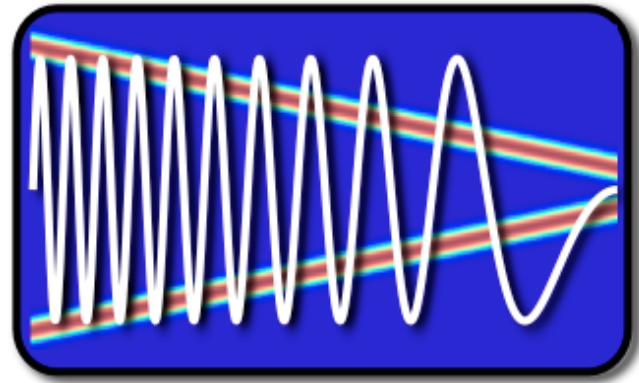


EE123



Digital Signal Processing

Lecture 26 Compressed Sensing

Lab + Frequency challenge

- Lab 4
 - Make sure you get good signal -- like the one I recorded
 - Think of detecting bursts -- a robust method will lead to good results in the last part
- Frequency challenge
 - Beacon in 5th floor, around 144.280MHz using 1ppb accurate GPSDO. Accurate up to 1/100 Hz.
 - Transmits my callsign in morse code 5 times then 2 minutes break.
 - Submit frequency on bcourses by Thursday 04/07
 - You can only use the rtl-sdr to participate -- no cheating!
 - Closest submission will win a radio!

Radios

- [https://inst.eecs.berkeley.edu/~ee123/
sp16/radio.html](https://inst.eecs.berkeley.edu/~ee123/sp16/radio.html)

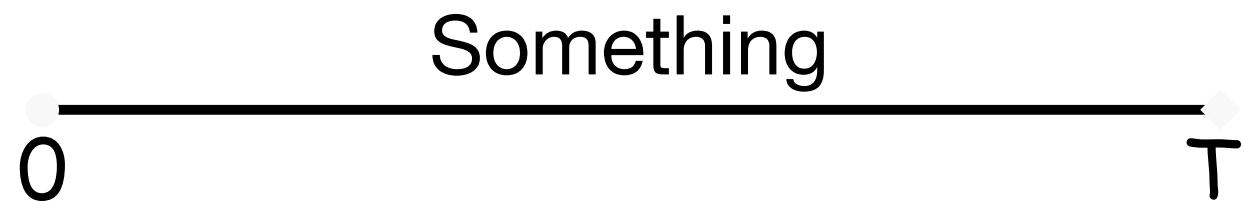
Compressive Sampling



Q: What is the rate you need to sample at?

A: At least Nyquist!

Compressive Sampling



Q: What is the rate you need to sample at?

A: Maybe less than Nyquist....

Image Compression

Images are compressible

Standard approach: First collect, then compress

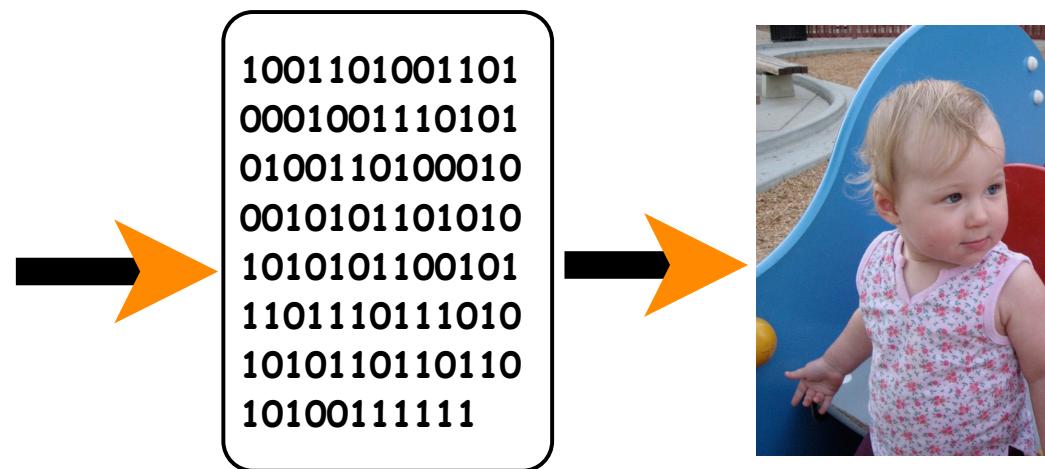
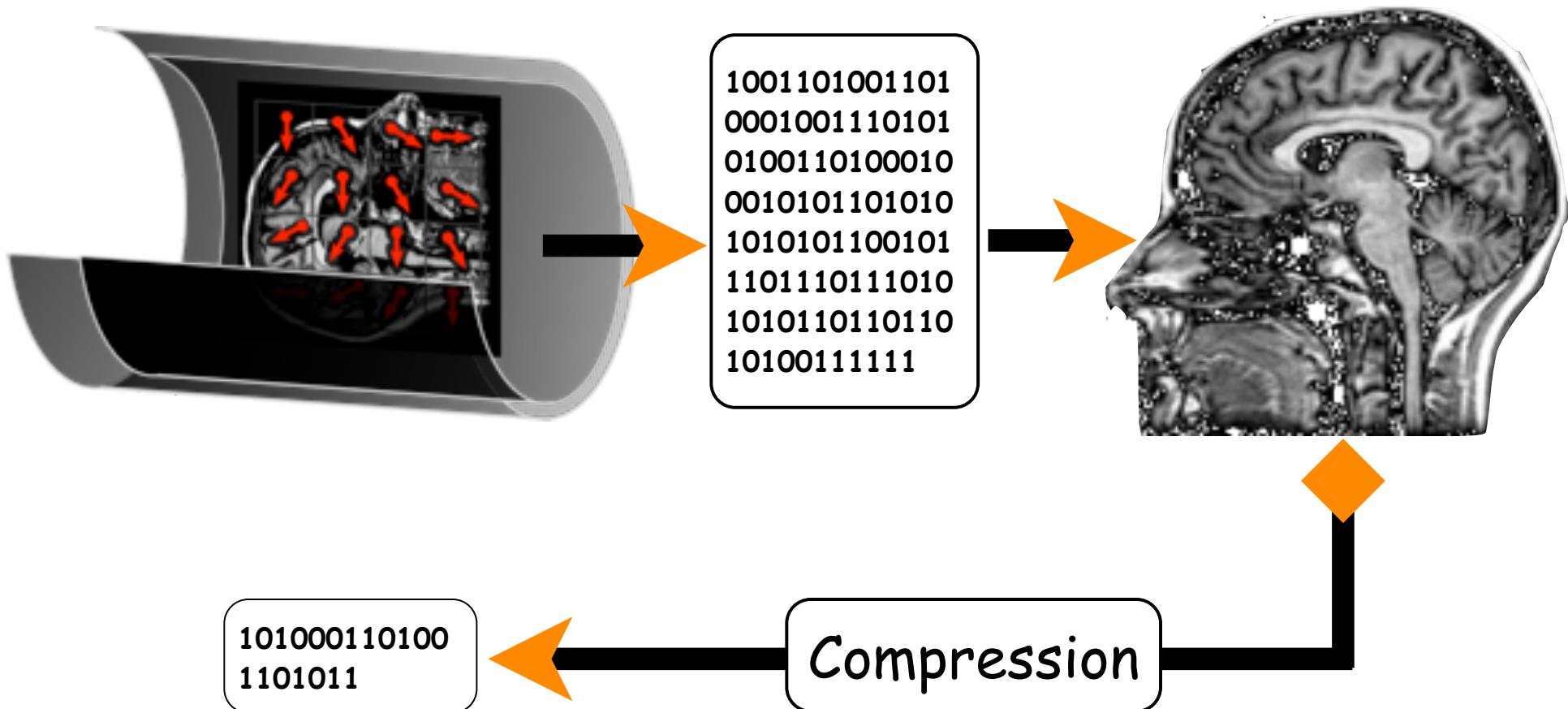


Image Compression

Medical images are compressible

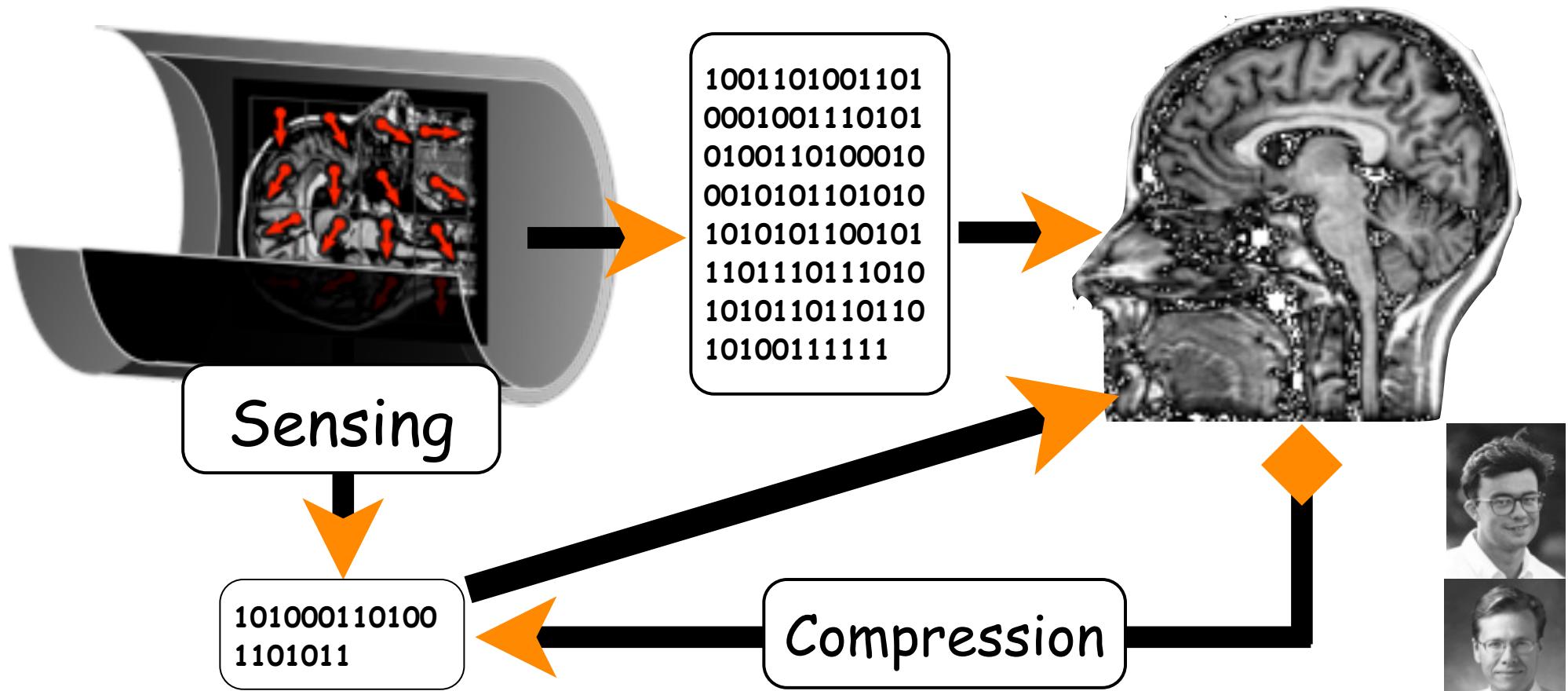
Standard approach: First collect, then compress



Compressed Sensing

Medical images are compressible

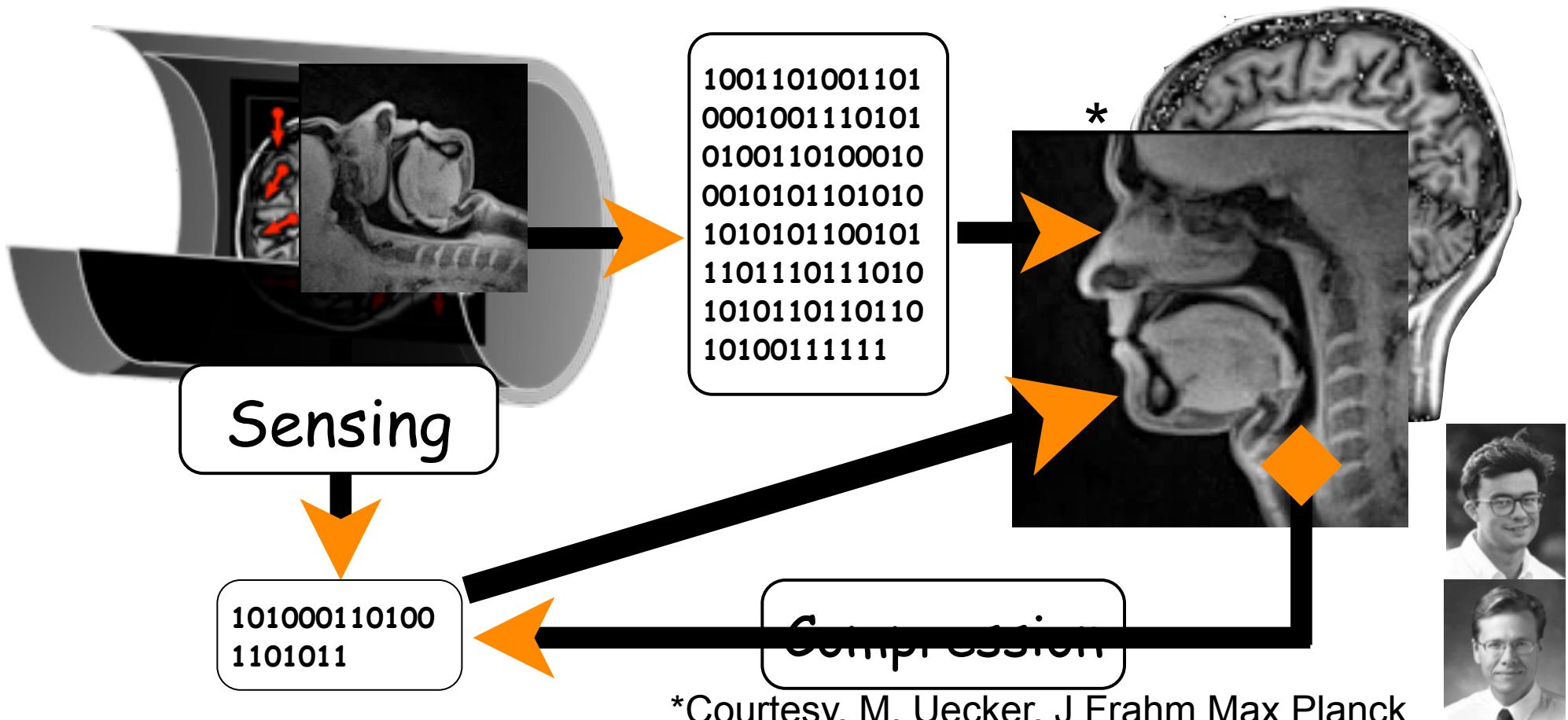
Standard approach: First collect, then compress



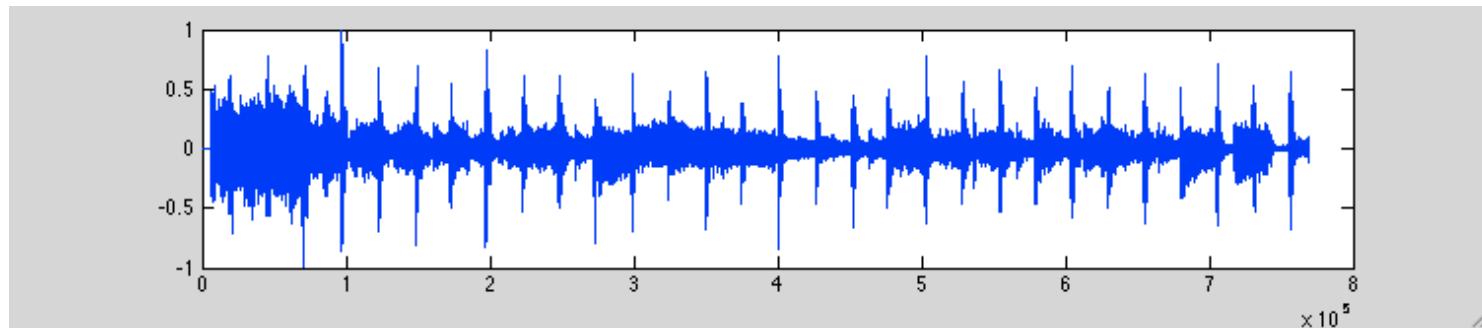
Compressed Sensing

Medical images are compressible

Standard approach: First collect, then compress



Example I: Audio



Raw audio: 44.1Khz, 16bit, stereo = 1378 Kbit/sec

MP3: 44.1Khz, 16bit, stereo = 128 Kbit/sec

10.76 fold!

Example II: Images



Raw image (RGB): 24 bit/pixel

JPEG : 1280x960, normal = 1.09 bit/pixel

22 fold!

Example III: Videos



Raw Video: (480x360)p x 24b/p x 24fps + 44.1Khz x 16b x 2 = 98,578 Kb/s

MPEG4 : 1300 Kb/s

75 fold!

Compression

Almost all compression algorithm use transform coding

mp3: DCT

JPEG: DCT

JPEG2000: Wavelet

MPEG: DCT & time-difference

Signal

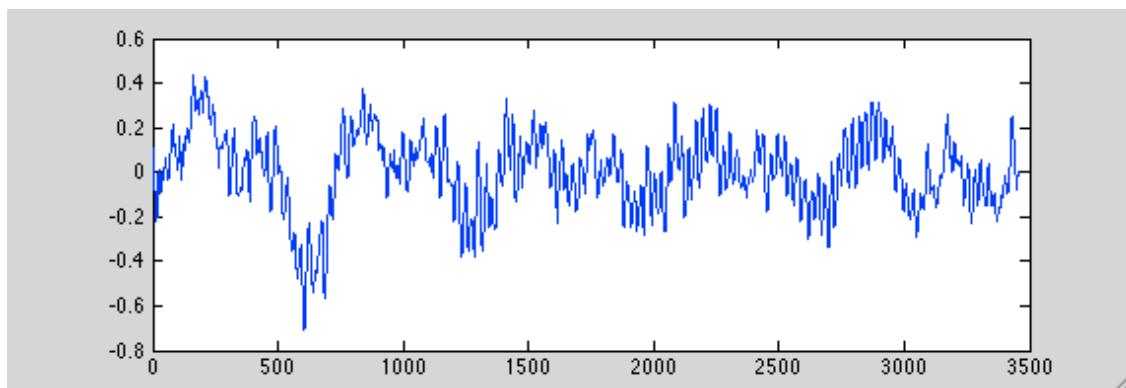
S p a r s e
Transform

QUANTIZATION

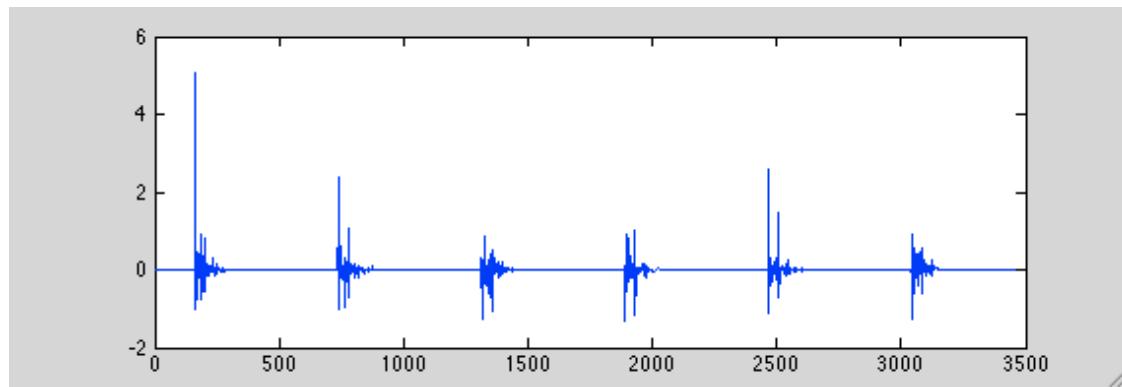
Entropy
encoding

Signal

Sparse Transform



DCT



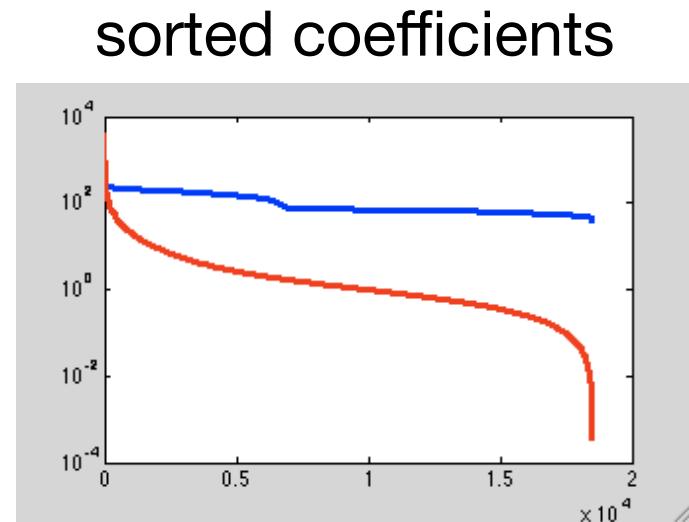
Signal
Sparse
Transform

QUANTIZATION

Entropy
encoding

Signal

Sparse Transform



Signal
Sparse
Transform

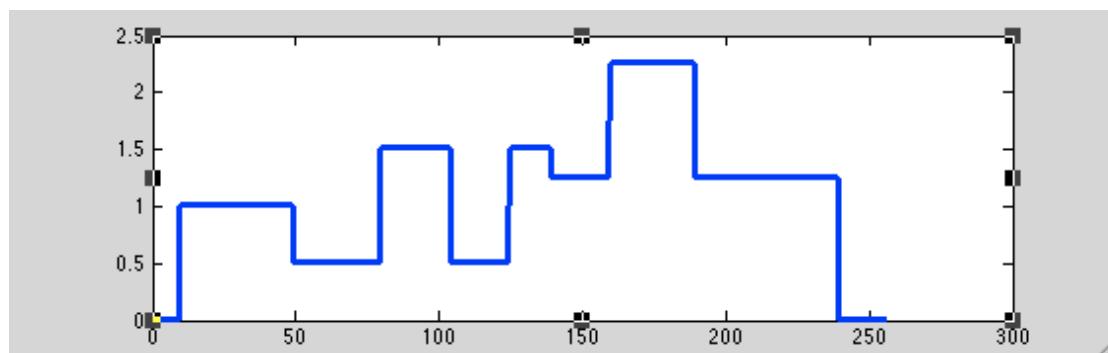
QUANTIZATION

Entropy
encoding

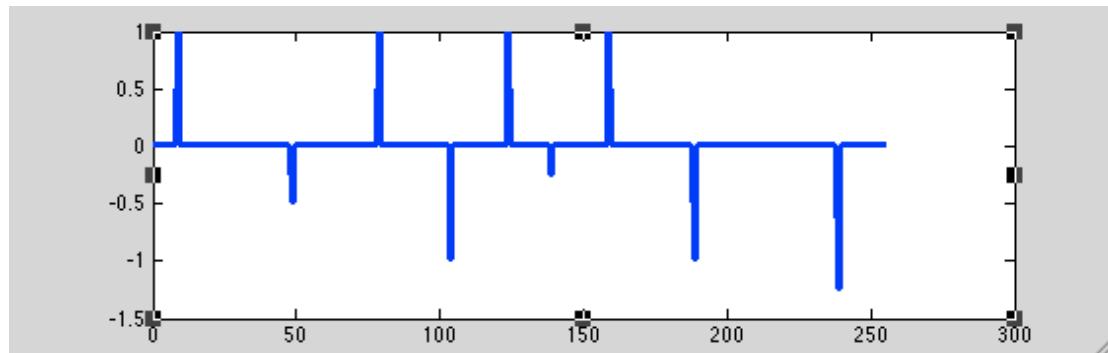
Signal

Sparse Transform

What sparsifying transform would you use here?



Difference



Signal
Sparse
Transform

QUANTIZATION

Entropy
encoding

Signal

S p a r s i t y

&

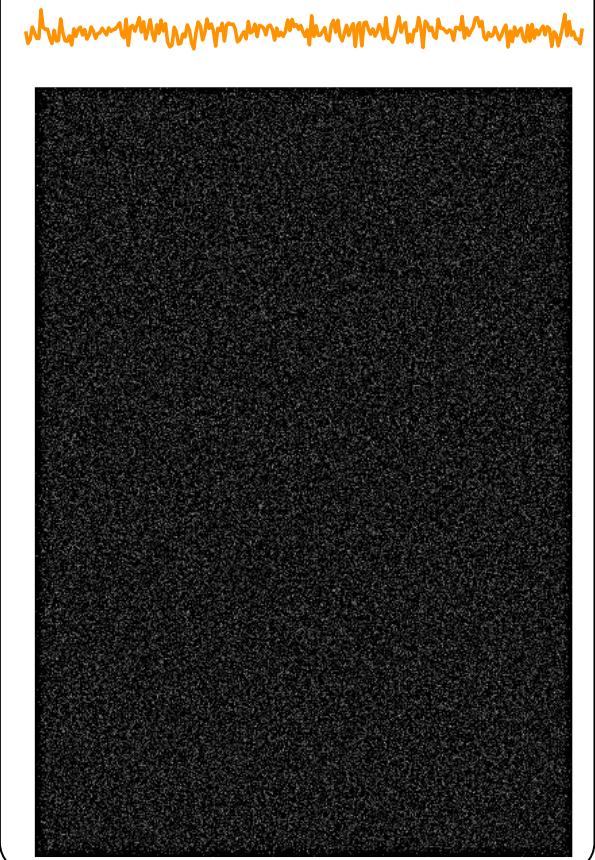
Compressibility

Sparsity and Noise

sparse

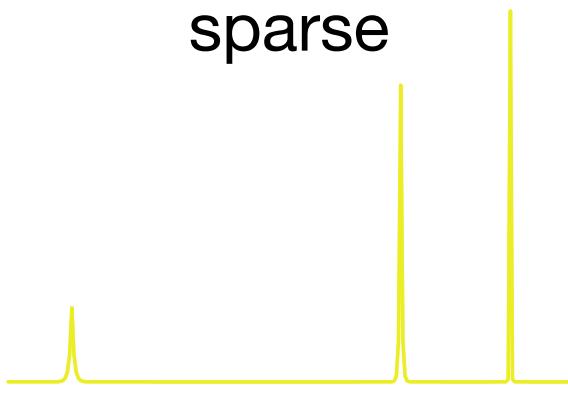


not sparse

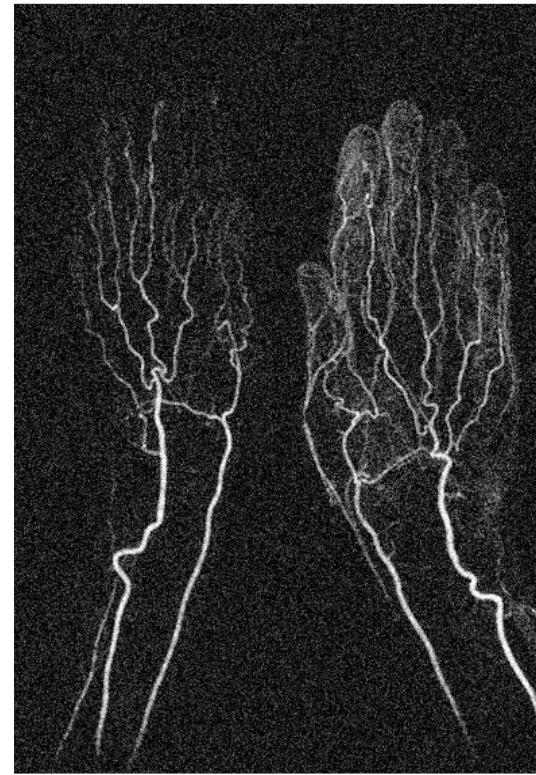
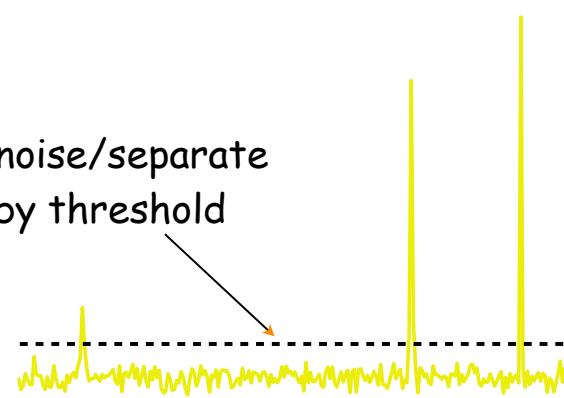


Sparsity and Noise

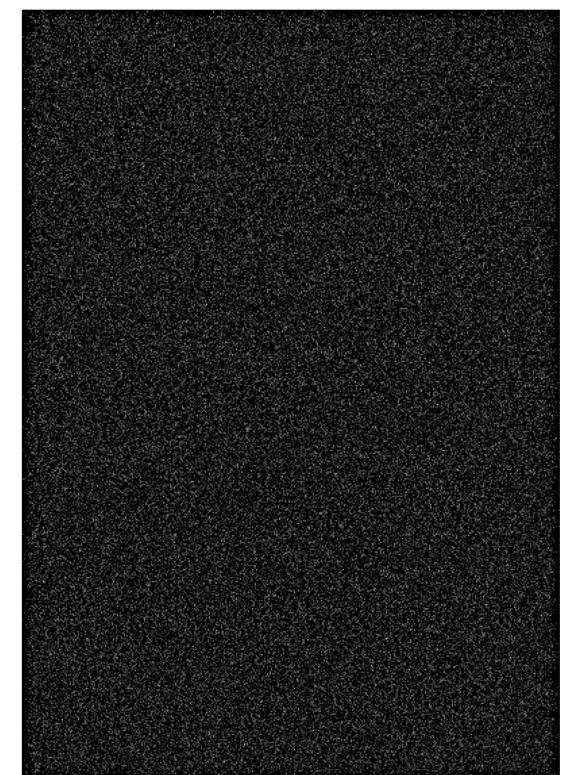
sparse

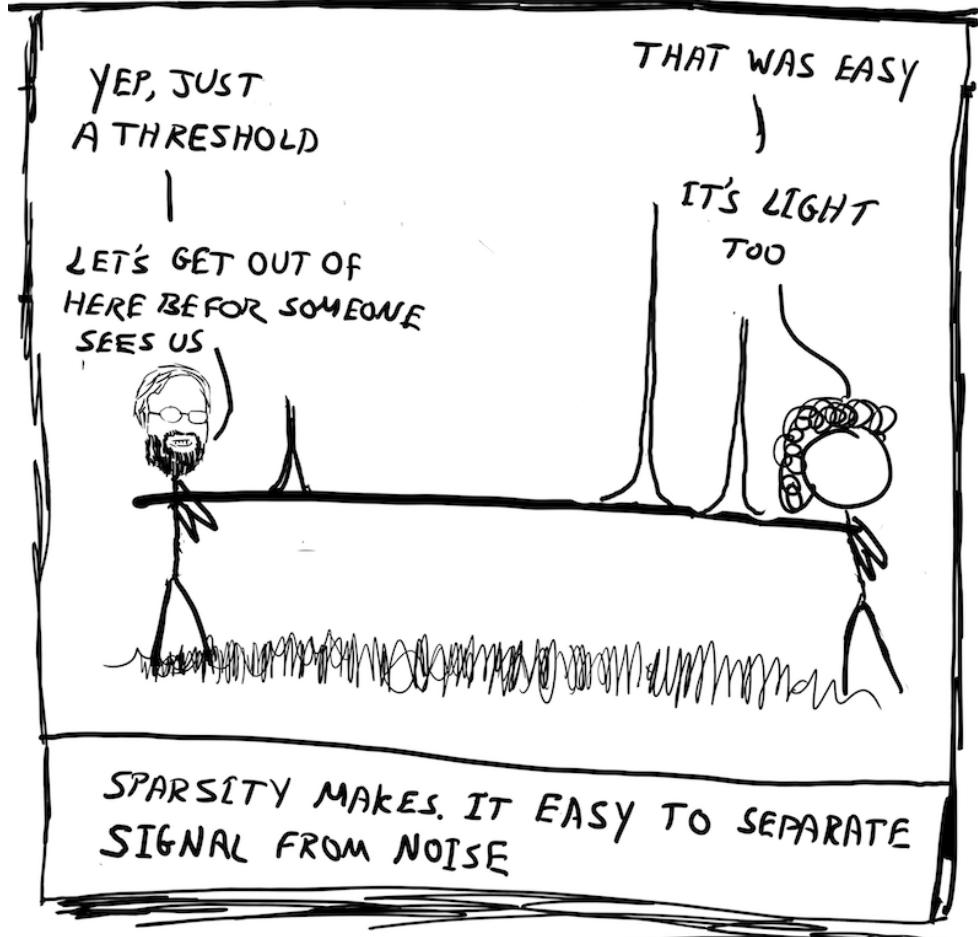
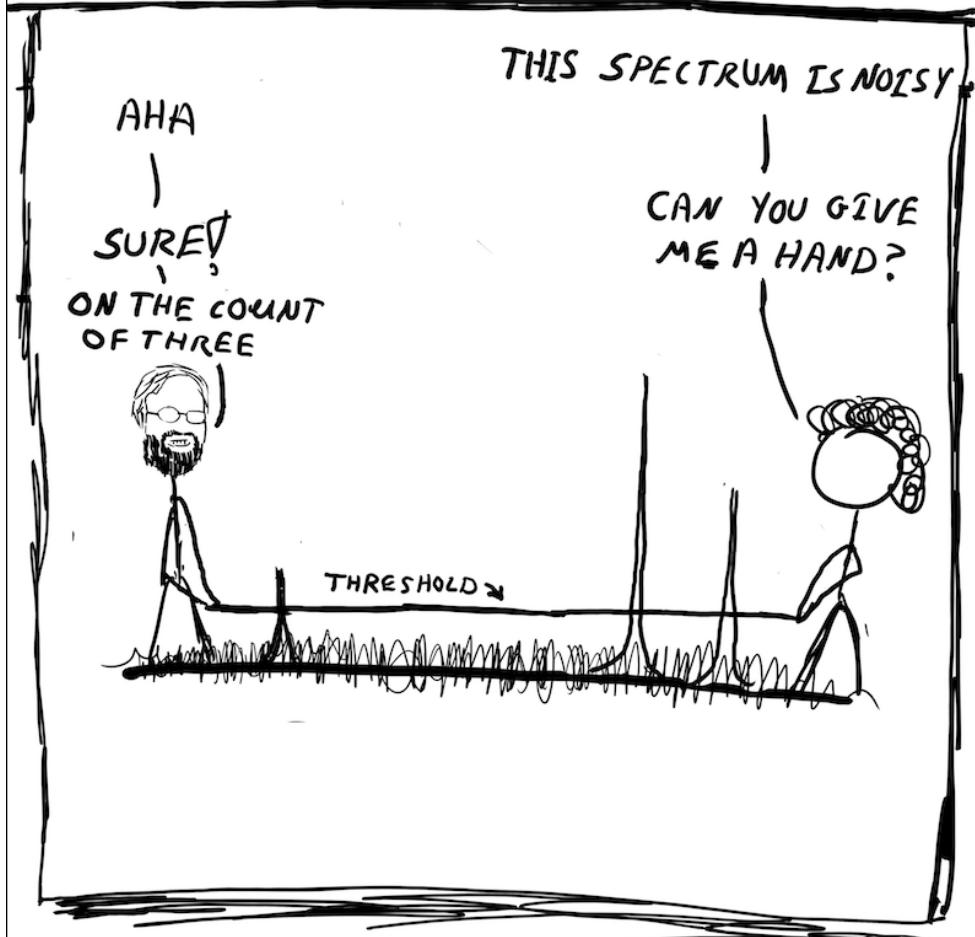


denoise/separate
by threshold



not sparse



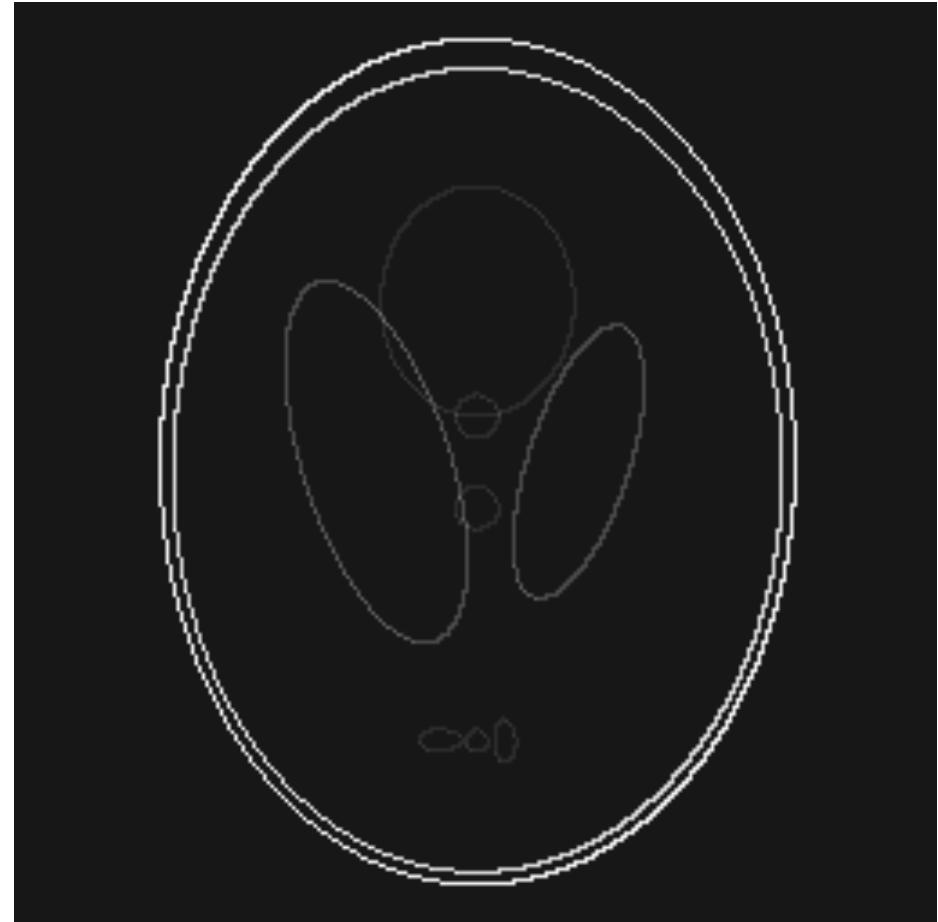


Transform Sparsity

not sparse

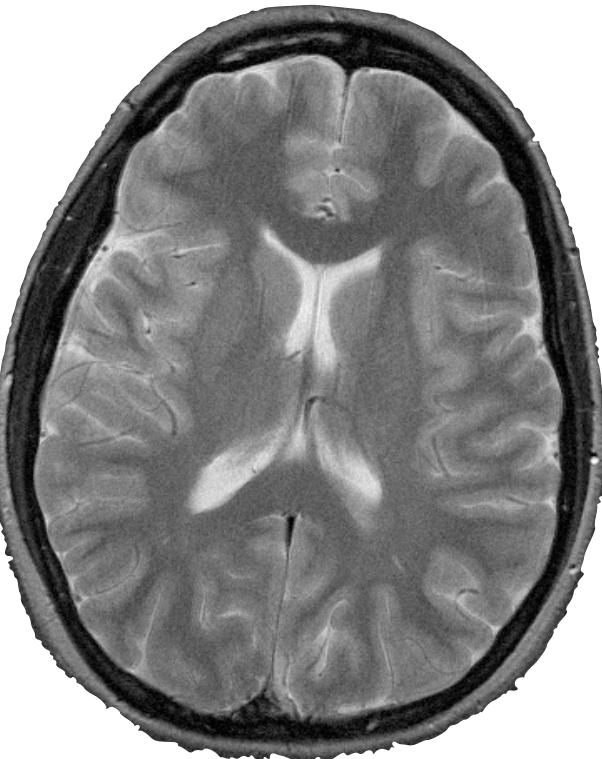


Sparse Edges



Transform Sparsity and Denoising

not sparse



sparse

wavelet transform

low-frequency

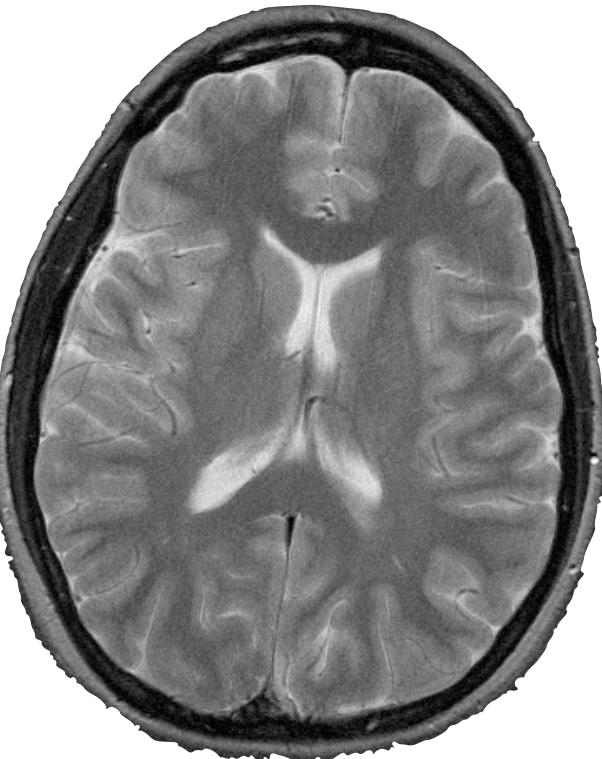
denoised

DL Donoho, I Johnstone Biometrika 1994;81(3):425-55

M. Lustig, EECS UC Berkeley

Transform Sparsity and Denoising

not sparse

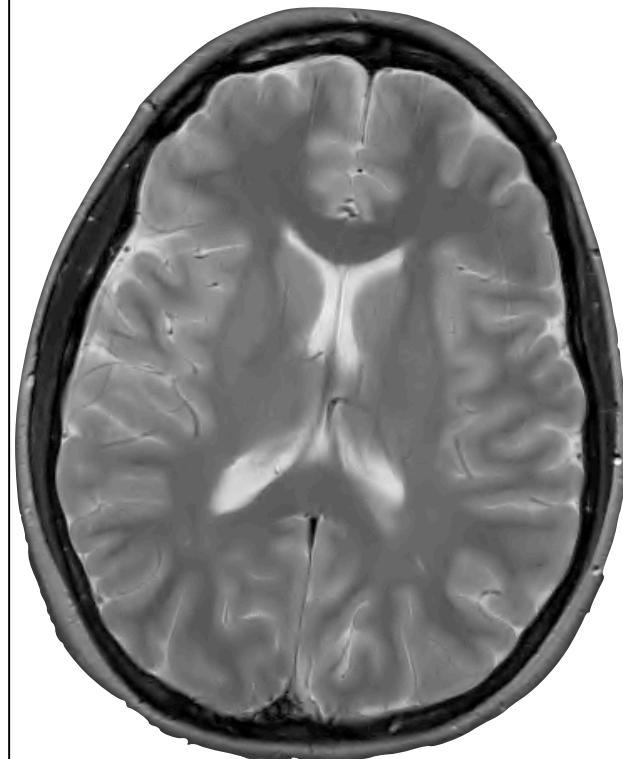


sparse

wavelet transform

low-frequency

denoised

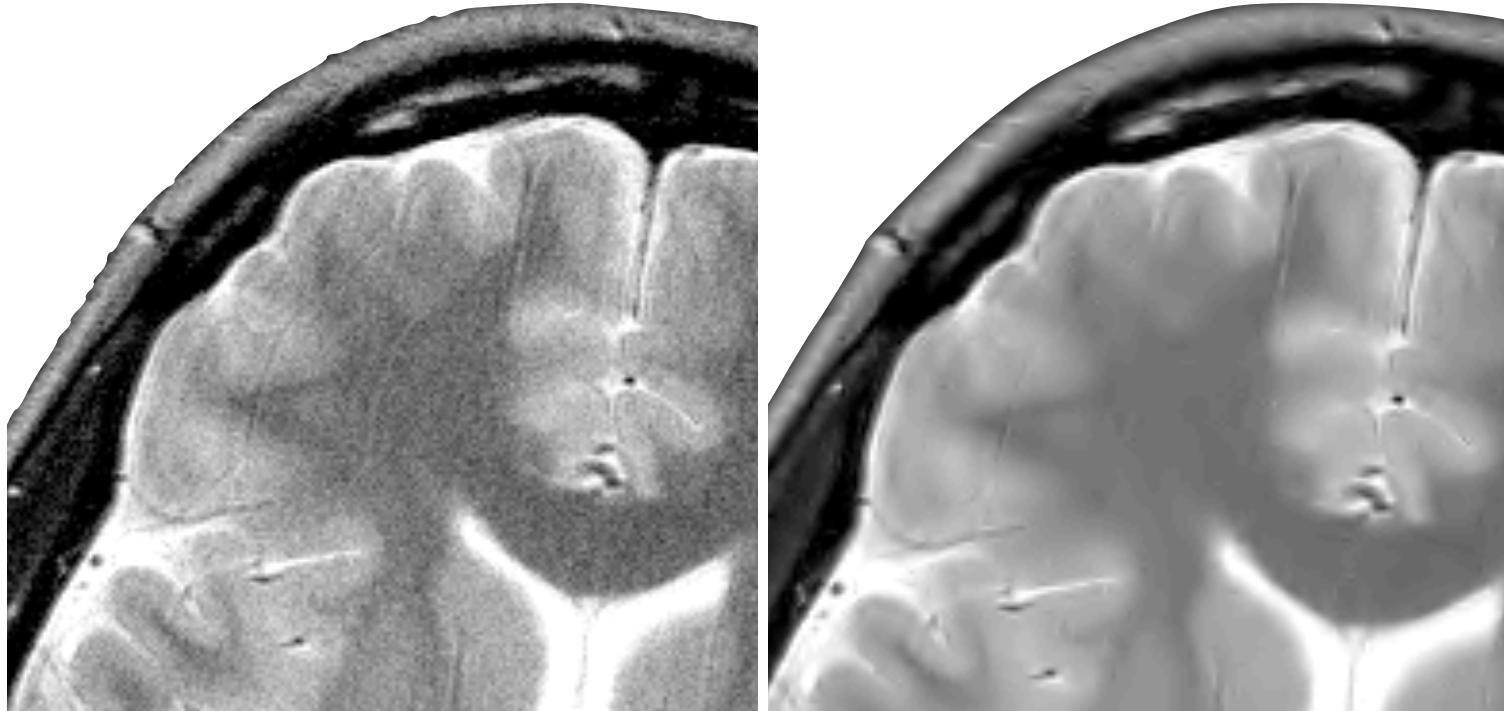


DL Donoho, I Johnstone Biometrika 1994;81(3):425-55

M. Lustig, EECS UC Berkeley

Transform Sparsity and Denoising

wavelet denoising

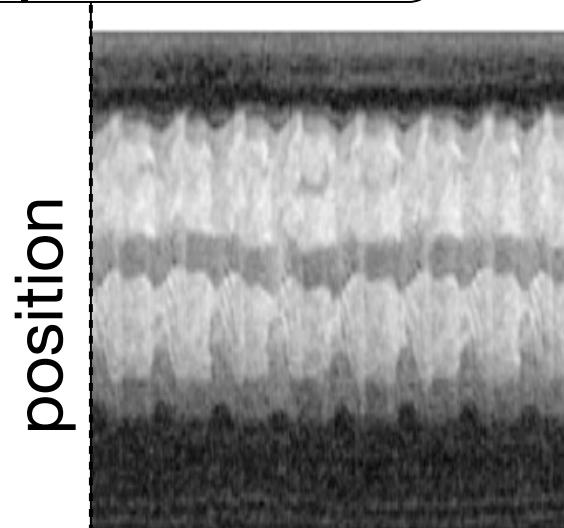
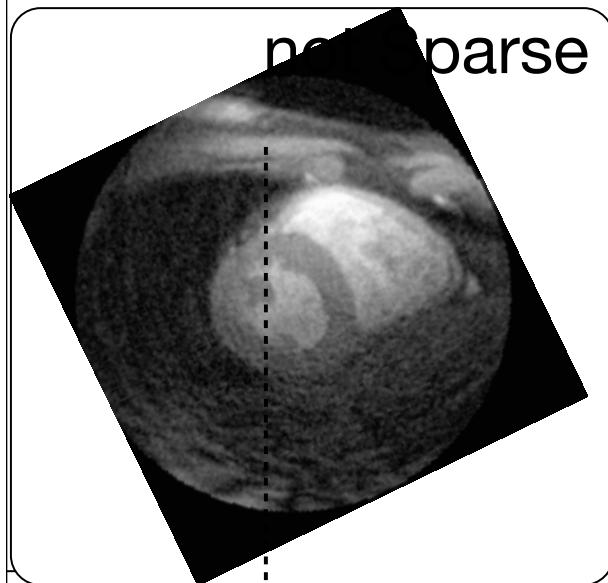


DL Donoho, I Johnstone Biometrika 1994;81(3):425-55

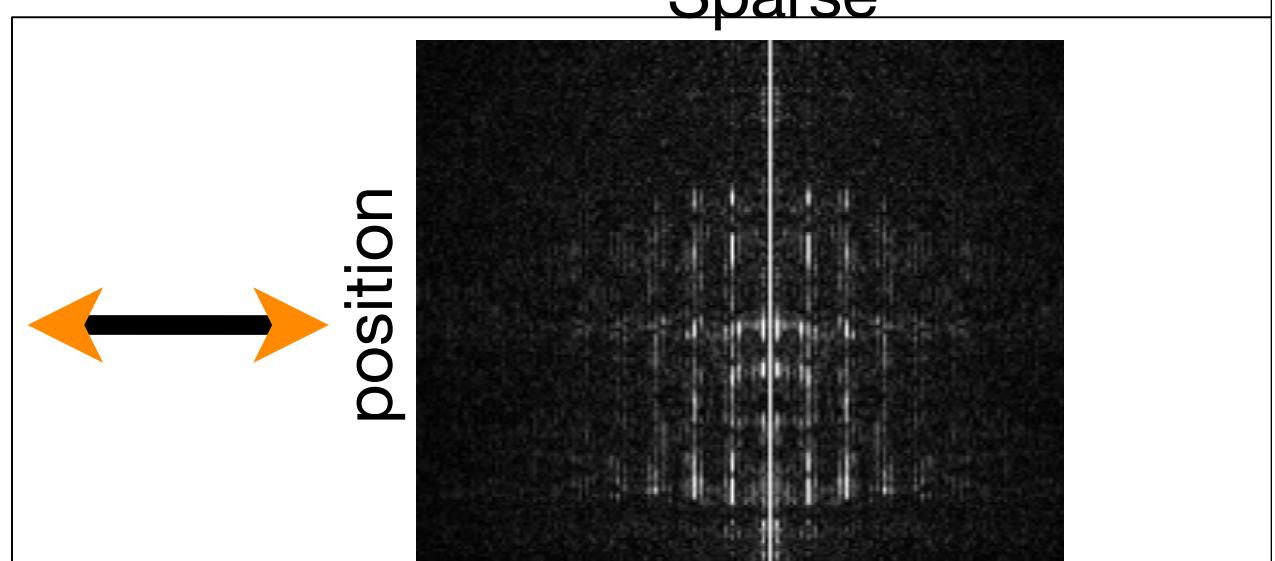
M. Lustig, EECS UC Berkeley

More Sparse Transforms

*Video courtesy of Juan Santos, Heart Vista



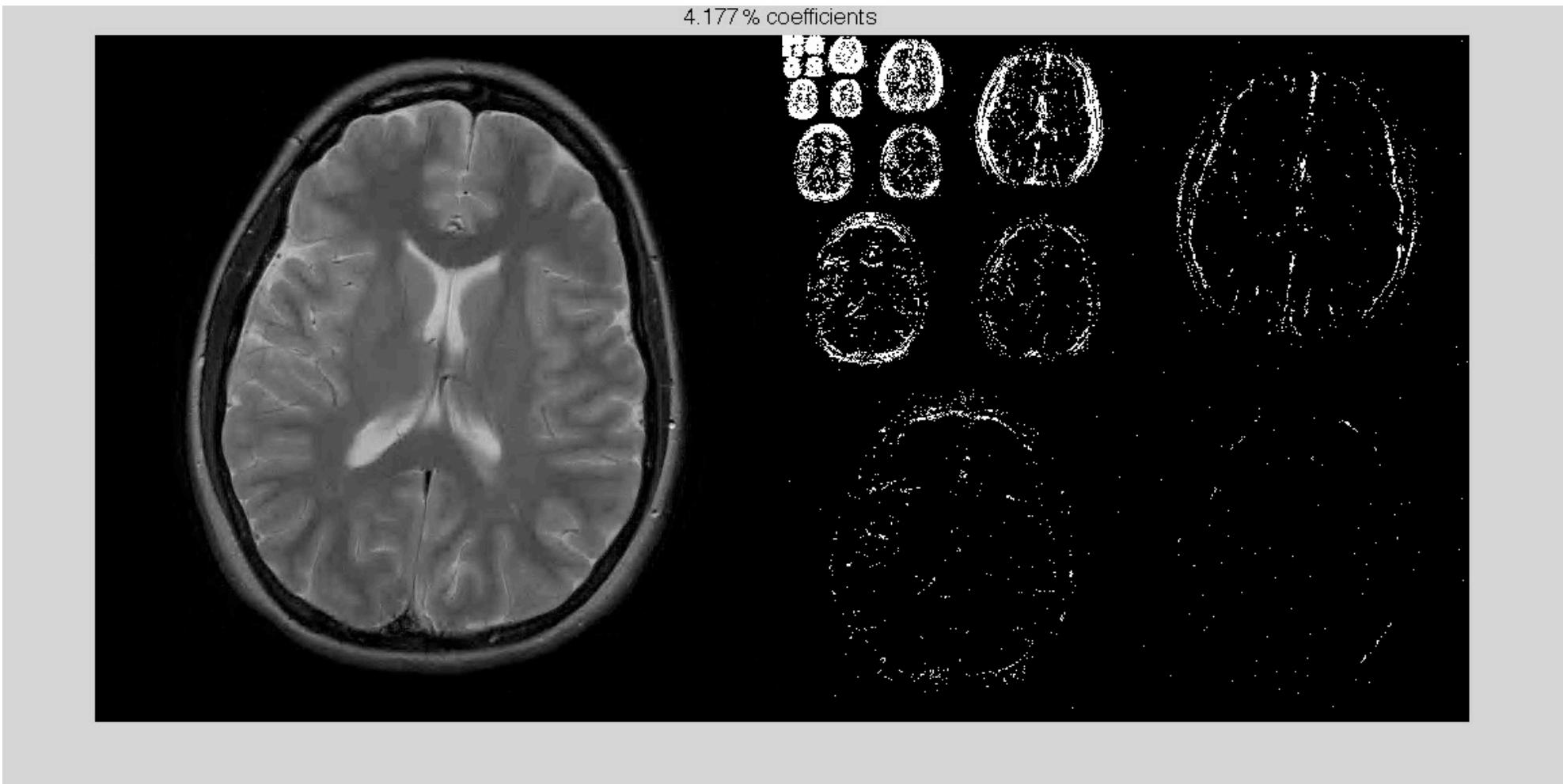
time



temporal frequency

Sparsity and Compression

- Only need to store non-zeros



From Samples to Measurements

- Shanon-Nyquist sampling
 - Worst case for ANY bandlimited data
- Compressive sampling (CS)

“Sparse signals statistics can be recovered from a small number of non-adaptive linear measurements”

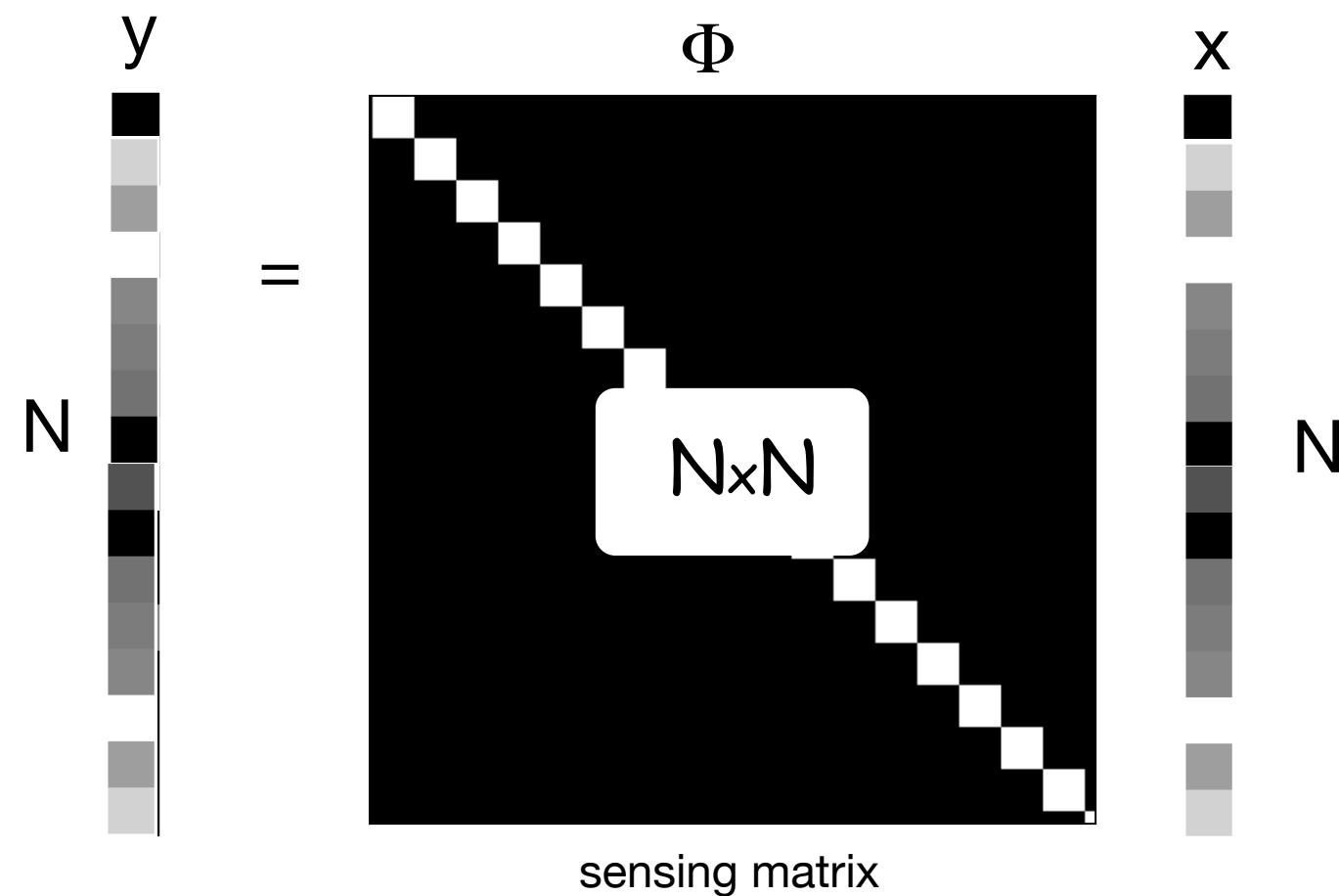
 - Integrated sensing, compression and processing.
 - Based on concepts of incoherency between signal and measurements



Traditional Sensing

- $x \in \mathbb{R}^N$ is a signal
- Make N linear measurements

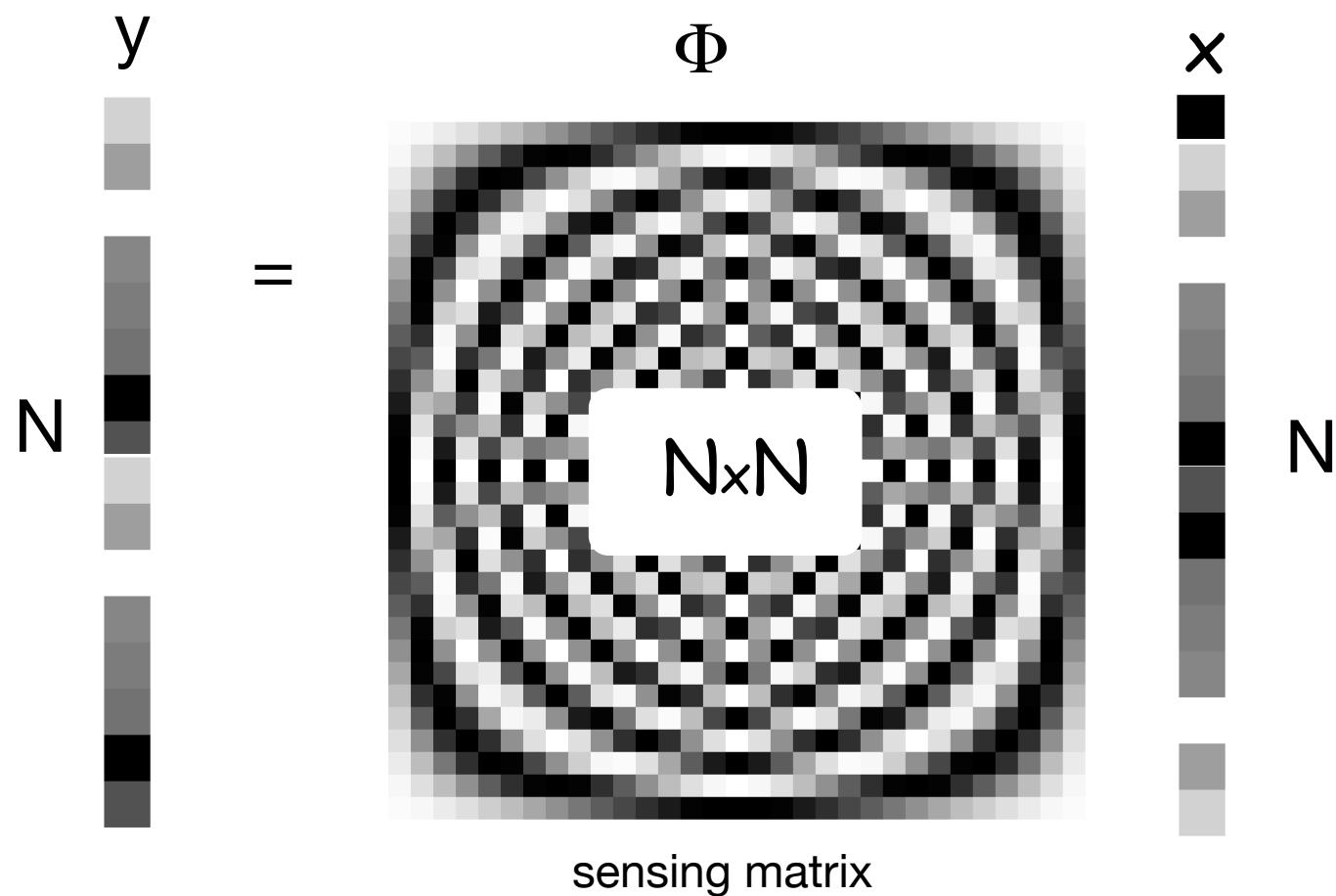
Desktop scanner/ digital camera sensing



Traditional Sensing

- $x \in \mathbb{R}^N$ is a signal
- Make N linear measurements

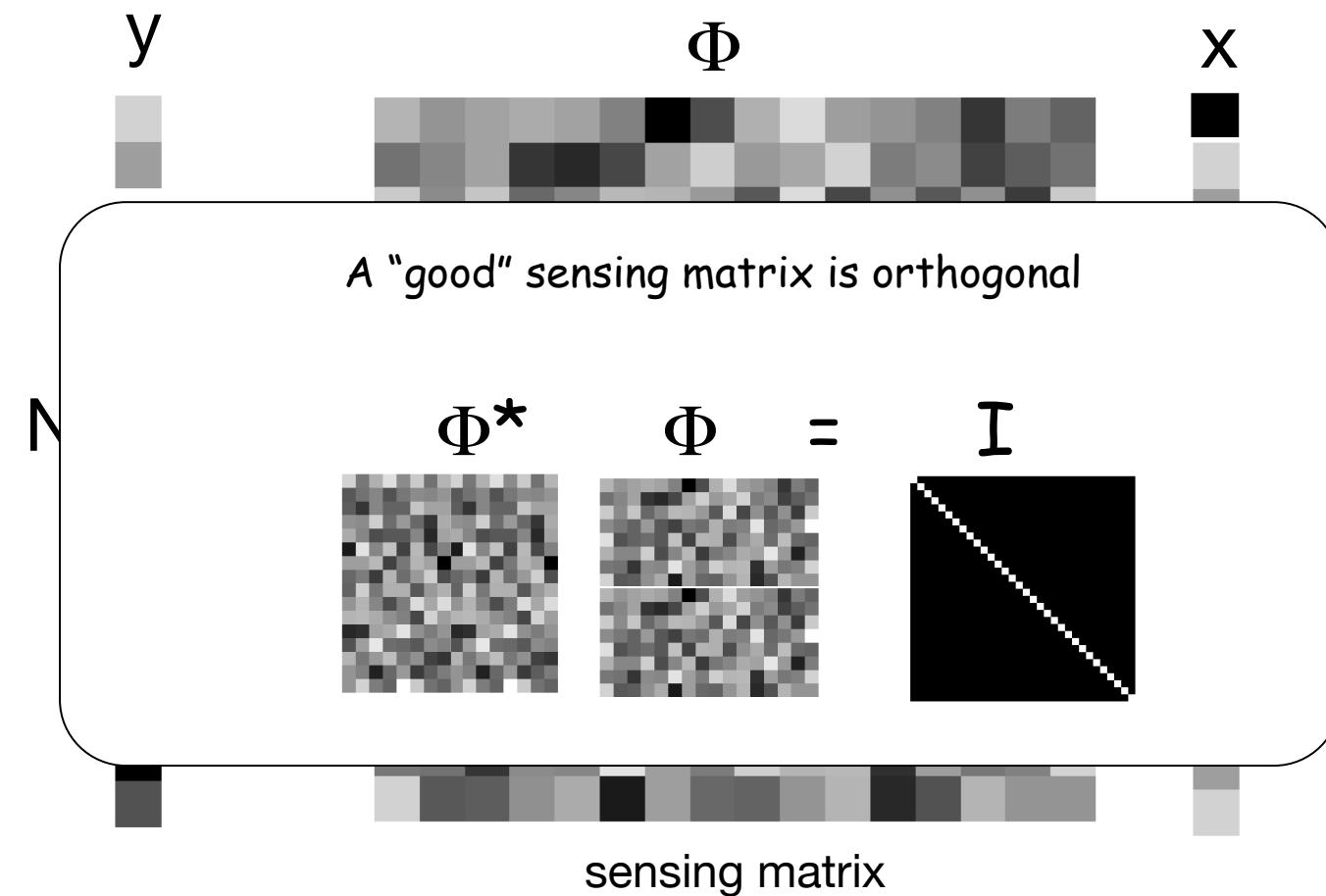
MRI Fourier Imaging



Traditional Sensing

- $x \in \mathbb{R}^N$ is a signal
- Make N linear measurements

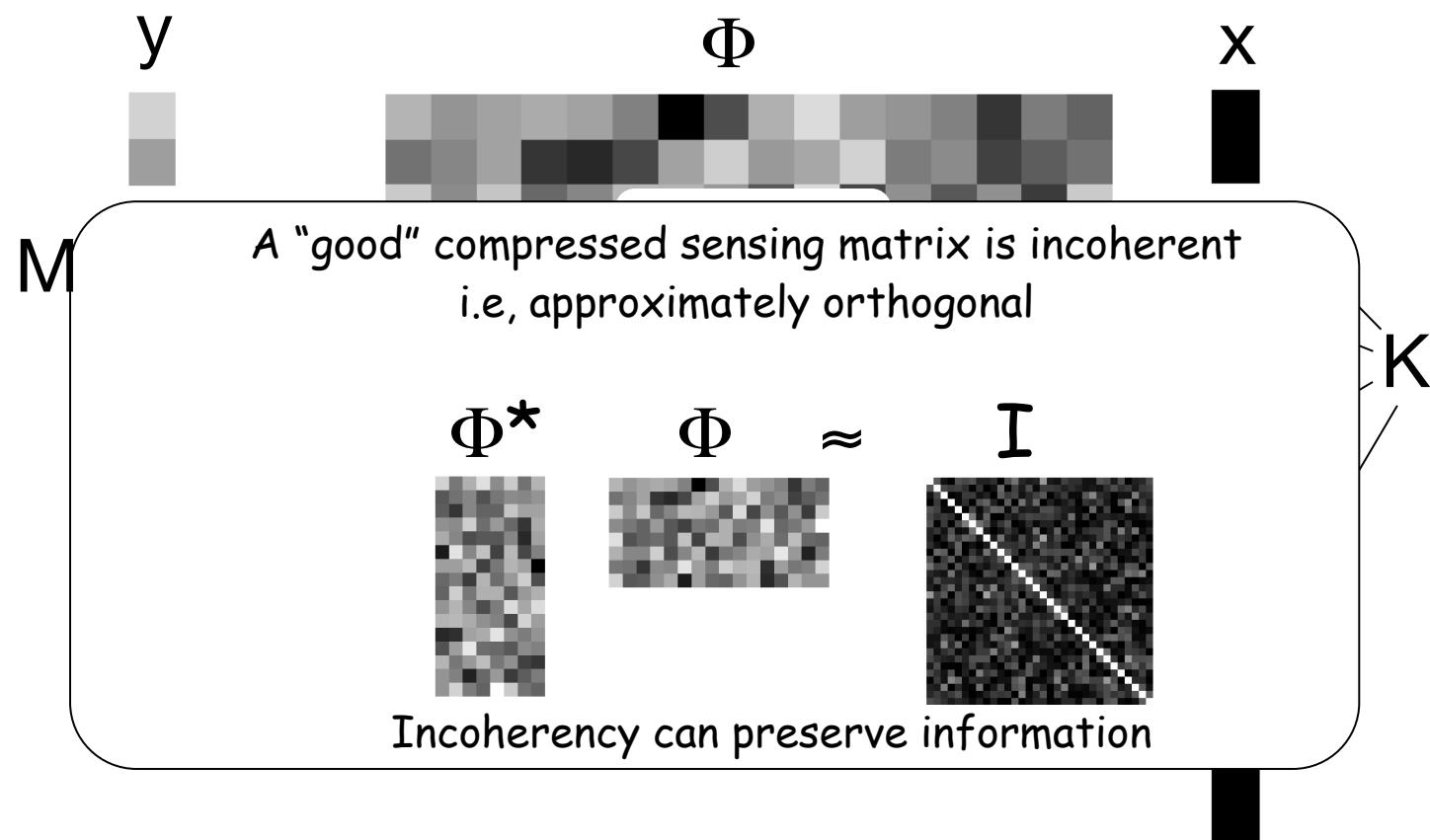
Arbitrary sensing



Compressed Sensing

(Candes, Romber, Tao 2006; Donoho 2006)

- $x \in \mathbb{R}^N$ is a K-sparse signal ($K \ll N$)
- Make M ($K < M < N$) incoherent linear projections



CS recovery

- Given $y = \Phi x$
find x
 - But there's hope, x is sparse!
- Under-determined

$$\begin{matrix} y \\ \vdots \\ \vdots \end{matrix} = \begin{matrix} \Phi \end{matrix} \begin{matrix} x \\ \vdots \\ \vdots \end{matrix}$$

CS recovery

- Given $y = \Phi x$
 - find x
 - But there's hope, x is sparse!
- }
- Under-determined

CS recovery

- Given $y = \Phi x$
 - find x
 - But there's hope, x is sparse!
- }
- Under-determined

minimize $\|x\|_2$

s.t. $y = \Phi x$

WRONG!

CS recovery

- Given $y = \Phi x$
 - find x
 - But there's hope, x is sparse!
- }
- Under-determined

minimize $\|x\|_0$

s.t. $y = \Phi x$

HARD!

CS recovery

- Given $y = \Phi x$
 - find x
 - But there's hope, x is sparse!
- }
- Under-determined

$$\text{minimize } \|x\|_1$$

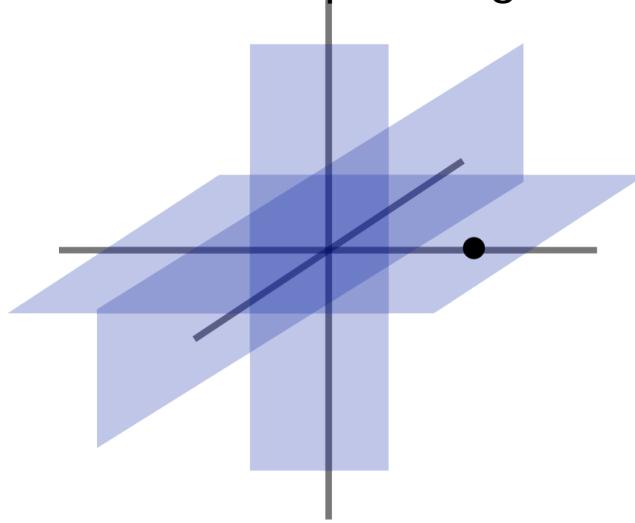
$$\text{s.t. } y = \Phi x$$

need $M \approx K \log(N) \ll N$

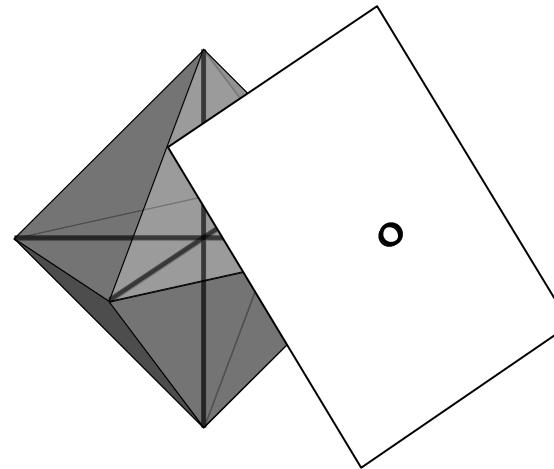
Solved by linear-programming

Geometric Interpretation

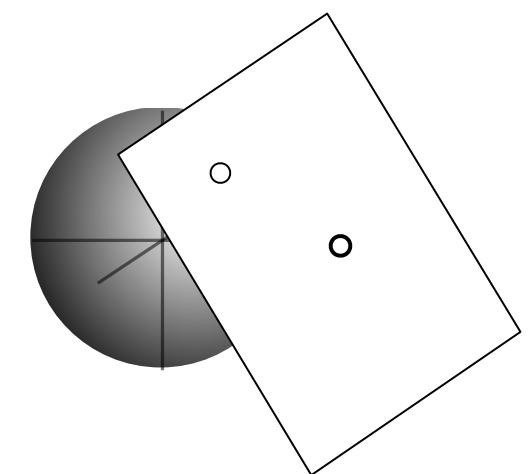
domain of sparse signals



minimum $\|x\|_1$



minimum $\|x\|_2$



$$\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [y_1]$$