Digital Signal Processing

Lecture 26
Compressed Sensing
Lab + Frequency challenge

- Lab 4
  - Make sure you get good signal -- like the one I recorded
  - Think of detecting bursts -- a robust method will lead to good results in the last part
- Frequency challenge
  - Beacon in 5th floor, around 144.280MHz using 1ppb accurate GPSDO. Accurate up to 1/100 Hz.
  - Transmits my callsign in morse code 5 times then 2 minutes break.
  - Submit frequency on bcourses by Thursday 04/07
  - You can only use the rtl-sdr to participate -- no cheating!
  - Closest submission will win a radio!
Radios

- [https://inst.eecs.berkeley.edu/~ee123/sp16/radio.html](https://inst.eecs.berkeley.edu/~ee123/sp16/radio.html)
Compressive Sampling

Q: What is the rate you need to sample at?
A: At least Nyquist!
Q: What is the rate you need to sample at?
A: Maybe less than Nyquist....
Image Compression

Images are compressible
Standard approach: First collect, then compress
Medical images are compressible
Standard approach: First collect, then compress
Medical images are compressible
Standard approach: First collect, then compress
Medical images are compressible
Standard approach: First collect, then compress

*Courtesy, M. Uecker, J Frahm Max Planck
Example I: Audio

Raw audio: 44.1Khz, 16bit, stereo = 1378 Kbit/sec
MP3: 44.1Khz, 16bit, stereo = 128 Kbit/sec
10.76 fold!
Example II: Images

Raw image (RGB): 24 bit/pixel

JPEG: 1280x960, normal = 1.09 bit/pixel

22 fold!
Example III: Videos

Raw Video: \((480\times360)p \times 24b/p \times 24fps + 44.1Khz \times 16b \times 2 = 98,578\) Kb/s

MPEG4 : 1300 Kb/s

75 fold!
Almost all compression algorithms use transform coding.

- mp3: DCT
- JPEG: DCT
- JPEG2000: Wavelet
- MPEG: DCT & time-difference
Sparse Transform

Signal

Sparse Transform

Quantization

Entropy encoding

DCT

Entropy encoding
Sparse Transform
What sparsifying transform would you use here?
Sparsity & Compressibility
Sparsity and Noise

*image courtesy of J. Trzasko*
Sparsity and Noise

sparse

not sparse

denoise/separate by threshold

*image courtesy of J. Trzasko
AHA
SURE!
ON THE COUNT OF THREE

THIS SPECTRUM IS NOISY
CAN YOU GIVE ME A HAND?

THRESHOLD

YEP, JUST A THRESHOLD
LET'S GET OUT OF HERE BEFORE SOMEONE SEES US

IT'S LIGHT TOO

SPARSITY MAKES IT EASY TO SEPARATE SIGNAL FROM NOISE

M. Lustig, EECS UC Berkeley
Transform Sparsity

not sparse

Sparse Edges
Transform Sparsity and Denoising

not sparse

sparse

wavelet transform

low-frequency

denoised

high frequency

Transform Sparsity and Denoising

not sparse

sparse

wavelet transform

low-frequency

denoised

high frequency


M. Lustig, EECS UC Berkeley
Transform Sparsity and Denoising

wavelet denoising

More Sparse Transforms

*Video courtesy of Juan Santos, Heart Vista

position

time

not Sparse

Sparse

temporal frequency
Sparsity and Compression

• Only need to store non-zeros
From Samples to Measurements

• Shannon-Nyquist sampling
  – Worst case for ANY bandlimited data

• Compressive sampling (CS)
  “Sparse signals statistics can be recovered from a small number of non-adaptive linear measurements”
  – Integrated sensing, compression and processing.
  – Based on concepts of incoherency between signal and measurements
Traditional Sensing

- $\mathbf{x} \in \mathbb{R}^N$ is a signal
- Make $N$ linear measurements

$$\mathbf{y} = \mathbf{\Phi} \mathbf{x}$$

Desktop scanner/ digital camera sensing

sensing matrix
Traditional Sensing

• $x \in \mathbb{R}^N$ is a signal

• Make N linear measurements

$y = \Phi x$

MRI Fourier Imaging

sensing matrix
Traditional Sensing

- $x \in \mathbb{R}^N$ is a signal
- Make N linear measurements

Arbitrary sensing

A “good” sensing matrix is orthogonal

sensing matrix
Compressed Sensing

(Candes, Romber, Tao 2006; Donoho 2006)

- $x \in \mathbb{R}^N$ is a $K$-sparse signal ($K << N$)
- Make $M$ ($K < M << N$) incoherent linear projections

A “good” compressed sensing matrix is incoherent i.e., approximately orthogonal

Incoherency can preserve information
CS recovery

- Given $y = \Phi x$
  find $x$

- But there’s hope, $x$ is sparse!

\[ y = \Phi x \]
CS recovery

- Given $y = \Phi x$
- find $x$

Under-determined

- But there’s hope, $x$ is sparse!
CS recovery

• Given $y = \Phi x$
  find $x$

• But there’s hope, $x$ is sparse!

\[
\text{minimize } ||x||_2 \\
\text{s.t. } y = \Phi x
\]

WRONG!
CS recovery

• Given $y = \Phi x$
  find $x$

• But there’s hope, $x$ is sparse!

\[
\begin{align*}
\text{minimize} & \quad ||x||_0 \\
\text{s.t.} & \quad y = \Phi x
\end{align*}
\]

HARD!
CS recovery

• Given $y = \Phi x$
  find $x$

{ Under-determined }

• But there’s hope, $x$ is sparse!

\[
\text{minimize } ||x||_1 \\
\text{s.t. } y = \Phi x
\]

need $M \approx K \log(N) \ll N$
Solved by linear-programming
Geometric Interpretation

- Domain of sparse signals
- Minimum $||x||_1$
- Minimum $||x||_2$

Matrix:
\[
\begin{bmatrix}
0 & 0 & 1 \\
3 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\]

Equation:
\[
\begin{bmatrix}
a_1 & a_2 & a_3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix} y_1 \end{bmatrix}
\]