Topics
• Did you sign up for the ham exam?
• Last time
  – D.T processing of C.T signals
  – C.T processing of D.T signals (ha?????)
  – Downsampling
• Today
  – Changing Sampling Rate via DSP
  – Upsampling
  – Rational resampling
  – Interchanging operations

Review DownSampling
• Much like C/D conversion
• Expect similar effects:
  – Aliasing
  – Mitigate by antialiasing filter
• Finely sampled signal ⇒ almost continuous
  – Downsample in that case is like sampling!

Changing Sampling-rate via D.T Processing

\[ X_d(e^{j\omega}) = \frac{1}{M} \sum_{i=0}^{M-1} X(e^{j(M\omega - 2\pi i)}) \]

\[ M=2 \]

Anti-Aliasing

\[ x[n] \xrightarrow{\text{LPF}} \tilde{x}[n] \xrightarrow{\downarrow M} \tilde{x}_d[n] = \tilde{x}[nM] \]
UpSampling

- Much like D/C converter
- Upsample by A LOT ⇒ almost continuous
- Intuition:
  - Recall our D/C model: x[n] ⇒ x_\text{s}(t)⇒x_\text{c}(t)
  - Approximate “x_\text{s}(t)” by placing zeros between samples
  - Convolve with a sinc to obtain “x_\text{c}(t)”

Up-sampling

\[ x[n] = X_e(nT) \]
\[ x_i[n] = X_e(nT') \quad \text{where } T' = \frac{T}{L}, \quad L \text{ integer} \]

Obtain \( x_i[n] \) from \( x[n] \) in two steps:

1. Generate:
   \[ x_e[n] = \begin{cases} 
   x[n/L] & n = 0, \pm L, \pm 2L, \cdots \\
   0 & \text{otherwise} 
   \end{cases} \]

2. Obtain \( x_i[n] \) from \( x_e[n] \) by bandlimited interpolation:
   \[ x_i[n] = x_e[n] \ast \text{sinc}(\frac{n}{L}) \]

Frequency Domain Interpretation

\[ x[n] \xrightarrow{\uparrow \text{L}} x_e[n] \xrightarrow{\text{LPF gain=L}} x_i[n] \]

\[ X_e(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_e[n] e^{-j\omega n} \]
\[ = \sum_{m=-\infty}^{\infty} x_e[mL] e^{-j\omega mL} = X(e^{j\omega L}) \]

Compress DTFT by a factor of L!
Practical Upsampling

- Can interpolate with simple, practical filters. What’s happening?
- Example: L=3, linear interpolation - convolve with triangle
Resampling by non-integer

- \( T' = T M / L \) (upsample \( L \), downsample \( M \))

\[
x[n] \xrightarrow{\uparrow L} \xrightarrow{\text{LPF}} \xrightarrow{\frac{\pi}{L}} \xrightarrow{\text{LPF}} \xrightarrow{\frac{\pi}{M}} \xrightarrow{\downarrow M}
\]

Or,

\[
x[n] \xrightarrow{\uparrow L} \xrightarrow{\text{gain} L} \xrightarrow{\text{LPF}} \xrightarrow{\min\{\pi / L, \pi / M\}} \xrightarrow{\downarrow M}
\]

- What would happen if change order?

Example:

- \( L = 2, M = 3, \) \( T' = 3 / 2 T \) (fig 4.30)

\[
\tilde{X}_i = H_d X_e
\]

\[
\begin{align*}
\text{expanding } L = 2 \\
X_e(j \omega) \\
\text{LP filtering} \\
\tilde{X}_i = H_d X_e
\end{align*}
\]

Multi-Rate Signal Processing

- What if we want to resample by 1.01\( T \)?
  - Expand by \( L = 100 \)
  - Filter \( n / 101 \) (\$\$\$\$\$)
  - Downsample by \( M = 101 \)

- Fortunately there are ways around it!
  - Called multi-rate
  - Uses compressors, expanders and filtering

Interchanging Operations

- \( \uparrow L \) \rightarrow \text{“expander”} \rightarrow \text{not LTI!} \rightarrow \downarrow M \) \rightarrow \text{“compressor”}

Note:

\[
x[n] \xrightarrow{H(z)} \xrightarrow{\uparrow L} \xrightarrow{y[n]} \xrightarrow{H(e^{j\omega})X(e^{j\omega})} \xrightarrow{x[n]} \xrightarrow{\uparrow L} \xrightarrow{H(z)} \xrightarrow{y[n]} \xrightarrow{H(e^{j\omega})X(e^{j\omega})}
\]

Interchanging Operations

- \( \uparrow L \) \rightarrow \text{“expander”} \rightarrow \text{not LTI!} \rightarrow \downarrow M \) \rightarrow \text{“compressor”}

Note:

\[
x[n] \xrightarrow{H(z)} \xrightarrow{\uparrow L} \xrightarrow{y[n]} \xrightarrow{H(e^{j\omega})X(e^{j\omega})} \xrightarrow{x[n]} \xrightarrow{\uparrow L} \xrightarrow{H(z)} \xrightarrow{y[n]} \xrightarrow{H(e^{j\omega})X(e^{j\omega})}
\]

\[
\equiv x[n] \xrightarrow{\uparrow L} \xrightarrow{H(z)} \xrightarrow{y[n]} \xrightarrow{H(e^{j\omega})X(e^{j\omega})}
\]
Interchanging Filter Expander

- Q: Can we move expander from Left to Right (with xform)?

\[ \uparrow L \rightarrow H(z) \rightarrow \oplus \rightarrow H(z) \rightarrow \uparrow L \]

- A: Yes, if \( H(z) \) is rational
  No, otherwise

Example: