Discussion Section 9: Midterm 2 Review

Giulia Fanti
April 1, 2015
Topics

• Transforms
  – STFT
    • Window types
  – DWT

• Sampling
  – Upsampling/downsampling

• Filterbanks
  – Polyphase decomposition
  – Multirate processing

• 2D signals
Tools we have studied

Sampling

Transforms

Filters

Filterbanks

Things we want

Reduce storage

Reduce computation on signals

Improve interpretability of signals

Modify signals
Draw the time-frequency tiling for the STFT and the DWT.

What are the relative advantages of one transform over the other?
Methods for computing the DWT

- Use the formula/project the signal onto each scaled and shifted wavelet
- Pyramidal decomposition
- Multiply by the DWT matrix (like a DFT matrix)
Sampling

• We have covered this topic extensively in class and discussion
• C/D sampling: Generates copies of the spectrum
• Downsampling: Stretches the spectrum
• Upsampling: Squeezes the spectrum
Consider a continuous to digital system that consists of a demodulation by $\Omega_d$, ideal analog low-pass filter with a cutoff $\Omega_c$, ideal C/D (without LP filter) with a sampling rate $T = \frac{1}{\Omega_s}$, and an ideal digital $\frac{L}{M}$ resampling (with an appropriate $\min\{\frac{\pi}{M}, \frac{\pi}{L}\}$ LP filter).

Also consider a signal with an unknown gain, $x_c(t)$, with the following spectrum, $X_c(j\Omega)$:

For each of the following possible outputs of the system, find the parameters $\Omega_d$, $\Omega_c$, $\Omega_s$ and $L/M$ that will produce it. Also, plot qualitatively the intermediate spectrums $X_{LP}(j\Omega)$ and $\tilde{X}(e^{j\omega})$. It is possible that a parameter combination that produces the output does not exist. In that case, mark the appropriate box and explain why.

The analog $H_{LP}$ filter has limits: $2\pi \leq \Omega_c \leq 8\pi$. When given a choice, choose the lowest $\Omega_c$. 

Practice problem:
The analog $H_{LP}$ filter has limits: $2\pi \leq \Omega_c \leq 8\pi$. When given a choice, choose the **lowest** $\Omega_c$.

The spectrum $X(e^{j\omega})$ is:

**Solution:**

$X(e^{j\omega})$ is centered at center so $\Omega_d$ is simply $\Omega_0$.

We then crop the spectrum at $2\pi$ to get the desired shape and sample at the Nyquist rate, $\Omega_s = 2 \cdot 2\pi$ to get the result. We don’t need any resampling.
The analog $H_{LP}$ filter has limits: $2\pi \leq \Omega_c \leq 8\pi$. When given a choice, choose the lowest $\Omega_c$.

b) The spectrum $X(e^{j\omega})$ is:

Solution:

$X(e^{j\omega})$ looks like the aliasing of the triangle and circle so we first want to shift the spectrum to the center of one of the shapes. We don't want to crop anything, so the lowest $\Omega_c$ is $6\pi$. To create aliasing, we sample at the same rate as the bandwidth, which is $4\pi$. We then upsample it by 4 to shrink the spectrum.
The analog $H_{LP}$ filter has limits: $2\pi \leq \Omega_c \leq 8\pi$. When given a choice, choose the **lowest** $\Omega_c$.

The spectrum $X(e^{j\omega})$ is:

\[ X(e^{j\omega}) \]

\[ \Omega_0 - 2\pi \quad \Omega_0 \quad \Omega_0 + 2\pi \]

Find the parameters $\Omega_d$, $\Omega_c$, $\Omega_s$ and $L/M$.

**Solution:**

We cannot get $X(e^{j\omega})$ because the ideal resampling always applies a filter to crop out the extra copies. We can flip the spectrum following part d, but there is no way to squeeze two copies in $-\pi$ to $\pi$. 
Filterbanks, Polyphase

Interchanging Operations

- ↑L ("expander")
- ↓M ("compressor")

Note:

1. $x[n] \rightarrow H(z) \rightarrow \uparrow L \rightarrow y[n]$
2. $x[n] \rightarrow \uparrow L \rightarrow H(z) \rightarrow y[n]$

Not LTI!
Do you agree with this statement?

Should say if $H(z^{1/L})$ is rational!
What are the conditions on this statement?

This always holds. How would you prove it?
Polyphase Decomposition

- We can decompose an impulse response to:

\[ h[n] = \sum_{k=0}^{M-1} h_k[n - k] \]
Polyphase Decomposition

Define:

\[ h_k[n] \rightarrow \downarrow M \rightarrow e_k[n] \]

\[ e_k[n] = h_k[nM] \]

So how do we get back from \( e_k[n] \) to \( h_k[n] \)?
How about in frequency domain?
Why is this interesting?

Can we switch M and H?

What is this picture representing?

Why is this interesting?
Reconstruction: Quadrature Mirror Filterbanks

• What is a Quadrature mirror filterbank?

• Quadrature mirror filters:
  - \( G_0(z) \) and \( G_1(z) \) are mirror images of \( H_0(z) \) and \( H_1(z) \), mirrored around \( \pi/2 \)
  - What kinds of filters are these?
4.53. Consider the analysis–synthesis system shown in Figure P4.53-1. The lowpass filter $h_0[n]$ is identical in the analyzer and synthesizer, and the highpass filter $h_1[n]$ is identical in the analyzer and synthesizer. The Fourier transforms of $h_0[n]$ and $h_1[n]$ are related by

$$H_1(e^{j\omega}) = H_0(e^{j(\omega + \pi)}).$$

![Diagram of the analysis–synthesis system with blocks labeled $h_0[n]$, $h_1[n]$, $r_0[n]$, $x_0[n]$, $g_0[n]$, $y_0[n]$, $r_1[n]$, $x_1[n]$, $g_1[n]$, $y_1[n]$, LPF, and HPF.]

(a) If $X(e^{j\omega})$ and $H_0(e^{j\omega})$ are as shown in Figure P4.53-2, sketch (to within a scale factor) $X_0(e^{j\omega})$, $G_0(e^{j\omega})$, and $Y_0(e^{j\omega})$. 

![Graph of $X(e^{j\omega})$ and $H_0(e^{j\omega})$ with $X_0(e^{j\omega})$, $G_0(e^{j\omega})$, and $Y_0(e^{j\omega})$ sketched to within a scale factor.]}
(a) First, $X(e^{j\omega})$ is plotted.

\[ X(e^{j\omega}) \]

The lowpass filter cuts off at $\frac{\pi}{4}$.

\[ R_0(e^{j\omega}) \]

The downsampler expands the frequency axis. Since $R_0(e^{j\omega})$ is bandlimited to $\frac{\pi}{4}$, no aliasing occurs.

\[ X_0(e^{j\omega}) \]

The upsampler compresses the frequency axis by a factor of 2.

\[ G_0(e^{j\omega}) \]

The lowpass filter cuts off at $\frac{\pi}{4} \Rightarrow Y_0(e^{j\omega}) = R_0(e^{j\omega})$ as sketched above.
(b) Write a general expression for \( G_0(e^{j\omega}) \) in terms of \( X(e^{j\omega}) \) and \( H_0(e^{j\omega}) \). Do not assume that \( X(e^{j\omega}) \) and \( H_0(e^{j\omega}) \) are as shown in Figure 4.53-2.

\[
(b) \quad G_0(e^{j\omega}) = \frac{1}{2} \left( X(e^{j\omega})H_0(e^{j\omega}) + X(e^{j(\omega+\pi)})H_0(e^{j(\omega+\pi)}) \right)
\]
(c) Determine a set of conditions on $H_0(e^{j\omega})$ that is as general as possible and that will guarantee that $y[n]$ is proportional to $x[n - n_d]$ for any stable input $x[n]$. Note: Analyzer-synthesizer filter banks of the form developed in this problem are very similar to quadrature mirror filter banks. For further reading, see Crochiere and Rabiner (1983), pp. 378–392.

\[ H_1(e^{j\omega}) = H_0(e^{j(\omega + \pi)}). \]

What is the relation between this figure and the QMF we have seen in class?
STOP

Hammer time.

radio