Compressed Sensing III

Traditional Sensing

- $x \in \mathbb{R}^N$ is a signal
- Make $N$ linear measurements

Compressed Sensing

- $x \in \mathbb{R}^N$ is a $K$-sparse signal ($K \ll N$)
- Make $M$ ($K < M < N$) incoherent linear projections

CS recovery

- Given $y = \Phi x$ find $x$
- But there’s hope, $x$ is sparse!

RADIOs

- [https://inst.eecs.berkeley.edu/~ee123/sp15/radio.html](https://inst.eecs.berkeley.edu/~ee123/sp15/radio.html)
- Interfaces and radios on Wednesday -- please come to pick up
- Midterm II this Friday -- same deal - open everything covers everything including 2D
CS recovery

• Given $y = \Phi x$
  find $x$

• But there’s hope, $x$ is sparse!

$$\minimize ||x||_2$$
$$\text{s.t. } y = \Phi x$$

Wrong!

CS recovery

• Given $y = \Phi x$
  find $x$

• But there’s hope, $x$ is sparse!

$$\minimize ||x||_0$$
$$\text{s.t. } y = \Phi x$$

Hard!

CS recovery

• Given $y = \Phi x$
  find $x$

• But there’s hope, $x$ is sparse!

$$\minimize ||x||_1$$
$$\text{s.t. } y = \Phi x$$

Need $M \approx K \log(N) \ll N$
Solved by linear-programming

Geometric Interpretation

Domain of sparse signals

Minimum $||x||_1$

Minimum $||x||_0$

$$\begin{bmatrix} 0 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \end{bmatrix}$$

A non-linear sampling theorem

• $f \in \mathbb{C}^N$ supported on a set $\Omega$ in Fourier

• Shannon:
  – $\Omega$ is known connected set, size $B$
  – Exact recovery from $B$ equispaced time samples
  – Linear reconstruction by sinc interpolation

• Non-linear sampling theorem
  – $\Omega$ is an arbitrary, unknown set of size $B$
  – Exact recovery from $\sim B \log(N)$ (almost) arbitrary placed samples
  – Nonlinear reconstruction by convex programming

Practicality of CS

• Can such sensing system exist in practice?
Practicality of CS

- Can such sensing system exist in practice?
- Randomly undersampled Fourier is incoherent

MRI samples in the Fourier domain!

\[ \Phi^* \Phi \approx I \]

Intuitive example of CS

Nyquist
**Intuitive example of CS**

![FFT](image1)

**sub-Nyquist**

**Ambiguity**

**Looks like “random noise”**

**But it’s not noise!**

M. Lustig, EECS UC Berkeley
Intuitive example of CS

Example inspired by Donoho et al, 2007

Question!

• What if this was the signal?
• Would CS still work?

Domains in Compressed Sensing

Not Sparse!

Signal

Sampling Domain

Sparse!

Sparse Domain

Incoherent
MRI

Signal
Sparse Domain
Not Sparse!

Sampling Domain
Sparse!
Incoherent

Sparse Domain
Acquired Data
Sparse "denoising"
Compressed Sensing Reconstruction

Undersampled Final Image

Tutorial & code available at http://www.mlustig.com

6 year old male abdomen. Fine structures (arrows) are buried in noise (artifactual + noise amplification) and are recovered by CS with L1-wavelets. x8 acceleration
Other Applications

- Compressive Imaging
- Medical Imaging
- Analog to information conversion
- Biosensing
- Geophysical Data Analysis
- Compressive Radar
- Astronomy
- Communications
- More ......

Resources

- CS + parallel imaging matlab code, examples
  http://www.eecs.berkeley.edu/~mlustig/software/

- Rice University CS page: papers, tutorials, codes, ....
  http://www.dsp.ece.rice.edu/cs/

- IEEE Signal Processing Magazine, special issue on compressive sampling 2008:25(2)

- March 2010 Issue Wired Magazine: "Filling the Blanks"

- Igor Caron Blog: http://nuit-blanche.blogspot.com/

Thank you!