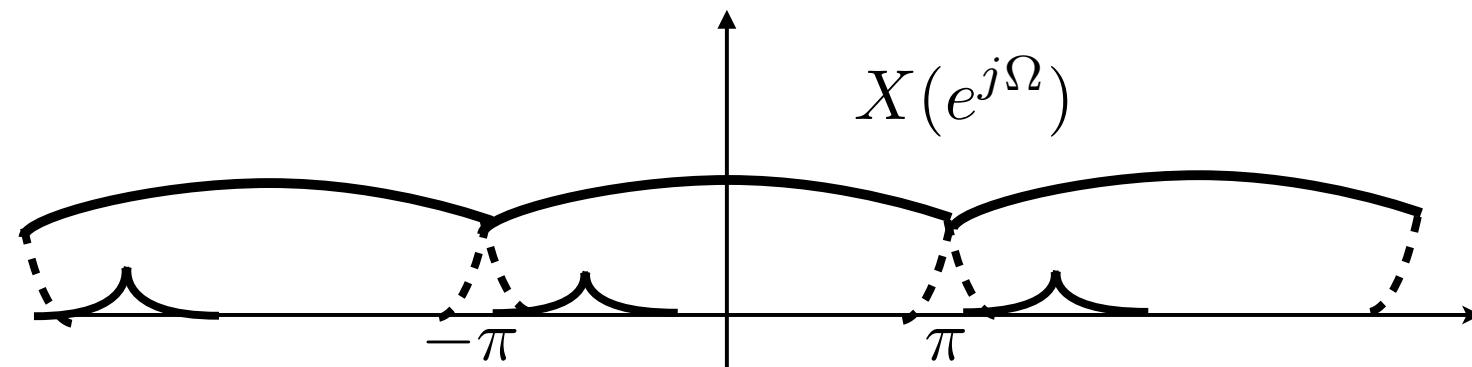
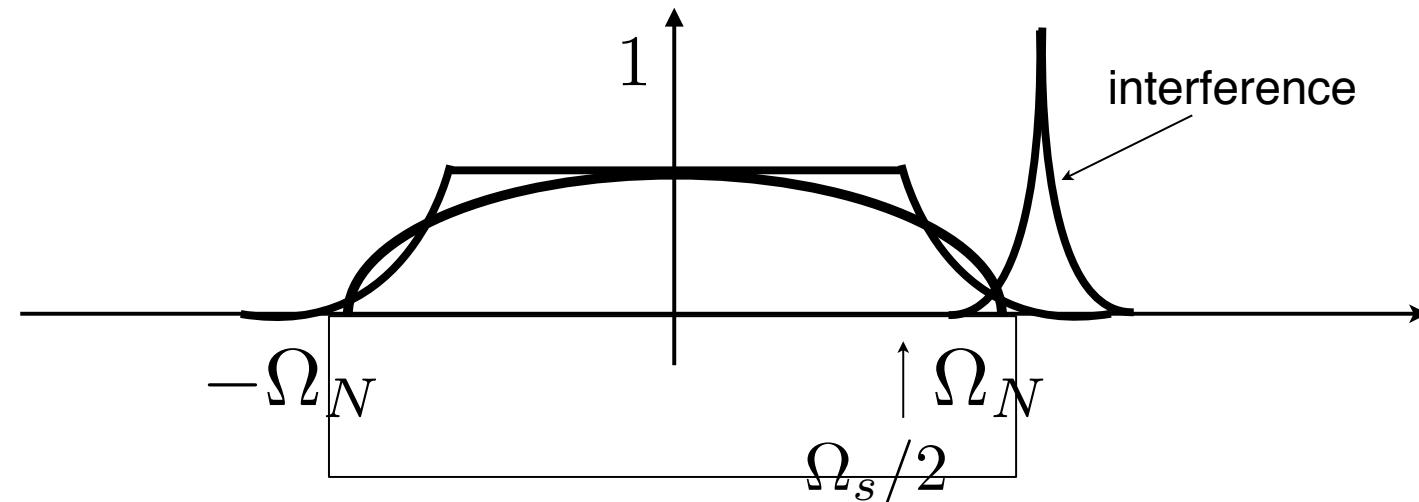
$$X_r(j\Omega) = \begin{cases} TX_s(j\Omega) & \text{if } |\Omega| \leq \Omega_s/2 \\ 0 & \text{otherwise} \end{cases}$$

Anti-Aliasing



Non Ideal Anti-Aliasing

$$X_c(j\Omega)H_{LP}(j\Omega)$$



SDR non-perfect anti-Aliasing Demo
