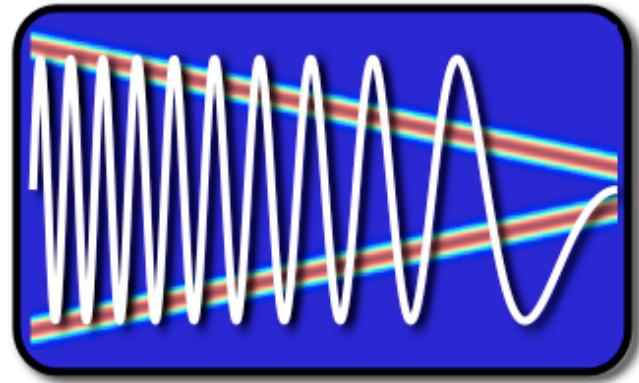


EE123



Digital Signal Processing

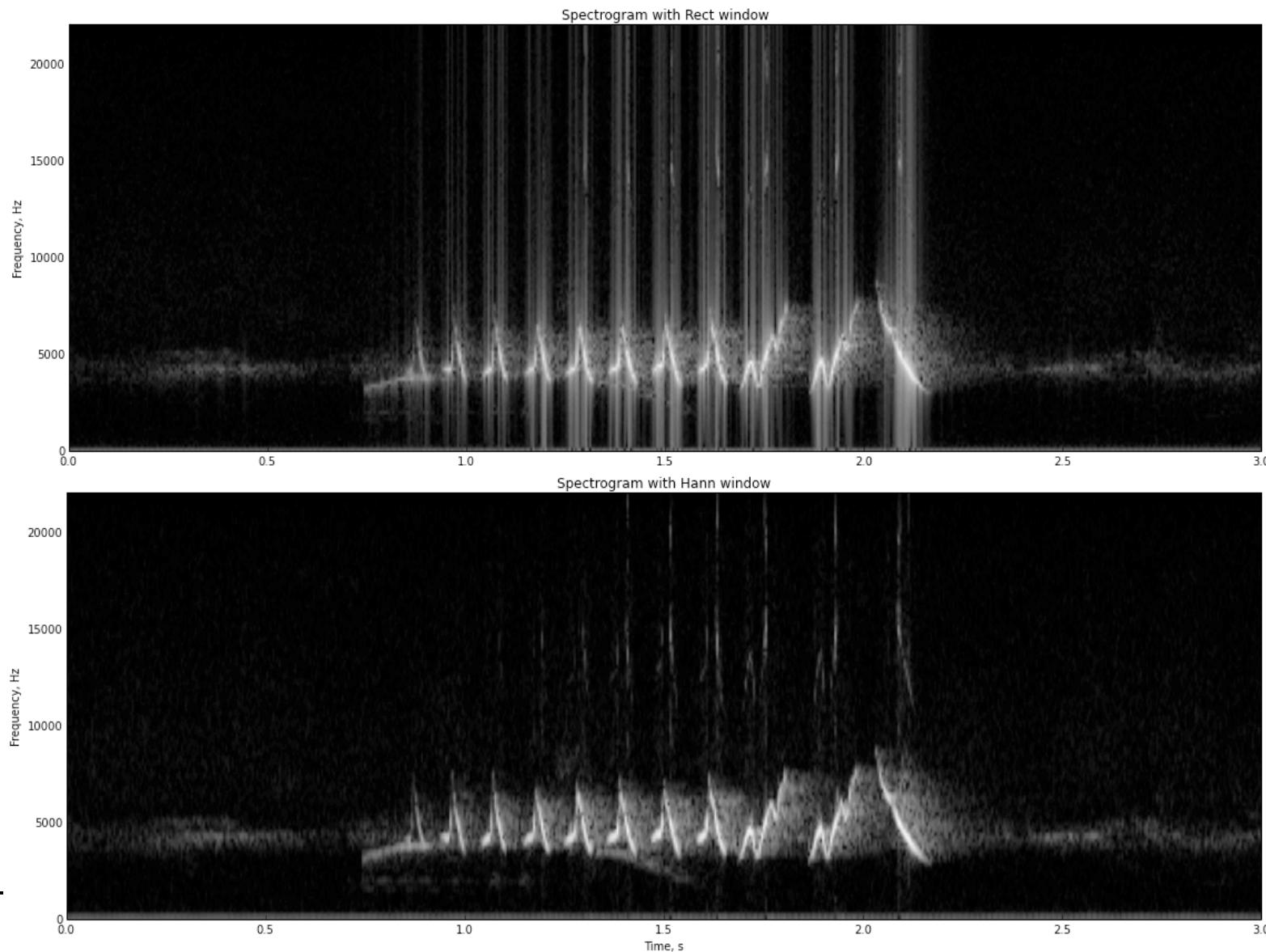
Lecture 17 Lab III Polyphase Filters

Topics

- Last time
 - Changing Sampling Rate via DSP
 - Upsampling
 - Rational resampling
- Today
 - Lab III
 - Interchanging Compressors/Expanders and filtering
 - Polyphase decomposition
 - Multi-rate processing

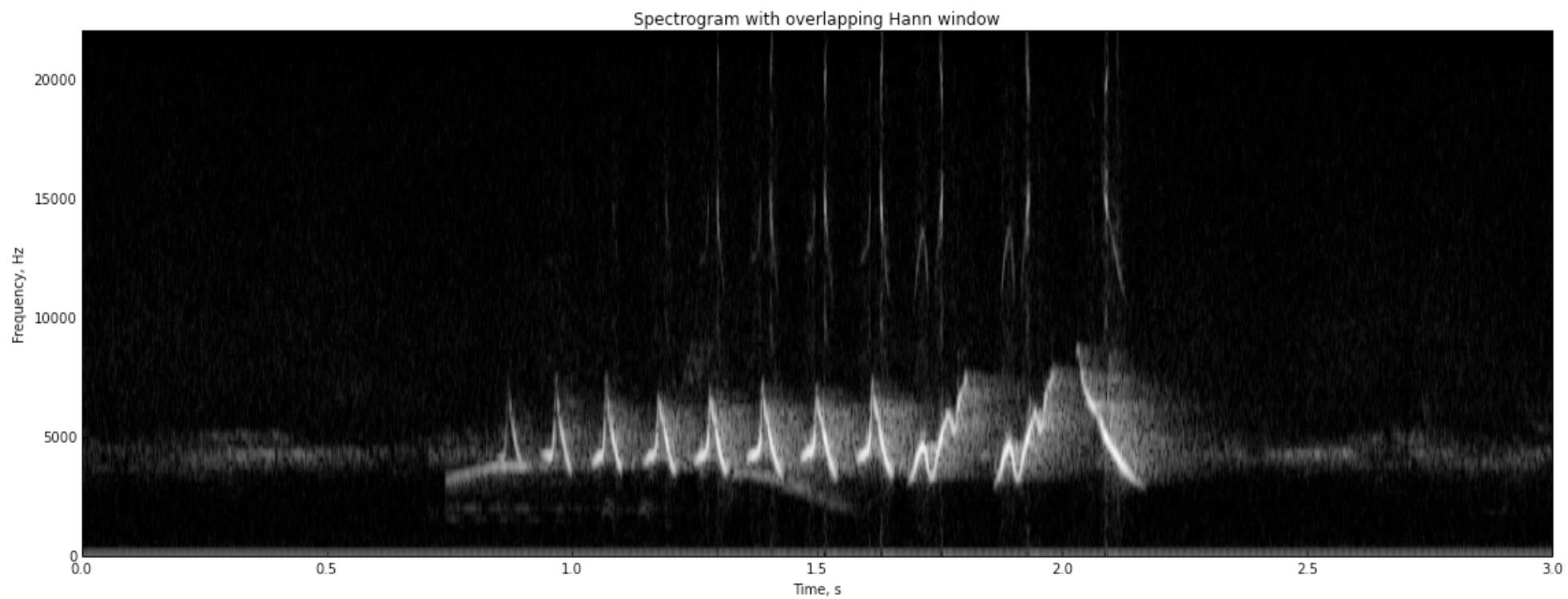
Lab III - Time-Frequency

- compute spectrograms with w/o windowing



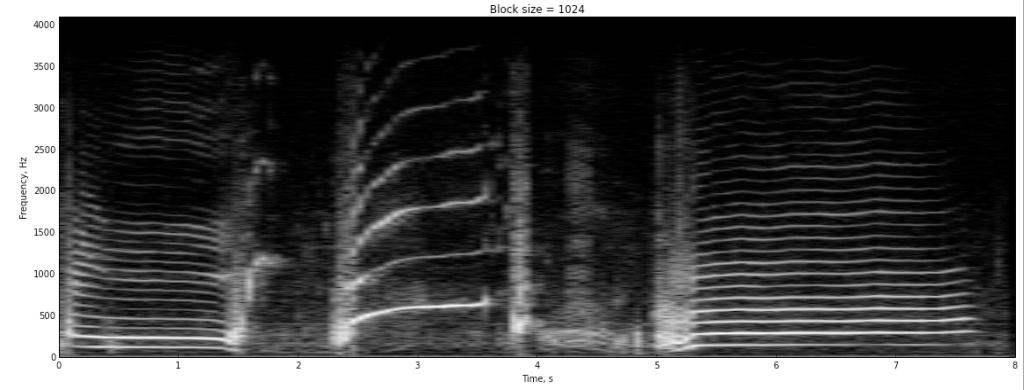
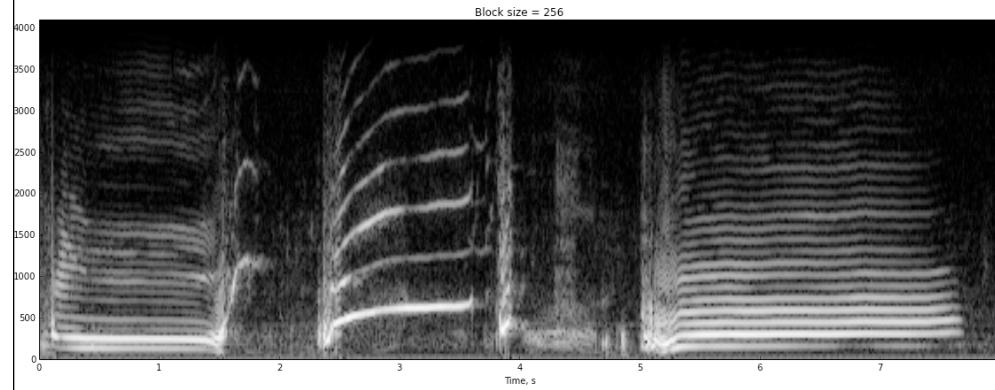
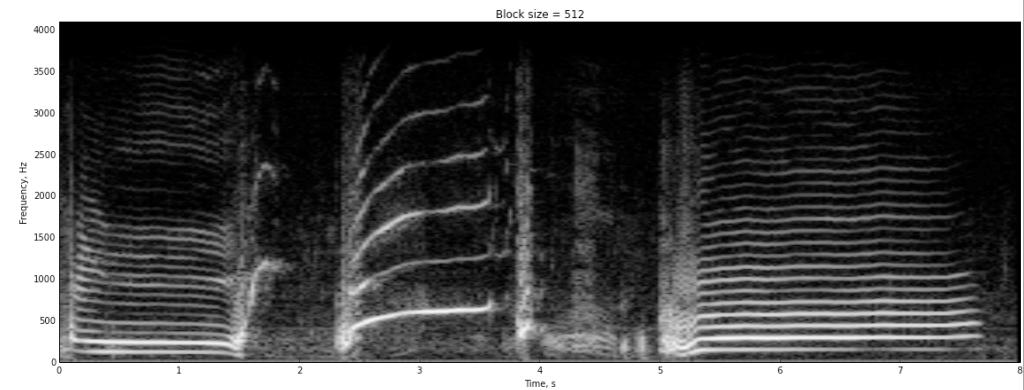
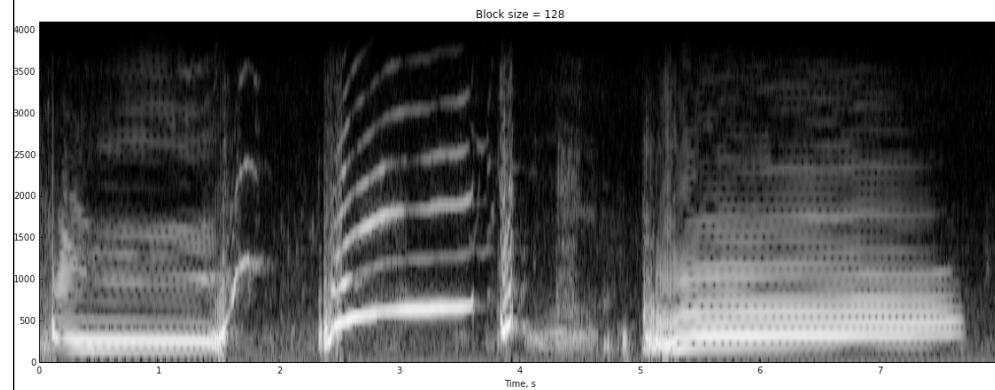
Lab III - Time-Frequency

- Compute with overlapping window

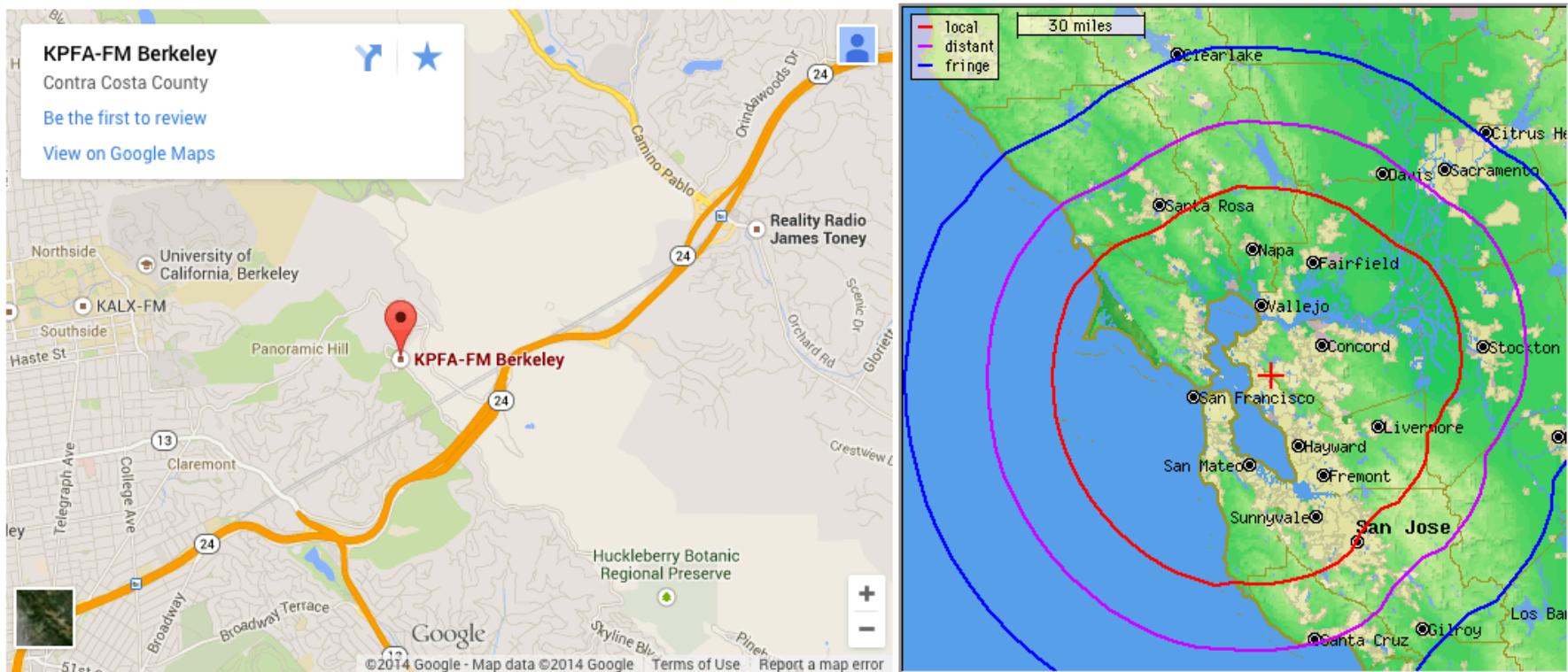


Lab III - Time-Frequency

- Look at temporal/frequency resolution tradeoffs:

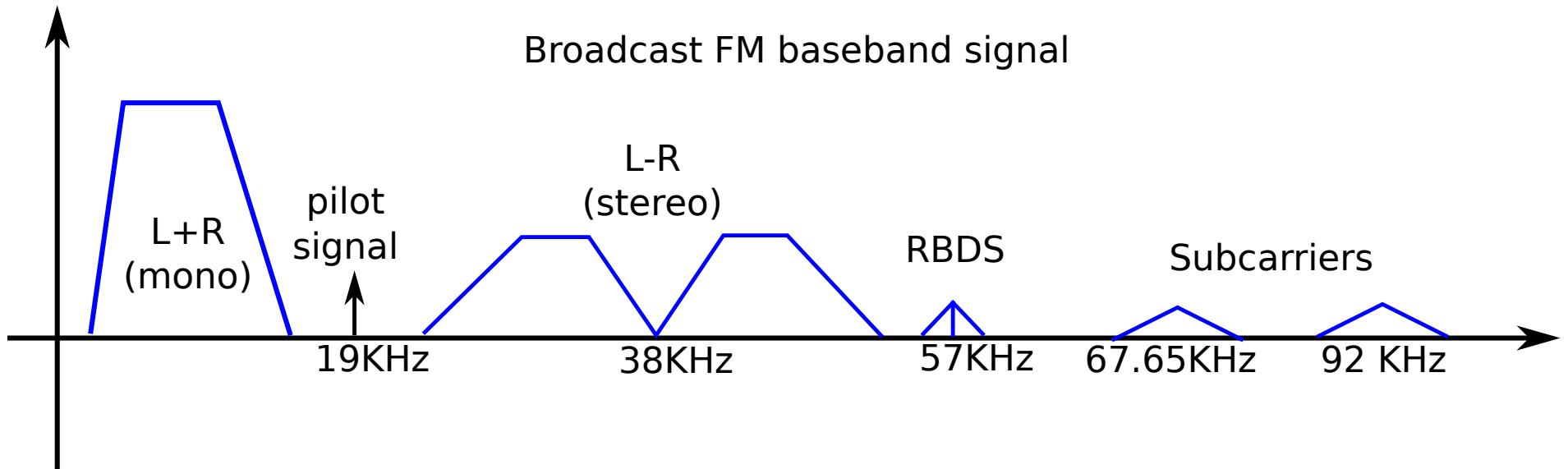


FM Broadcast Radio - KPFA 94.1MHz



FM Modulation

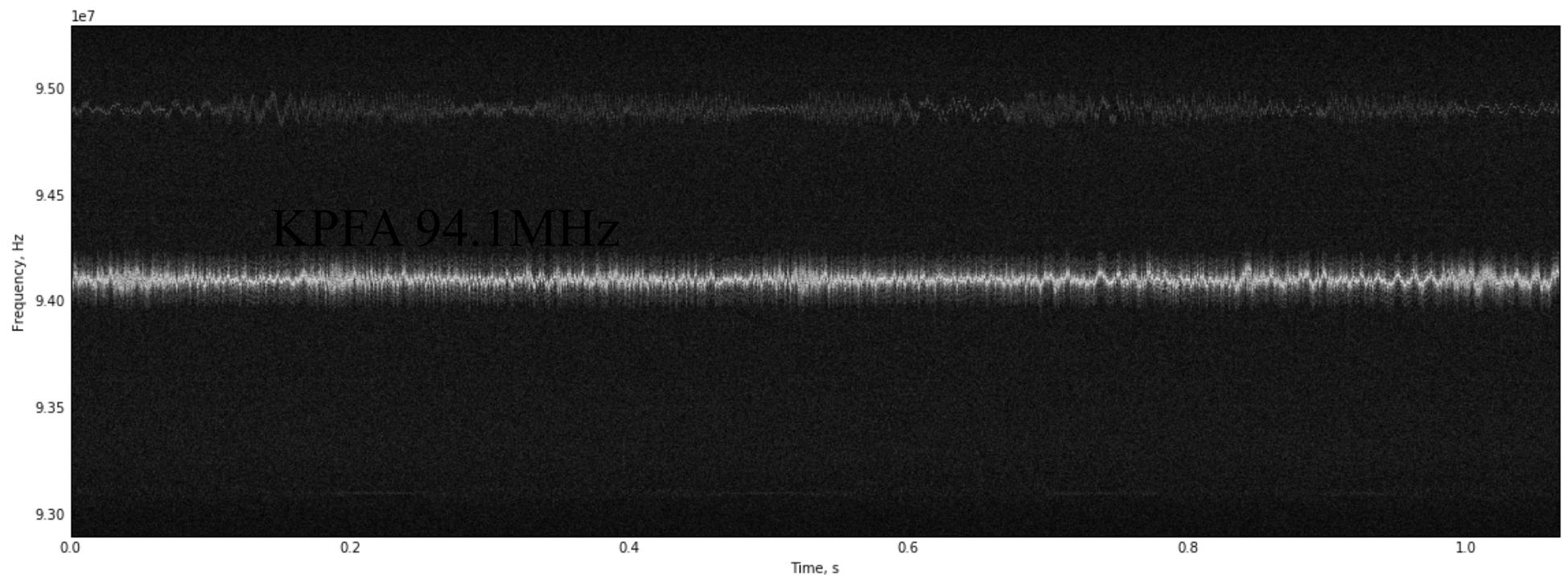
$$x(t) = \underbrace{(L + R)}_{\text{mono}} + \underbrace{0.1 \cdot \cos(2\pi f_p t)}_{\text{pilot}} + \underbrace{(L - R) \cos(2\pi(2f_p)t)}_{\text{stereo}} + \underbrace{0.05 \cdot \text{RBDS}(t) \cos(2\pi(3f_p)t)}_{\text{digital RBDS}} + \text{subcarriers}$$



$$y_c(t) = A \cos \left(2\pi f_c t + 2\pi \Delta f \int_0^t x(\tau) d\tau \right)$$

Look at Spectrum of Broadcast FM

- Capture 1 second, @ $f_s=2.4\text{MHz}$
plot spectrogram

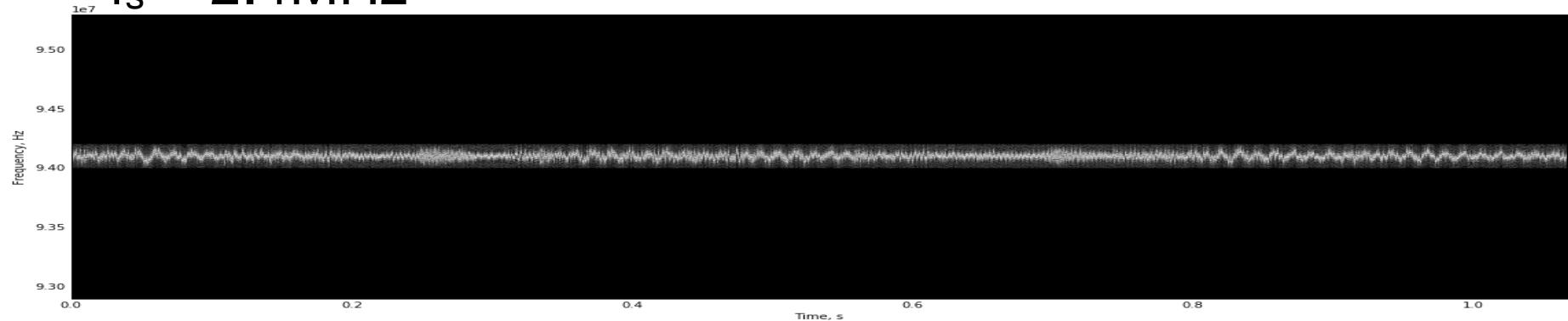


Filter Downsample

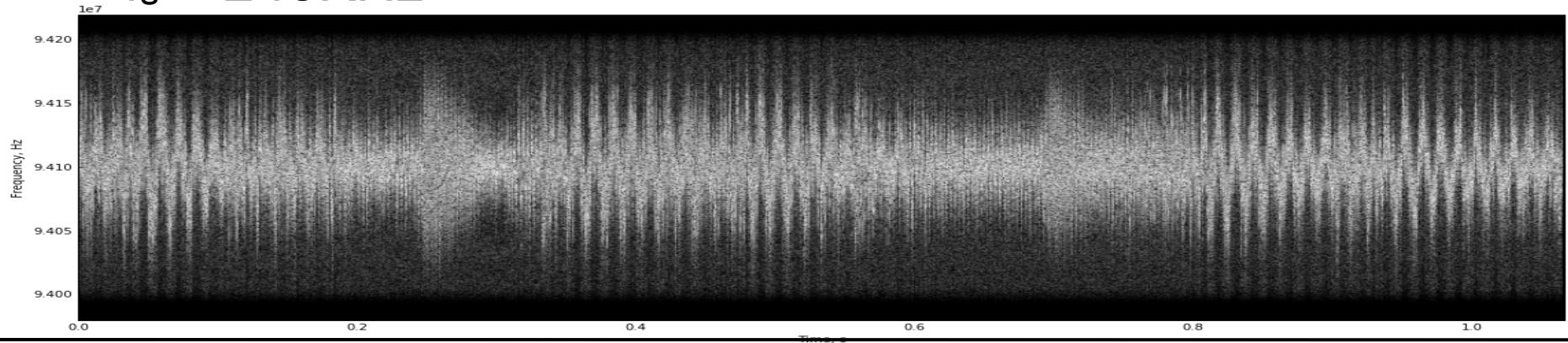


- `h = signal.firwin(513,100000.0,nyq=2400000.0/2,window='hanning')`

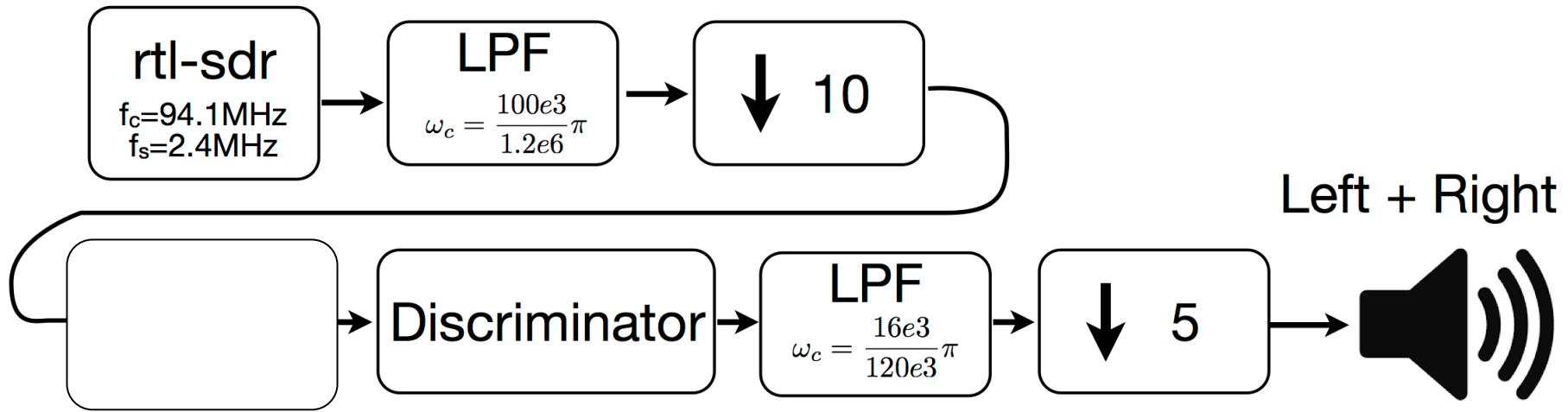
$$f_s = 2.4\text{MHz}$$



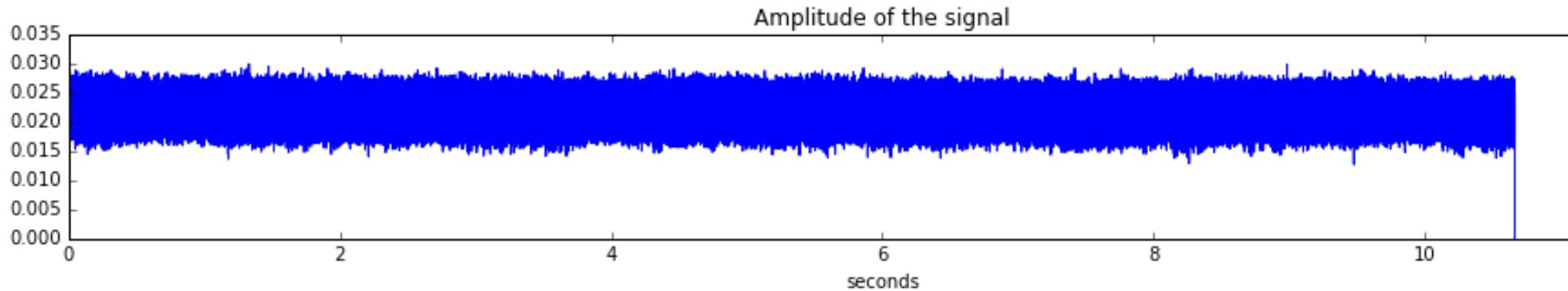
$$f_s = 240\text{KHz}$$



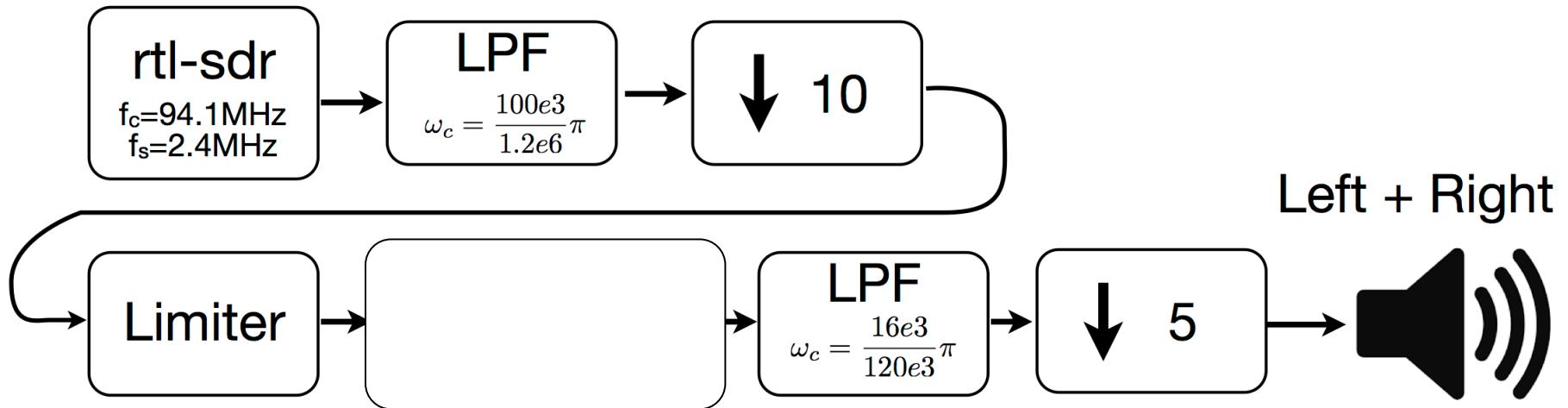
FM demodulation



- Limiter removes unwanted amplitude variation



FM demodulation

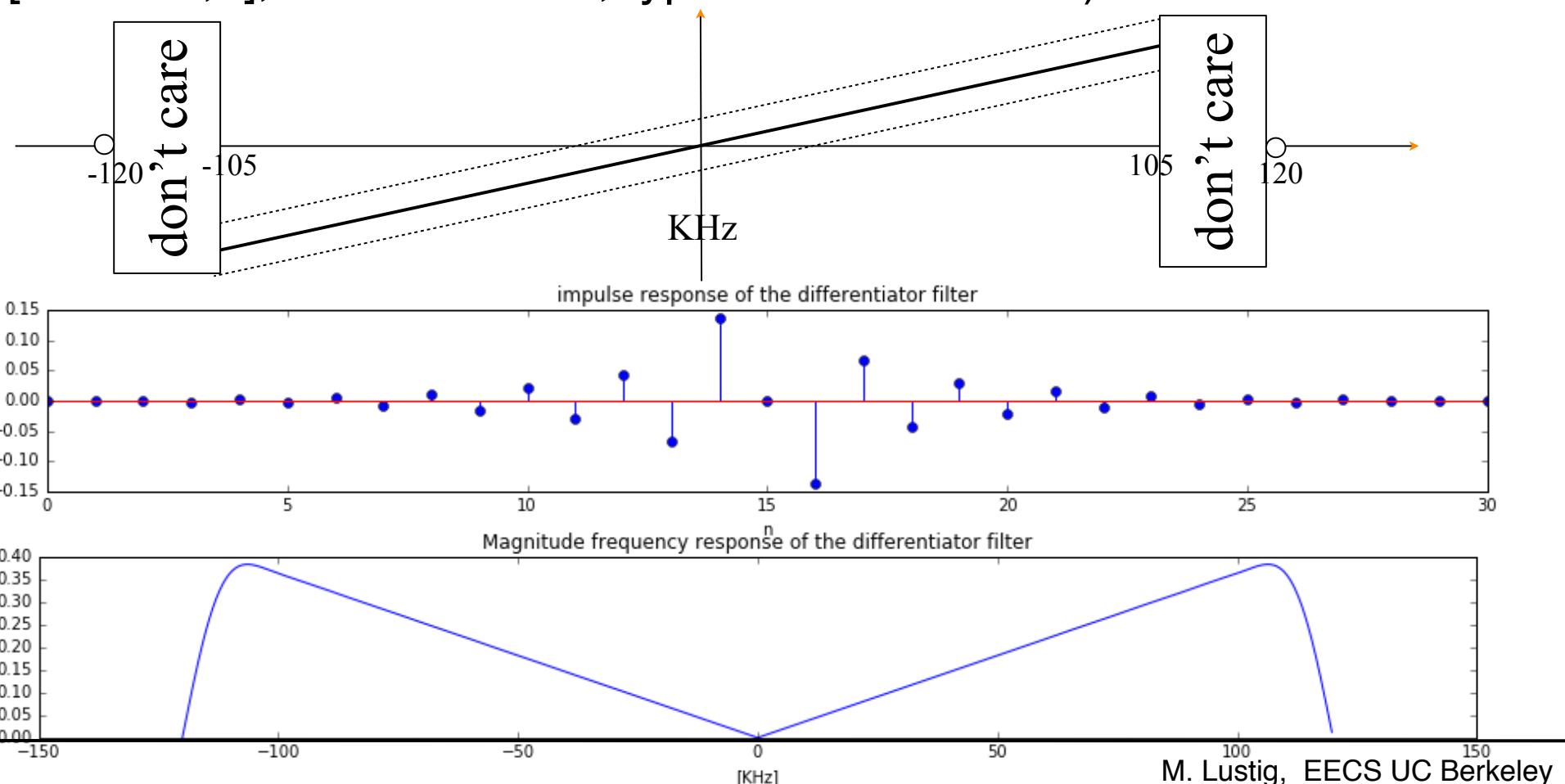


- Discriminator converts frequency deviations to amplitude
 - Several implementations possible. Here we use this:

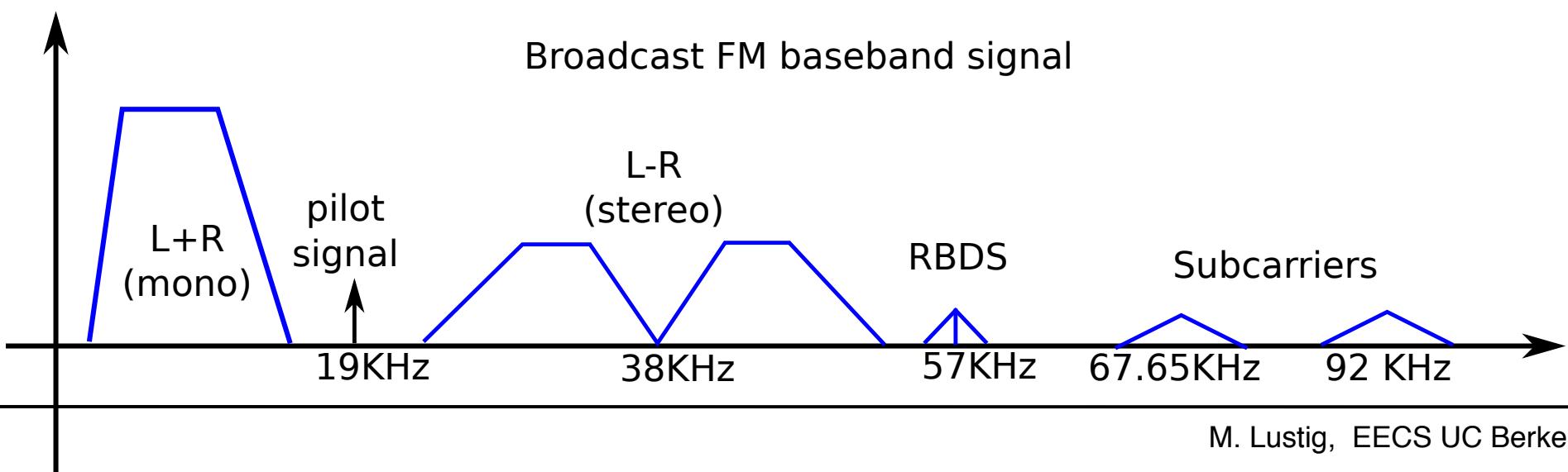
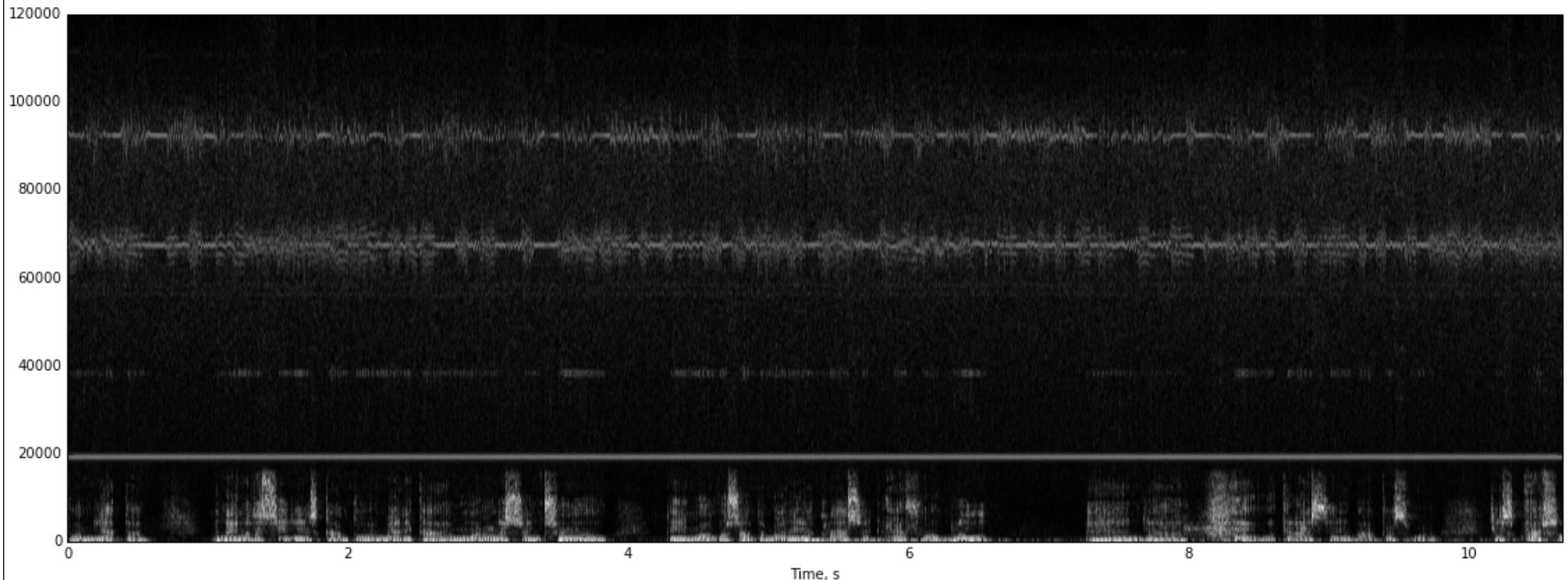
$$\begin{aligned} \left(\frac{d}{dt} y_b(t) \right) y_b^*(t) &= j2\pi\Delta f \cdot x(t) \cdot e^{j2\pi\Delta f \int_0^t x(\tau)d\tau} \cdot e^{-j2\pi\Delta f \int_0^t x(\tau)d\tau} \\ &= j2\pi\Delta f \cdot x(t) \end{aligned}$$

Differentiator

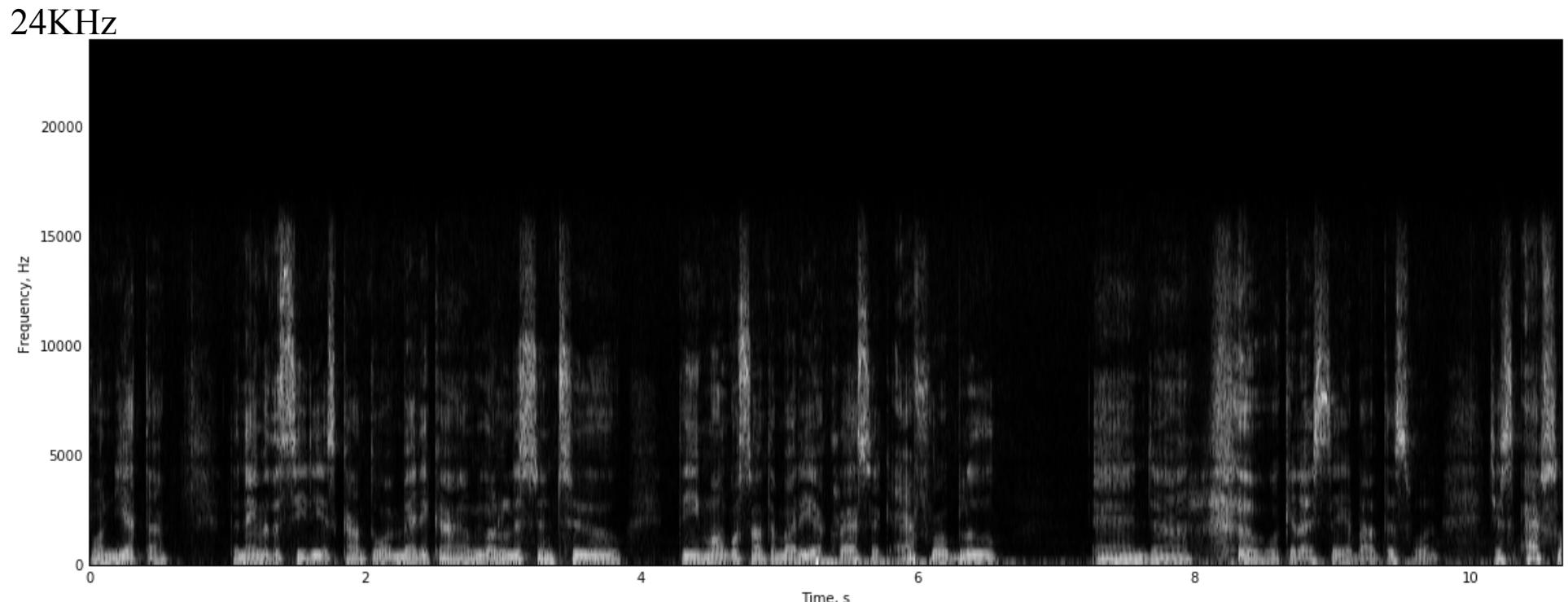
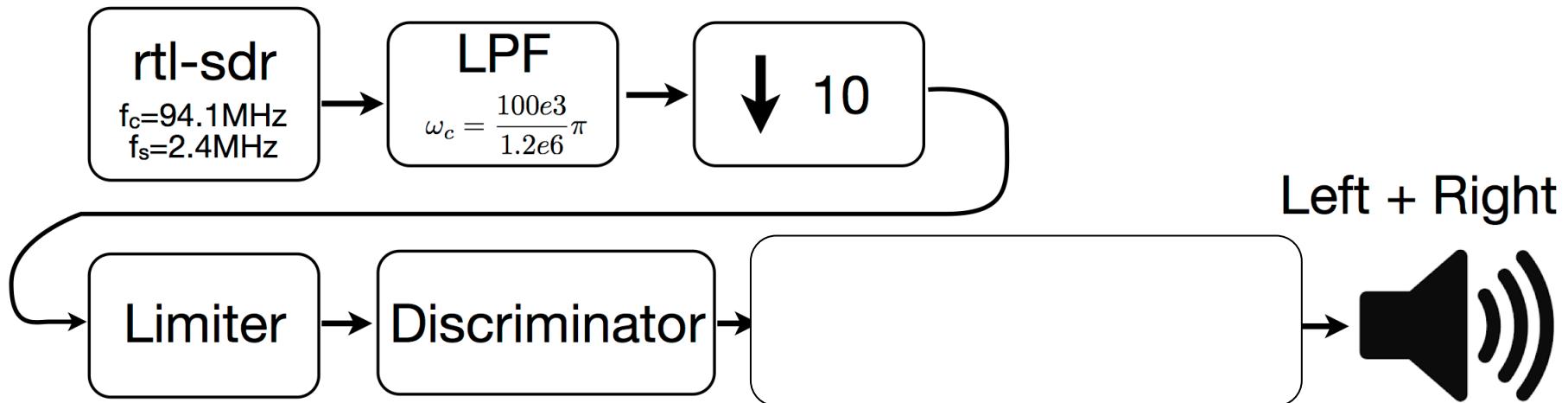
- Use Parks-McClellan algorithm (Remez exchange algorithm)
- Equiripple optimal filter design
- `h_diff = signal.remez(31,[0.0,105000.0,120000.0,120000.0], [1.05/1.2,0],Hz = 240000.0, type='differentiator')`



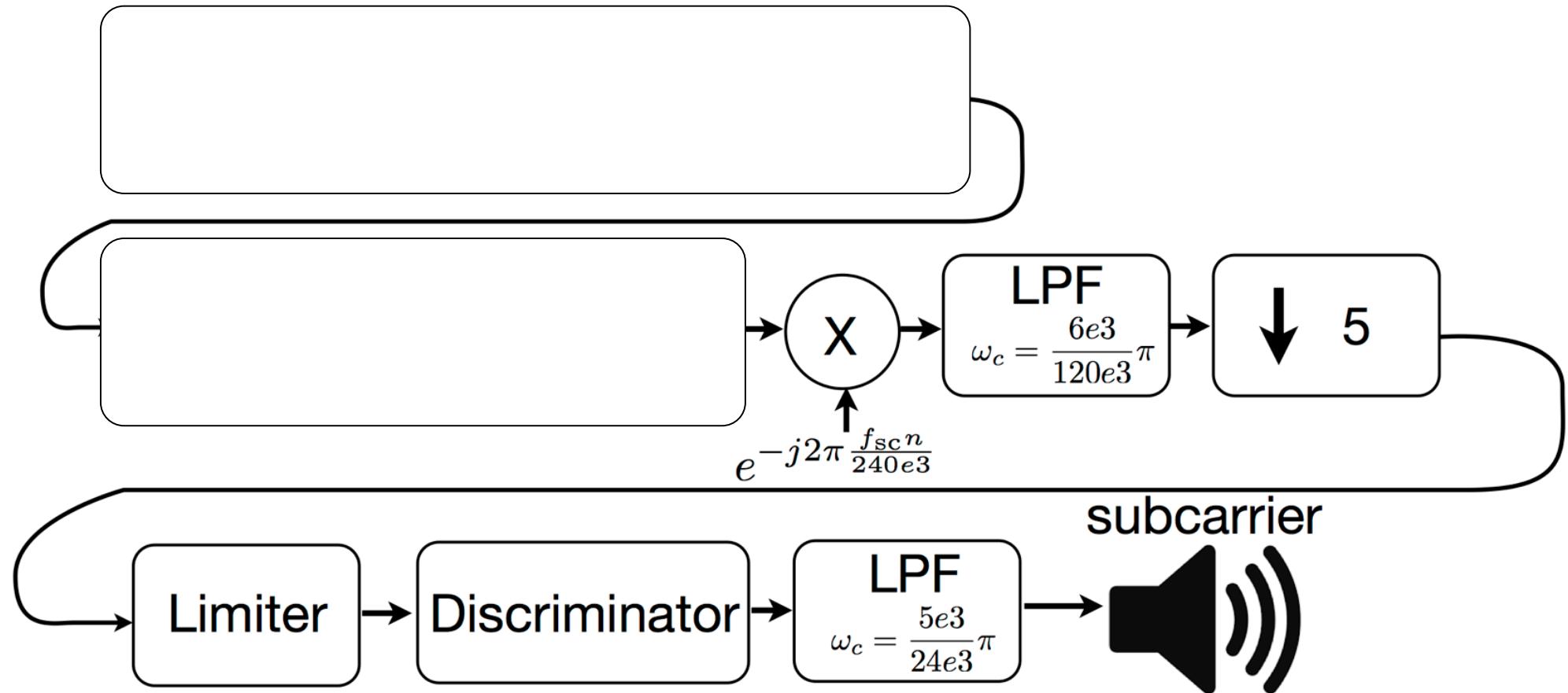
FM Baseband after Demodulation



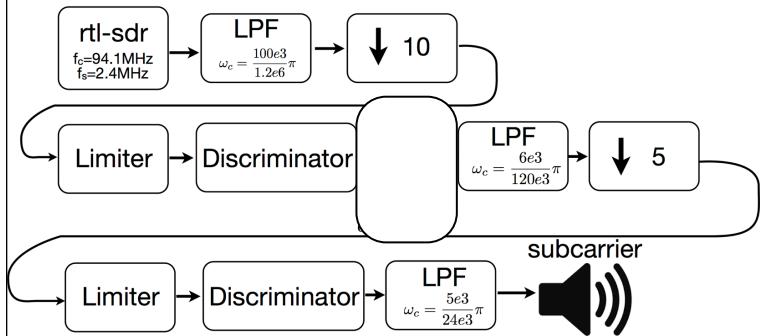
Mono (Left + Right Channels)



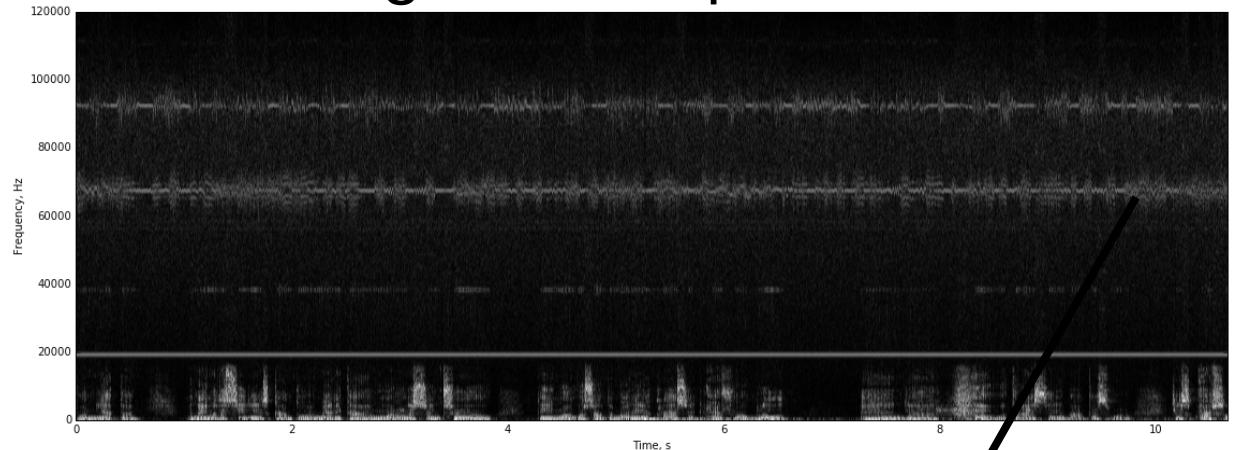
Demodulating Subcarriers



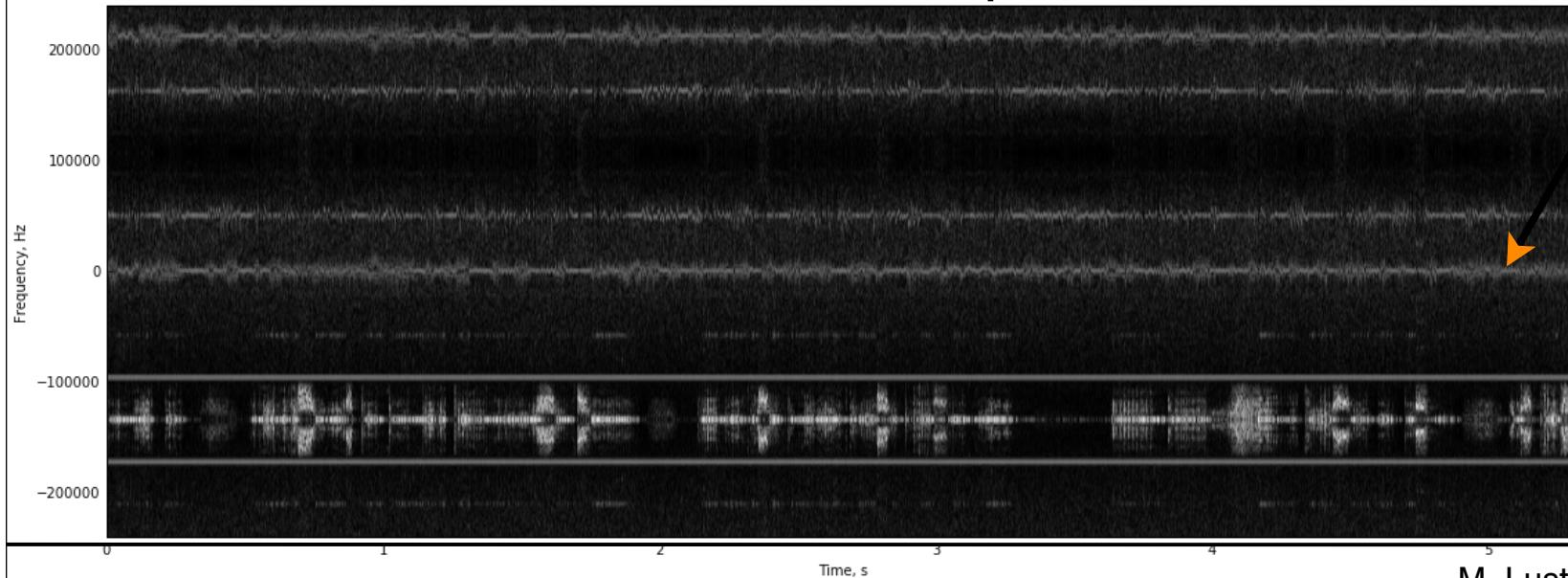
Demodulating Subcarriers



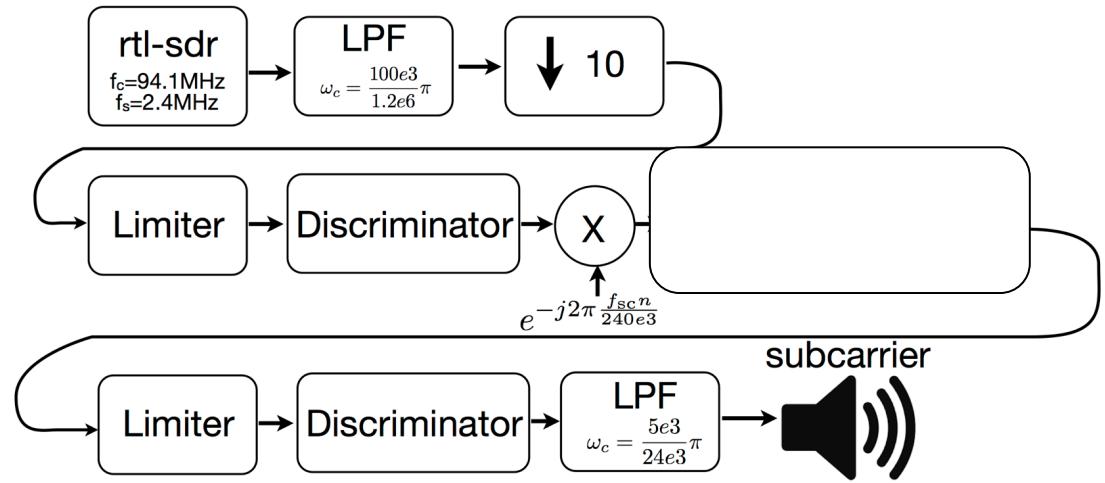
single sided spectrum



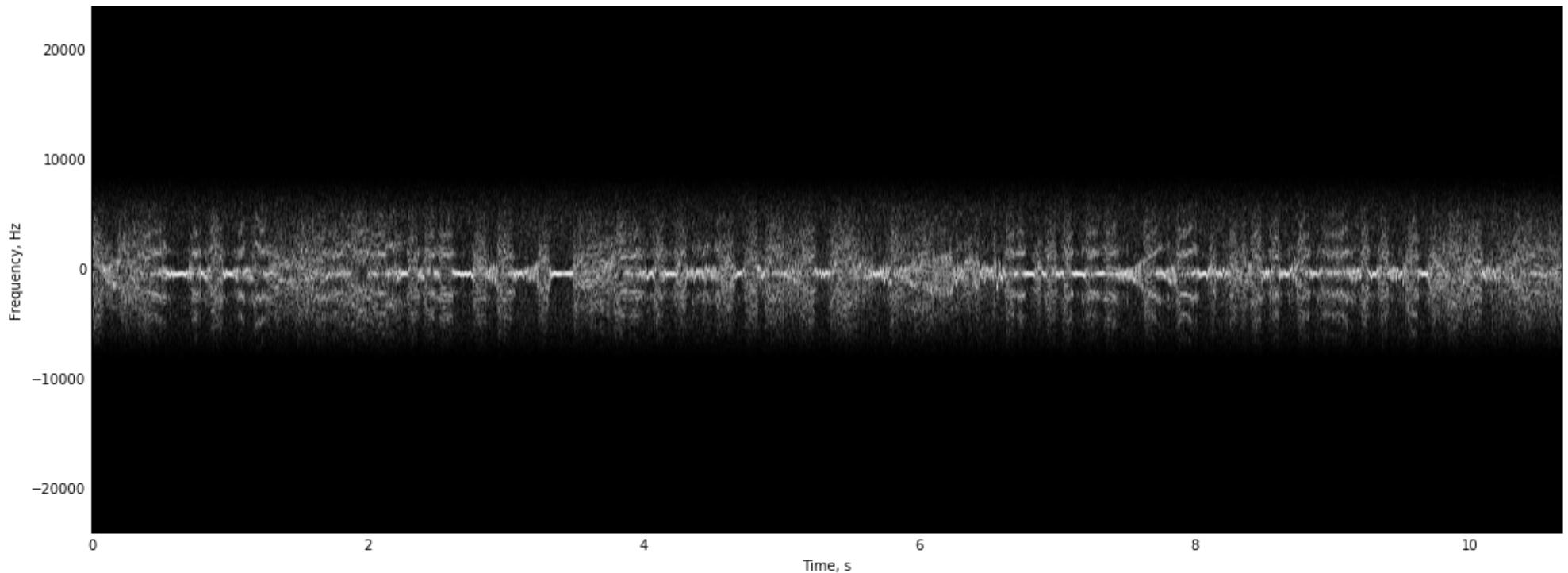
double sided spectrum



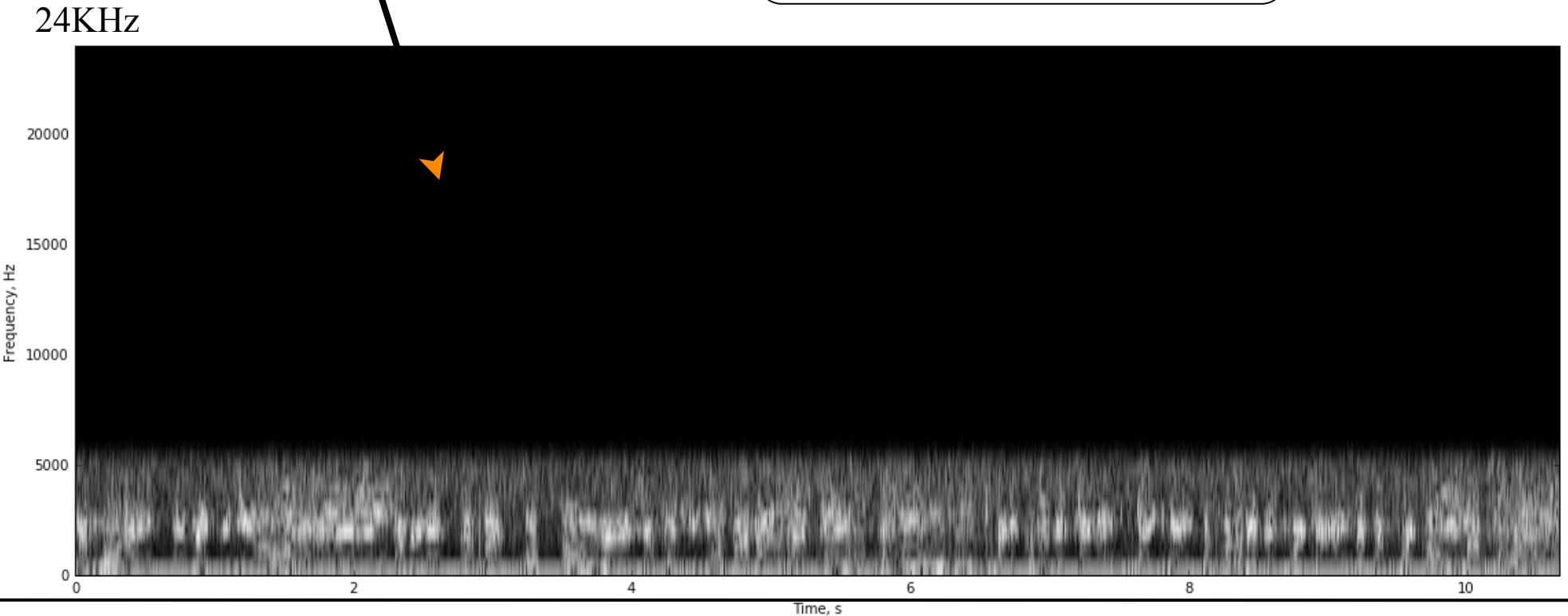
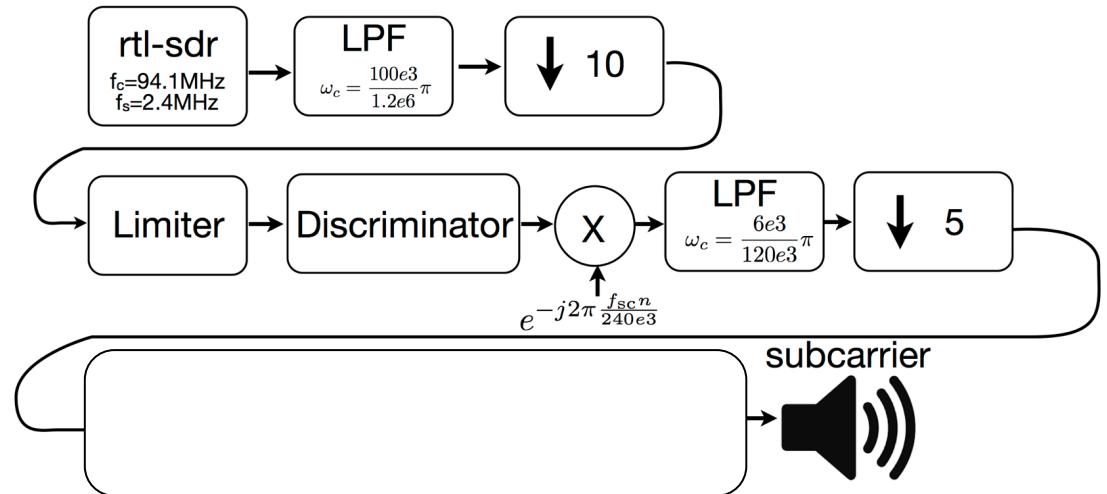
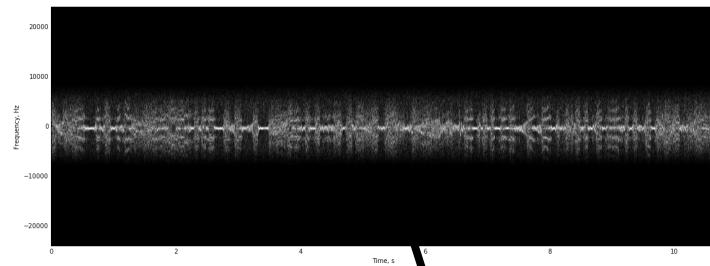
Demodulating Subcarriers



24KHz



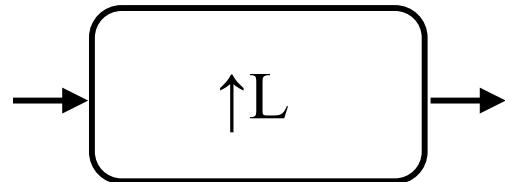
Demodulating Subcarriers



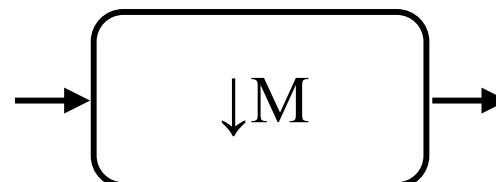
Multi-Rate Signal Processing

- What if we want to resample by $1.01T$?
 - Expand by $L=100$
 - Filter $\pi/101$ (\$\$\$\$\$)
 - Downsample by $M=101$
- Fortunately there are ways around it!
 - Called multi-rate
 - Uses compressors, expanders and filtering

Interchanging Operations



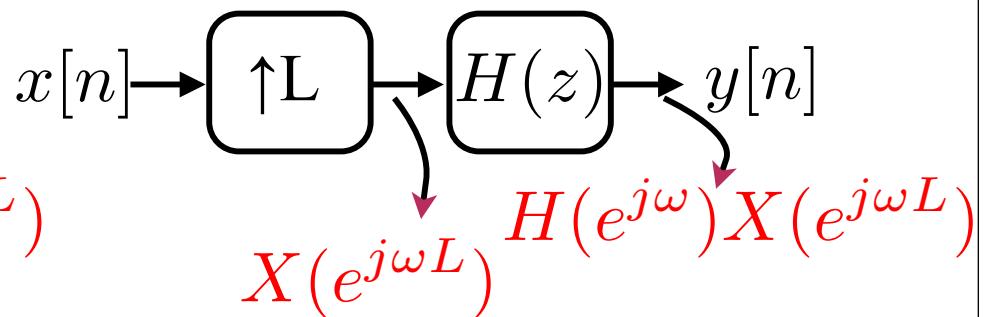
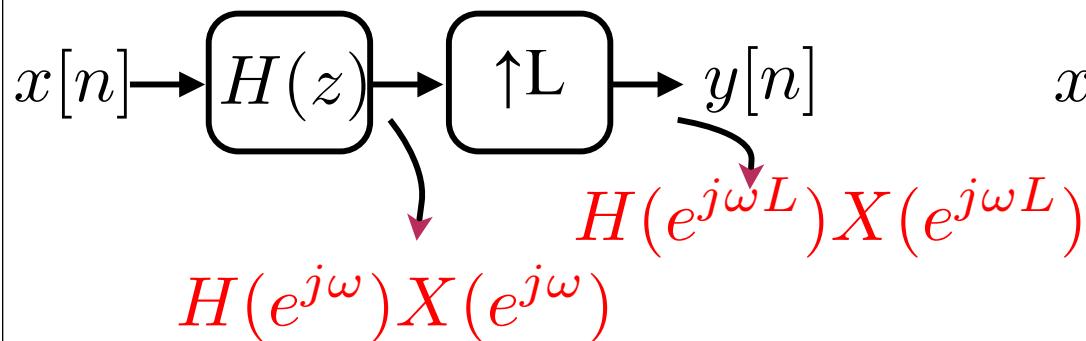
“expander”



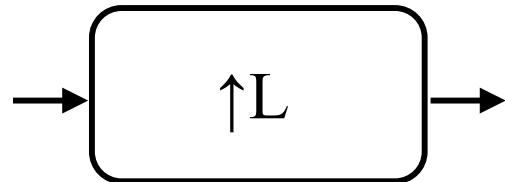
“compressor”

not LTI!

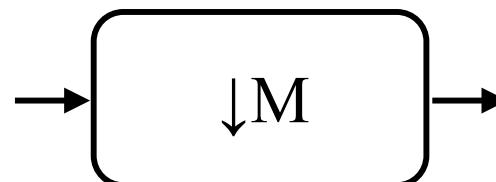
Note:



Interchanging Operations



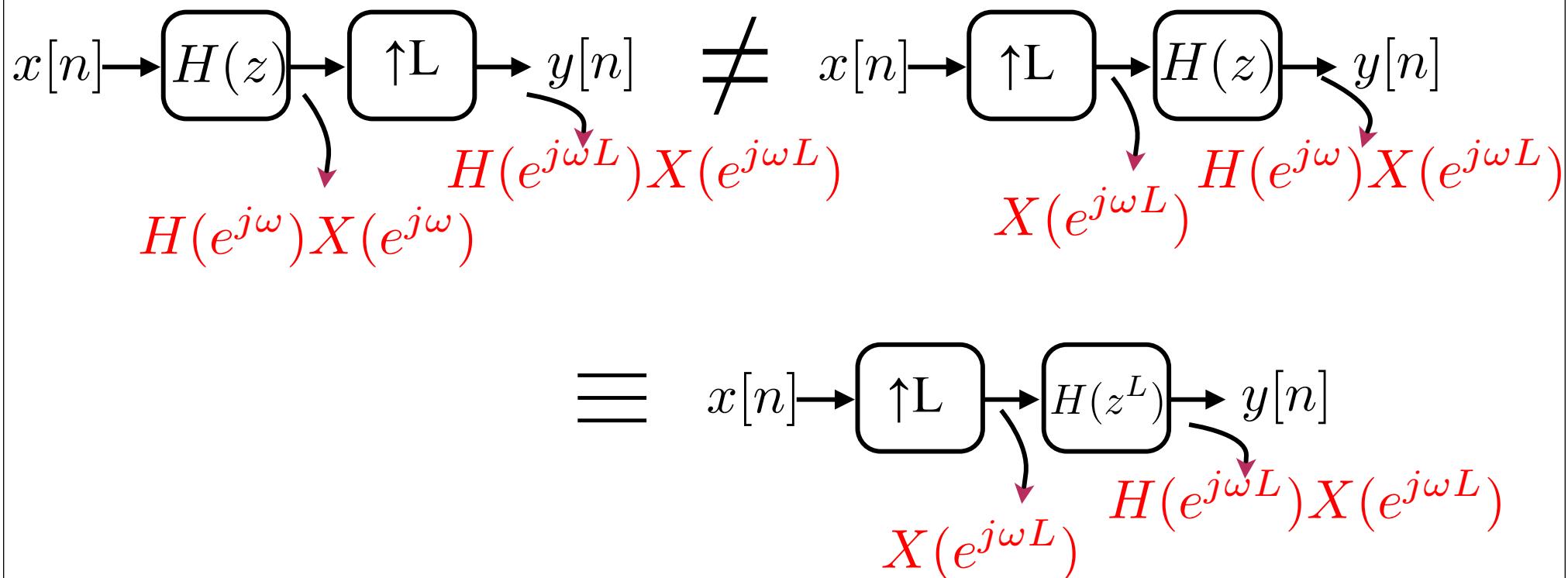
“expander”



“compressor”

not LTI!

Note:



Interchanging Filter Expander

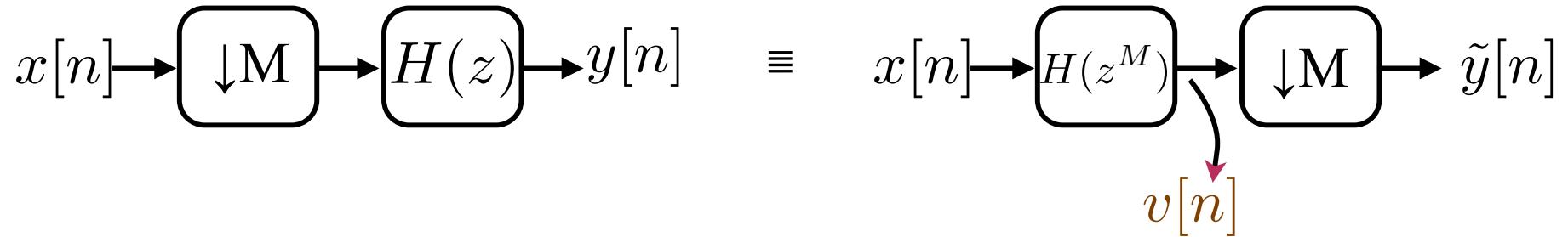
- Q: Can we move expander from Left to Right (with xform)?



- A: Yes, if $H(z)$ is rational
No, otherwise

Compressor

Claim:



Proof:

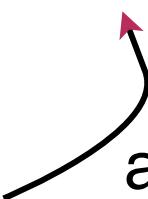
Compressor

Proof:

$$\begin{aligned} Y(e^{i\omega}) &= H(e^{j\omega}) \left(\frac{1}{M} \sum_{i=0}^{M-1} X \left(e^{j(\frac{\omega}{M} - \frac{2\pi i}{M})} \right) \right) \\ &= \frac{1}{M} \sum_{i=0}^{M-1} \underbrace{H \left(e^{j(\omega - 2\pi i)} \right)}_{H(e^{j\omega})} X \left(e^{j(\frac{\omega}{M} - \frac{2\pi i}{M})} \right) \\ &= \frac{1}{M} \sum_{i=0}^{M-1} H \left(e^{jM(\frac{\omega}{M} - \frac{2\pi i}{M})} \right) X \left(e^{j(\frac{\omega}{M} - \frac{2\pi i}{M})} \right) \end{aligned}$$

$$V(e^{j\omega}) = H(e^{j\omega M})X(e^{j\omega})$$

after compressor



Compressor

Claim:



Proof:

$$\begin{aligned} Y(e^{i\omega}) &= H(e^{j\omega}) \left(\frac{1}{M} \sum_{i=0}^{M-1} X \left(e^{j(\frac{\omega}{M} - \frac{2\pi i}{M})} \right) \right) \\ &= \frac{1}{M} \sum_{i=0}^{M-1} \underbrace{H \left(e^{j(\omega - 2\pi i)} \right)}_{H(e^{j\omega})} X \left(e^{j(\frac{\omega}{M} - \frac{2\pi i}{M})} \right) \\ &= \frac{1}{M} \sum_{i=0}^{M-1} H \left(e^{jM(\frac{\omega}{M} - \frac{2\pi i}{M})} \right) X \left(e^{j(\frac{\omega}{M} - \frac{2\pi i}{M})} \right) \end{aligned}$$

after compressor

Q: Move Compressor from right to left?

A: Only if $H(z^{1/M})$ is rational!

$$V(e^{j\omega}) = H(e^{j\omega M})X(e^{j\omega})$$

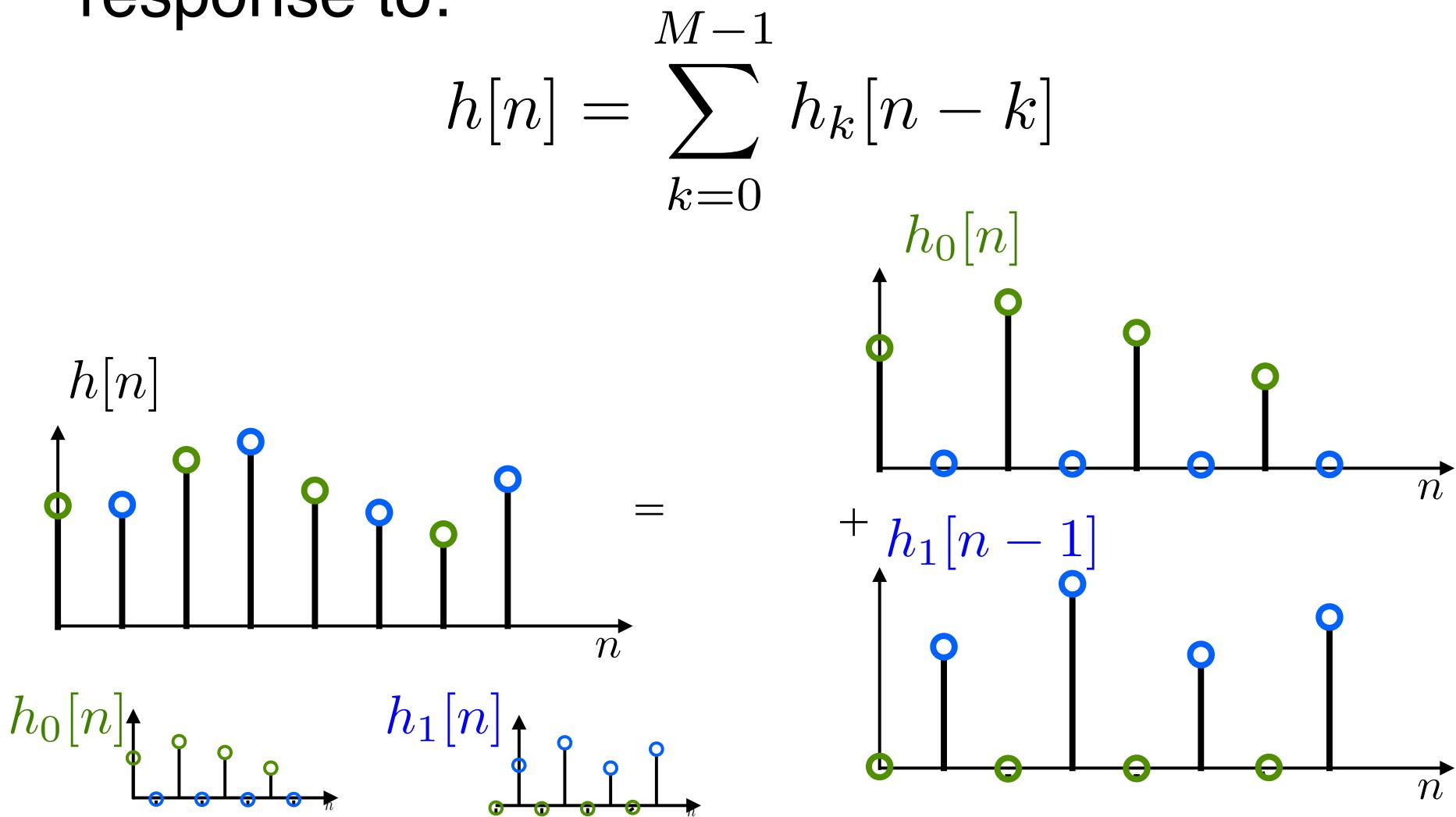
Interchanging Operations

$$x[n] \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow L} \rightarrow y[n] \quad \equiv \quad x[n] \rightarrow \boxed{\uparrow L} \rightarrow \boxed{H(z^L)} \rightarrow y[n]$$

$$x[n] \rightarrow \boxed{\downarrow M} \rightarrow \boxed{H(z)} \rightarrow y[n] \quad \equiv \quad x[n] \rightarrow \boxed{H(z^M)} \rightarrow \boxed{\downarrow M} \rightarrow y[n]$$

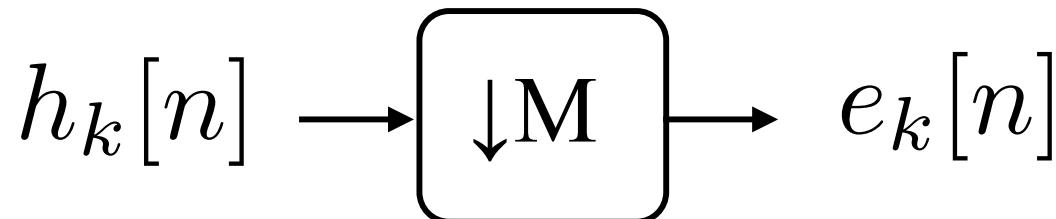
Polyphase Decomposition

- We can decompose an impulse response to:

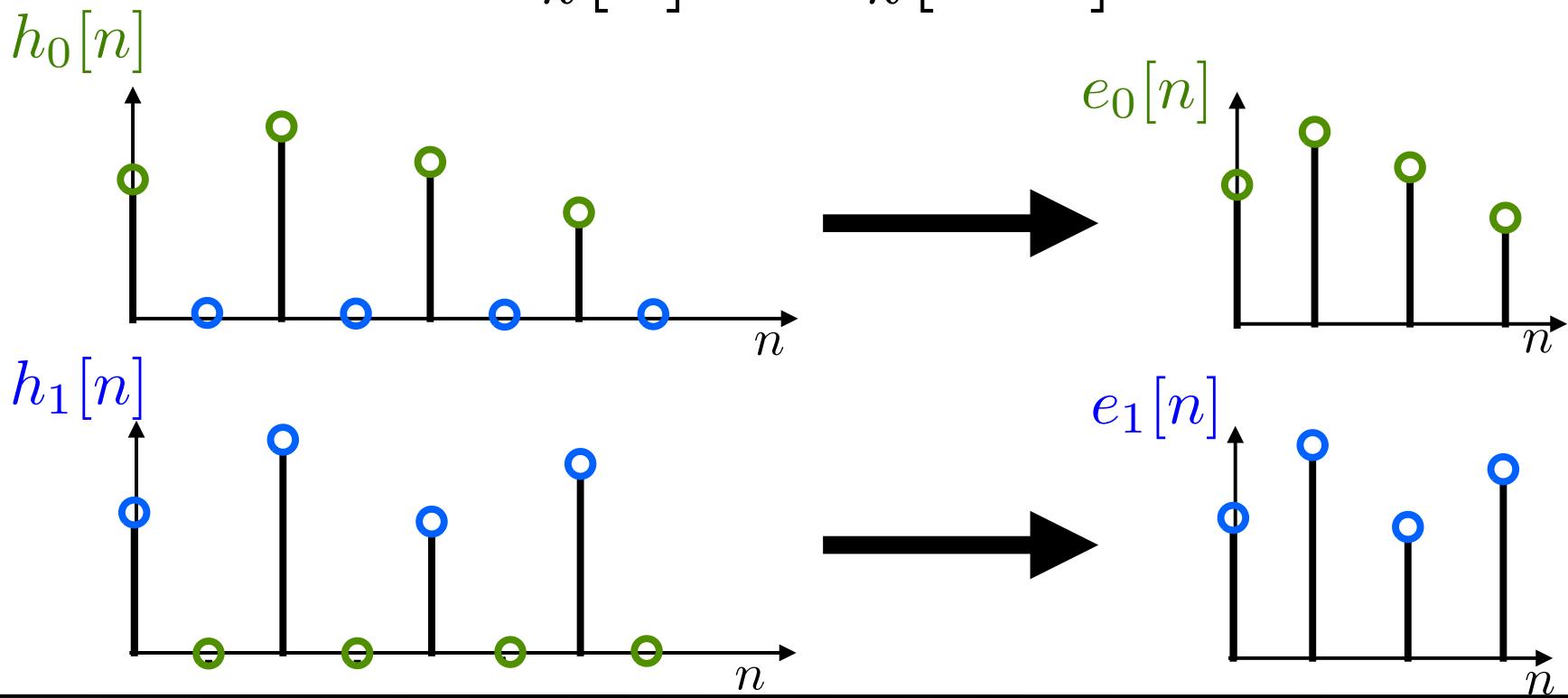


Polyphase Decomposition

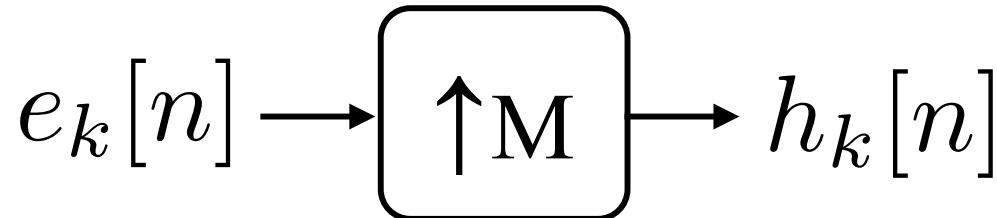
- Define:



$$e_k[n] = h_k[nM]$$



Polyphase Decomposition



recall upsampling \Rightarrow scaling

$$H_k(z) = E_k(z^M)$$

Also, recall:

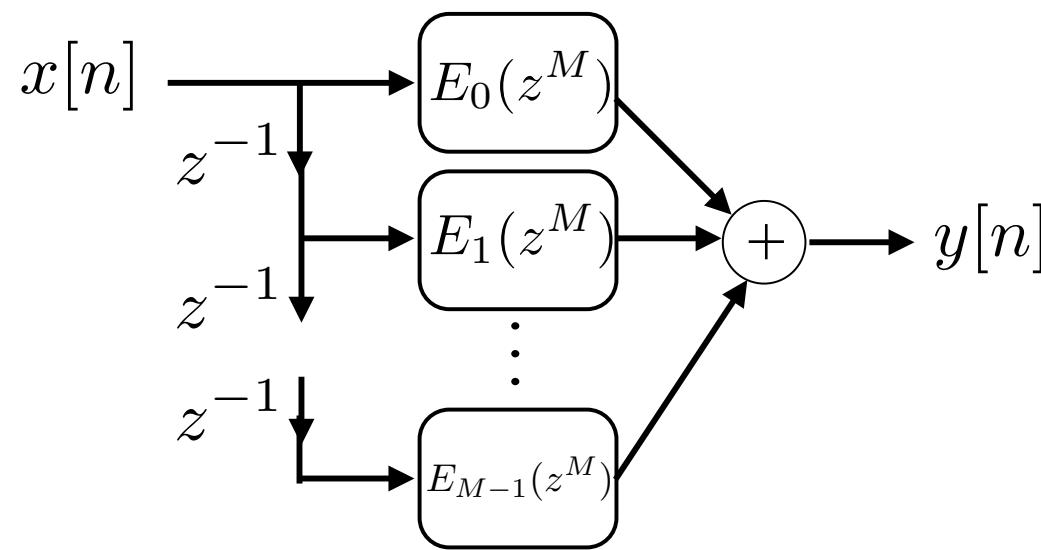
$$h[n] = \sum_{k=0}^{M-1} h_k[n - k]$$

So,

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$

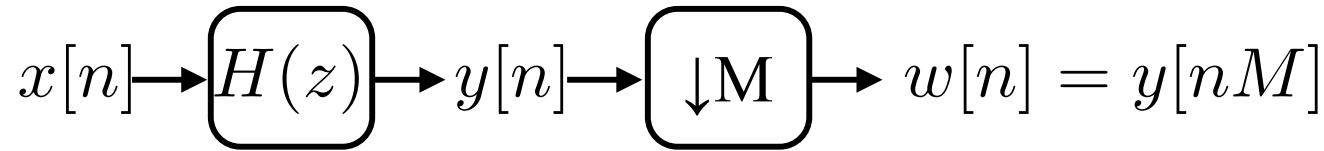
Polyphase Decomposition

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$



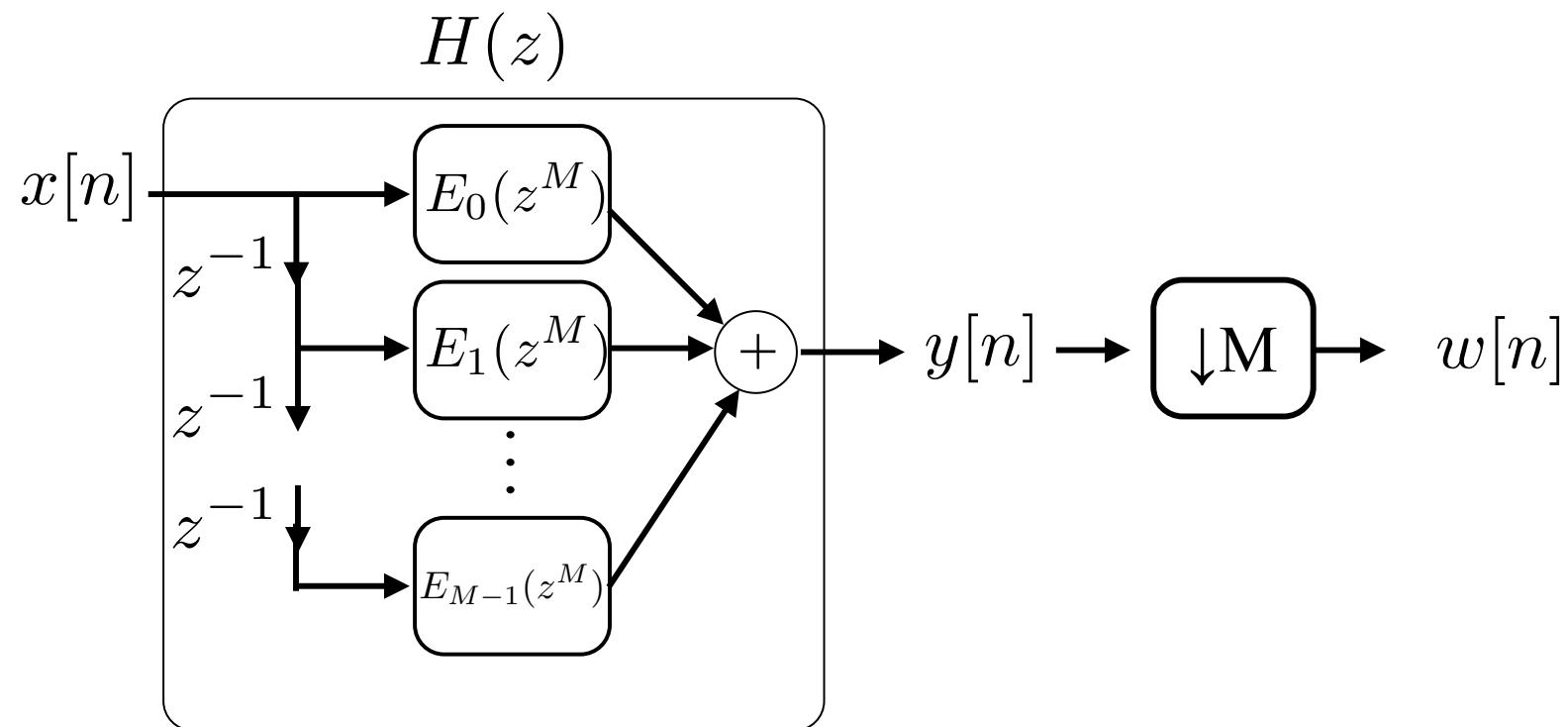
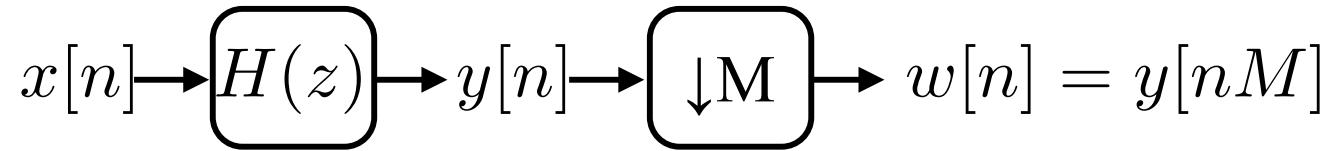
Why should you care?

Polyphase Implementation of Decimation

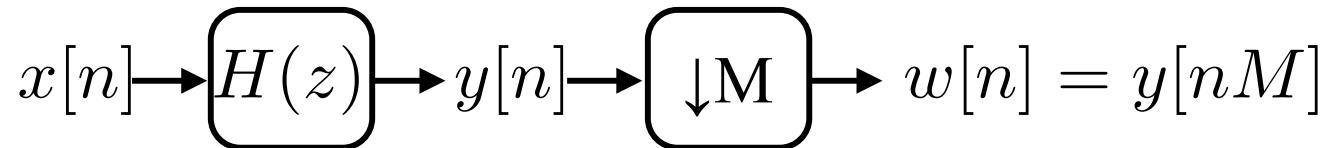


- Problem:
 - Compute all $y[n]$ and then throw away -- wasted computation!
 - For FIR length $N \Rightarrow N$ mults/unit time
 - Can interchange Filter with compressor?
 - Not in general!

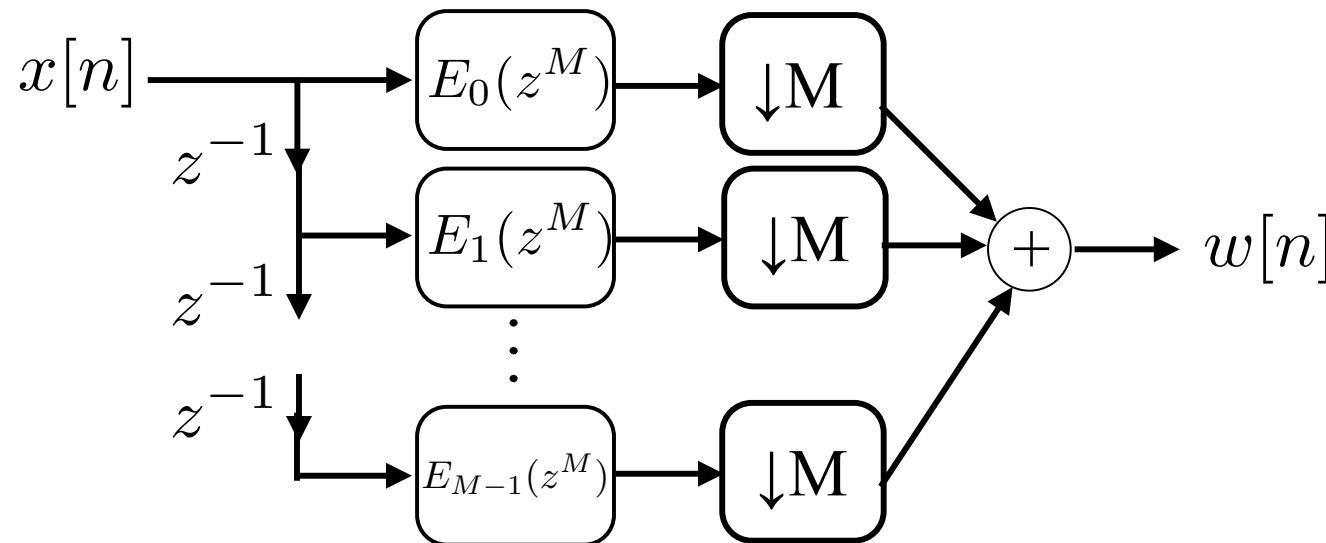
Polyphase Implementation of Decimation



Polyphase Implementation of Decimation

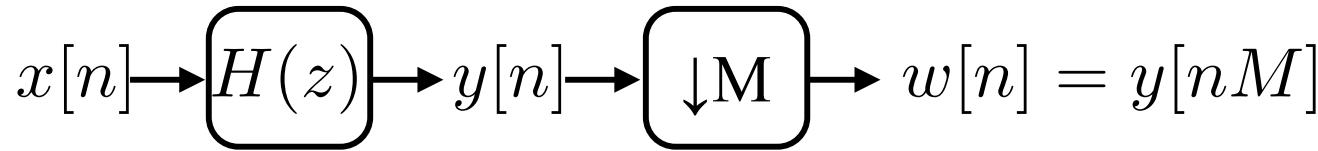


Interchange sum with decimation

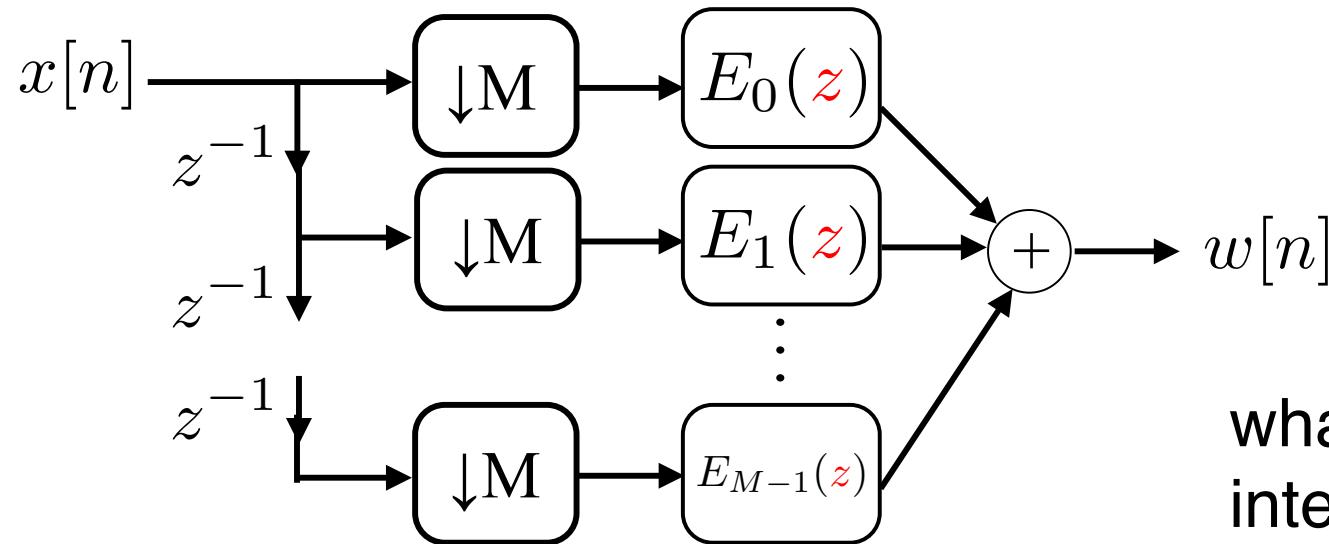


now, what can we do?

Polyphase Implementation of Decimation



Interchange filter with decimation



Computation:

Each Filter: $N/M * (1/M)$ mult/unit time

Total: N/M mult/unit time

what about
interpolation?
(great midterm Q)