**Ideal Anti-Aliasing**

\[ x_c(t) \quad \text{ADC A/D} \]

\[ x_s(j\Omega) \quad \text{sampler} \quad x[n] = x_s(nT) \quad \text{Quantizer} \]

\[ X_c(j\Omega) \quad \Omega_s < 2\Omega_N \]

\[ X_s(j\Omega) \quad \text{and} \quad \Omega_s < 2\Omega_N \]

\[ X_s(j\Omega) H_{\text{LP}}(j\Omega) \quad \text{and} \quad \Omega_s < 2\Omega_N \]

**Non Ideal Anti-Aliasing**

\[ X_c(j\Omega) H_{\text{LP}}(j\Omega) \]

- Problem: Hard to implement sharp analog filter
- Tradeoff:
  - Crop part of the signal
  - Suffer from noise and interference (See lab II!)

**Oversampled ADC**

\[ x_c(t) \quad \text{ADC A/D} \]

\[ x_s(j\Omega) \quad \text{sampler} \quad x[n] = x_s(nT) \quad \text{Quantizer} \]

\[ X_s(j\Omega) \quad \Omega_s < 2\Omega_N \]

\[ X_s(j\Omega) H_{\text{LP}}(j\Omega) \quad \text{and} \quad \Omega_s < 2\Omega_N \]

\[ X_s(j\Omega) \quad \text{after oversampling x2} \]

\[ \hat{X}(e^{j\omega}) \quad \text{aliased noise} \]

\[ \Omega_N \]

\[ \Omega_s/2 \]
Oversampled ADC

after oversampling x2

\[ \hat{X}(e^{j\omega}) \]

\( T \)

after digital LP and decimation

\[ \hat{X}_d(e^{j\omega}) \]

\( T_d = MT \)

Sampling and Quantization

\[ x_c(t) \]

\[ \frac{C}{D} \]

\[ x[n] = x_c(nT) \]

\[ \hat{x}[n] \]

Quantization Error

\[ x[n] \]

\[ \hat{x}[n] = x[n] + e[n] \]

\[ e[n] \]

\[ -\Delta/2 \leq e[n] < \Delta/2 \]

\[ -X_m - \Delta/2 < x[n] \leq (X_m - \Delta/2) \]

Noise Model for Quantization Error

\[ \Delta = \frac{2X_m}{2B+1} = \frac{X_m}{2^B} \]

\[ \hat{x}[n] = X_m \hat{x}_B[n] \]

\[ 2X_m \]

\[ \Delta \]

Quantization Noise

\[ \sigma_e^2 = \frac{\Delta^2}{12} \]

\[ \sigma_{\hat{x}}^2 = \frac{2^{-2B}X_m^2}{12} \text{ since } \Delta = 2^{-B}X_m \]

Assumptions:

- Model \( e[n] \) as a sample sequence of a stationary random process
- \( e[n] \) is not correlated with \( x[n] \), e.g., \( E[e[n]x[n]] = 0 \)
- \( e[n] \) not correlated with \( e[m] \), e.g., \( E[e[n]e[m]] = 0 \) \( m \neq n \)
  - \( e[n] \sim U[-\Delta/2, \Delta/2] \)
- Variance is: \( \sigma_e^2 = \frac{\Delta^2}{12} \text{, or } \sigma_{\hat{x}}^2 = \frac{2^{-2B}X_m^2}{12} \text{ since } \Delta = 2^{-B}X_m \)
- Assumptions work well for signals that change rapidly, are not clipped and for small \( \Delta \)
SNR of Quantization Noise

For uniform B+1 bits quantizer:

\[ SNR_Q = 10 \log_{10} \left( \frac{\sigma_x^2}{\sigma_e^2} \right) \]

\[ = 10 \log_{10} \left( \frac{12 \cdot 2^B \sigma_x^2}{X_m^2} \right) \]

\[ SNR_Q = 6.02B + 10 \cdot 8 \cdot 20 \log_{10} \left( \frac{X_m}{\sigma_x} \right) \]

Quantizer range

\[ \text{Quantizer range} \]

\[ \text{rms of amp} \]

Improvement of 6dB with every bit

- The range of the quantization must be adapted to the rms amplitude of the signal
  - Tradeoff between clipping and noise!
  - Often use pre-amp
  - Sometimes use analog auto gain controller (AGC)
  - If \( \sigma_x = X_m/4 \) then \( SNR_Q \approx 6B - 1.25dB \)

so SNR of 90-96 dB requires 16-bits (audio)

Energy of \( x_d[n] \) equals energy of \( x[n] \)
- No filtering of signal!

Quantization noise in Oversampled ADC

\[ x_c(t) \xrightarrow{\text{C/D}} x[n] \rightarrow e[n] = x[n] + \epsilon[n] + \epsilon(t) \rightarrow \text{LPF} \xrightarrow{\omega_c} \text{downsampled} \rightarrow x_d[n] = x_d[n] + e_d[n] \rightarrow \Delta M \]

\[ X_c(j\Omega) \xrightarrow{\text{LPF}} \tilde{X}(e^{j\Omega}) \xrightarrow{\text{downsampled}} \Delta M \]

\[ X_d(e^{j\Omega}) = \tilde{X}(e^{j\Omega}) \]

\[ \sigma_e^2 = \sigma^2/\Delta M \]

SNR in Oversampled ADC

\[ SNR = 6.02B + 10 \cdot 8 \cdot 20 \log_{10} \left( \frac{X_m}{\sigma_x} \right) + 10 \log_{10} M \]

Energy of \( x_d[n] \) equals energy of \( x[n] \)

- No filtering of signal!

Practical ADC

\[ x[n] = x(t)|_{t=nT} \rightarrow \text{sinc pulse generator} \rightarrow x_r(t) = \sum_{n=-\infty}^{\infty} x[n] \text{sinc} \left( \frac{t-nT}{T} \right) \]

- Scaled train of sinc pulses
- Difficult to generate sinc ⇒ Too long!
Practical ADC

Zero-Order-Hold interpolation

\[ x_0(t) = \sum_{n=-\infty}^{\infty} x[n] h_0(t - nT) = x_0(t) * x_s(t) \]

Taking a FT:

\[ X(j\Omega) = H_0(j\Omega) X_s(j\Omega) \]
\[ = H_0(j\Omega) \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\Omega - k\Omega_s)) \]

Ideally:

\[ X_s(j\Omega) H_{LF}(j\Omega) \]

Practically:

\[ X_s(j\Omega) H_0(j\Omega) \]

\[ X_s(j\Omega) H_0(j\Omega) \]

Practically:

\[ X_s(j\Omega) H_0(j\Omega) H_r(j\Omega) \]
Easier Implementation with Digital upsampling

\[ x[n] \overset{\uparrow L}{\rightarrow} x_L[n] \overset{\text{LPP gain} = \frac{L}{L}}{\rightarrow} \]

Practically:

\[ X_s(j\Omega) H_0(j\Omega) H_r(j\Omega) \]

... ... ...

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Easier Implementation with Digital upsampling

... ... ...

easier implementing with analog components

Need analog components made of Nonobtainium

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