EE123 Spring 2016
Discussion Section 4

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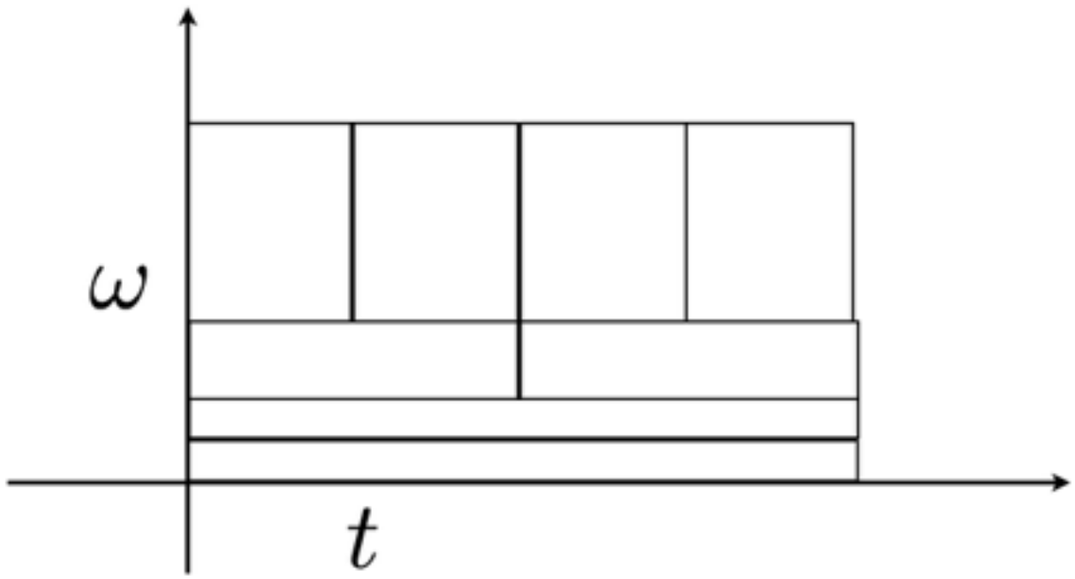
Plan

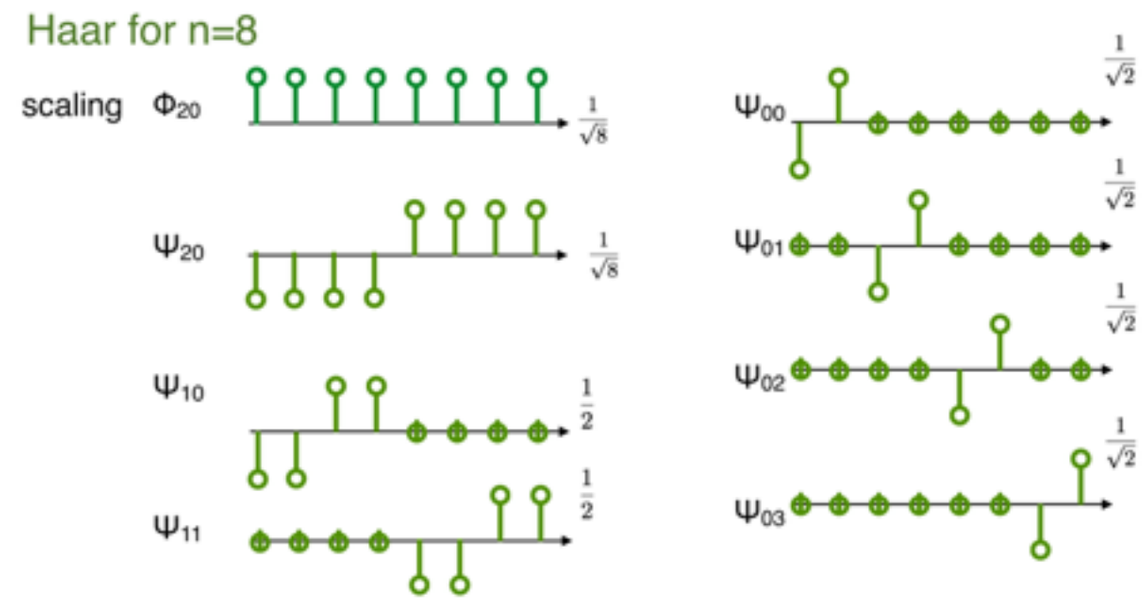
- Wavelet Transform Review
- Sampling

Three Views of the Wavelet Transform

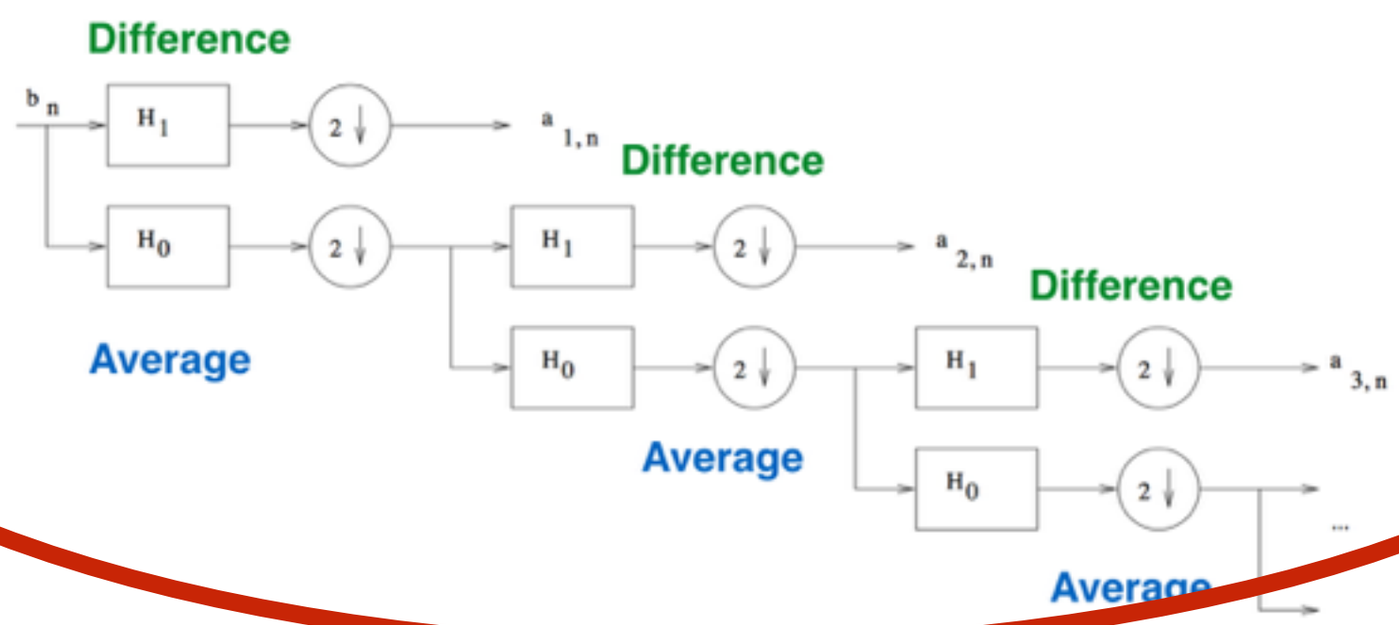
Multi-scale Time-Frequency Tiling



Wavelet Basis functions



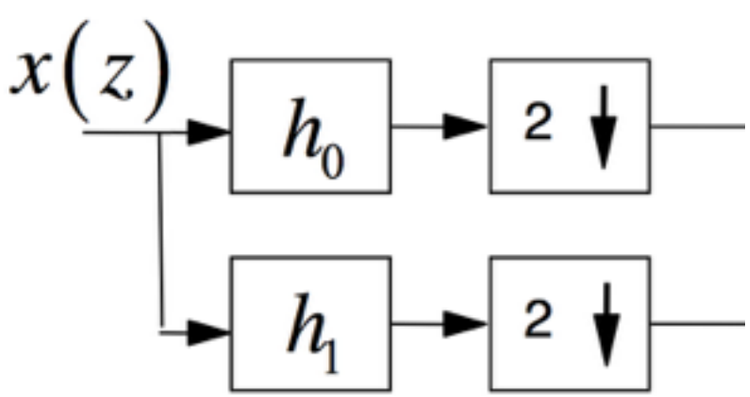
Fast Wavelet Transform



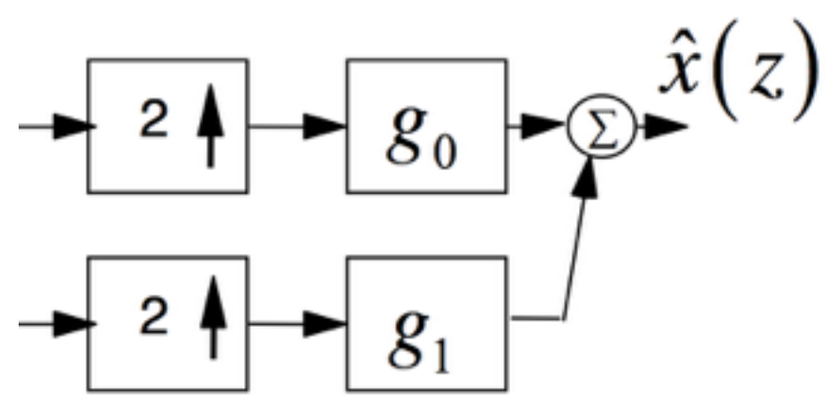
2-Channel Filter Bank

- Corresponds to one-level of the fast wavelet transform
- Once you understand it, you understand wavelet transform

Decomposition



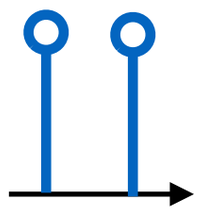
Reconstruction



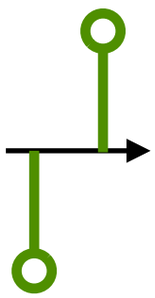
Haar filters

- For the Haar wavelet, the filters are:

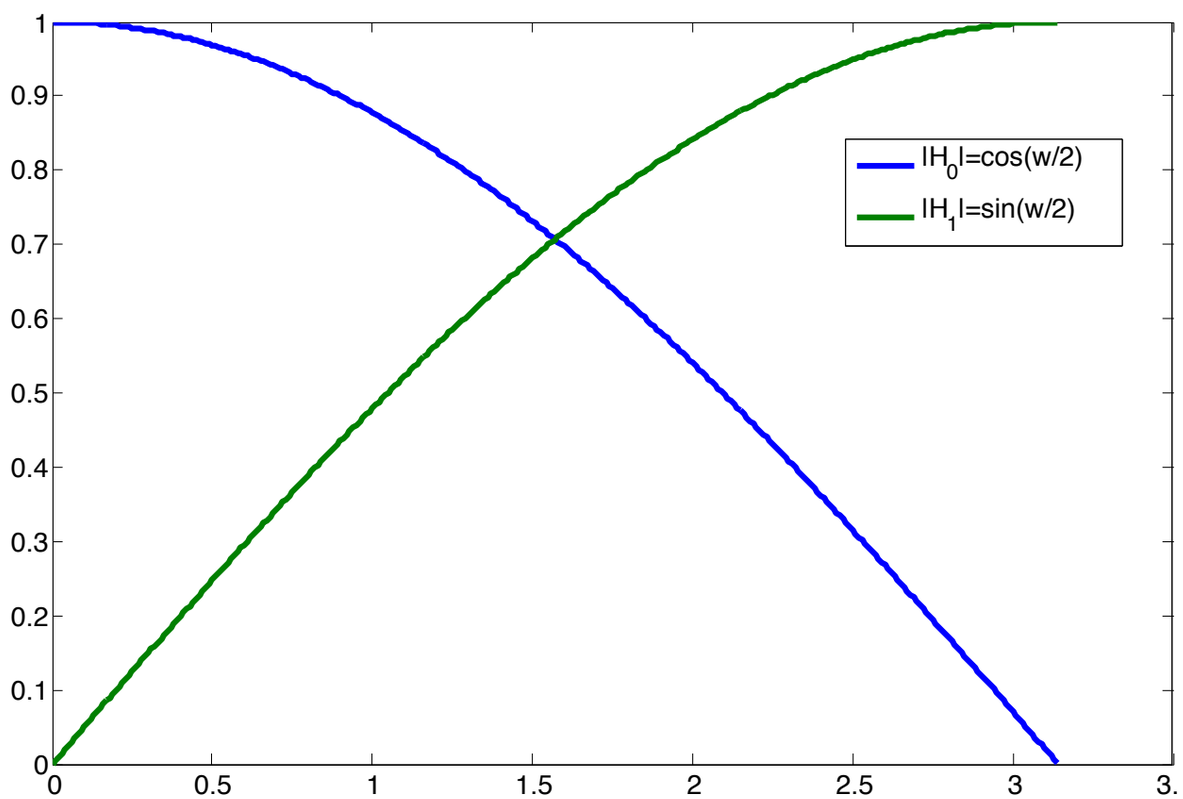
Average filter h_0



Difference filter h_1

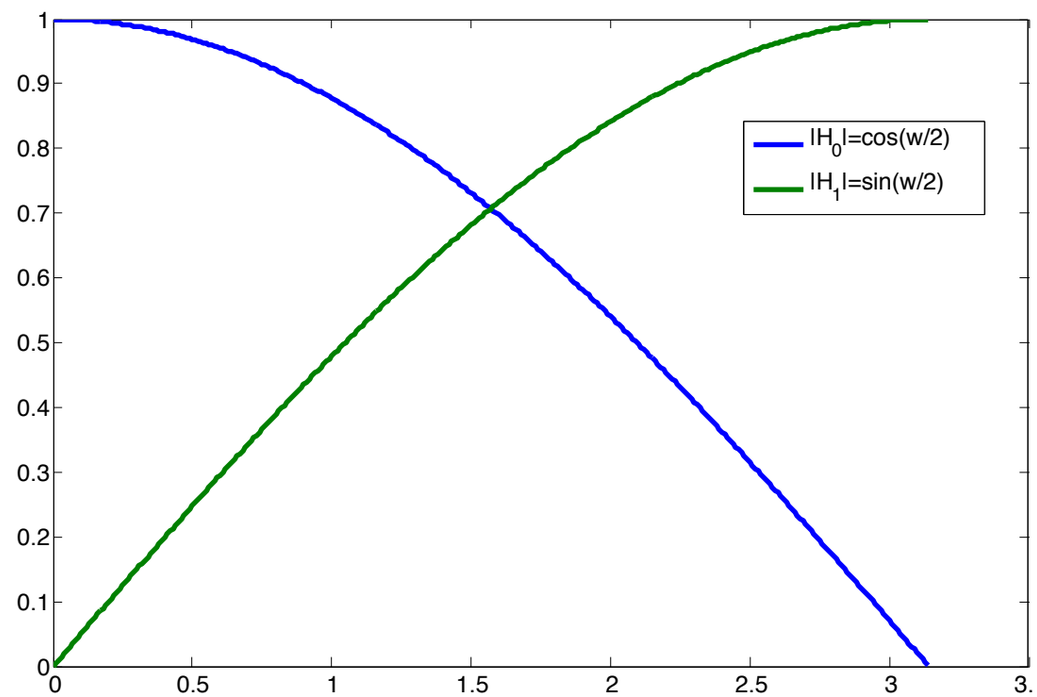


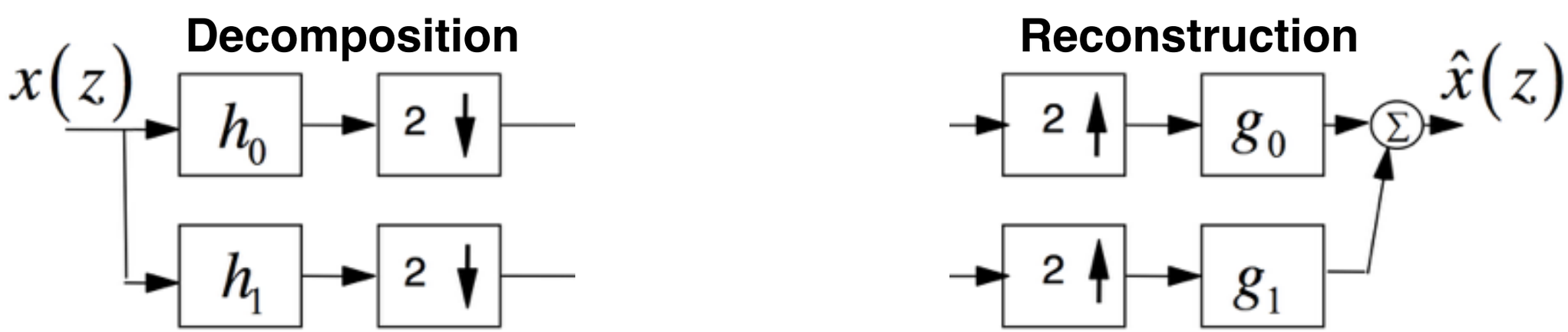
- And their magnitude responses are:



The filters have complementary frequency responses

Haar filters

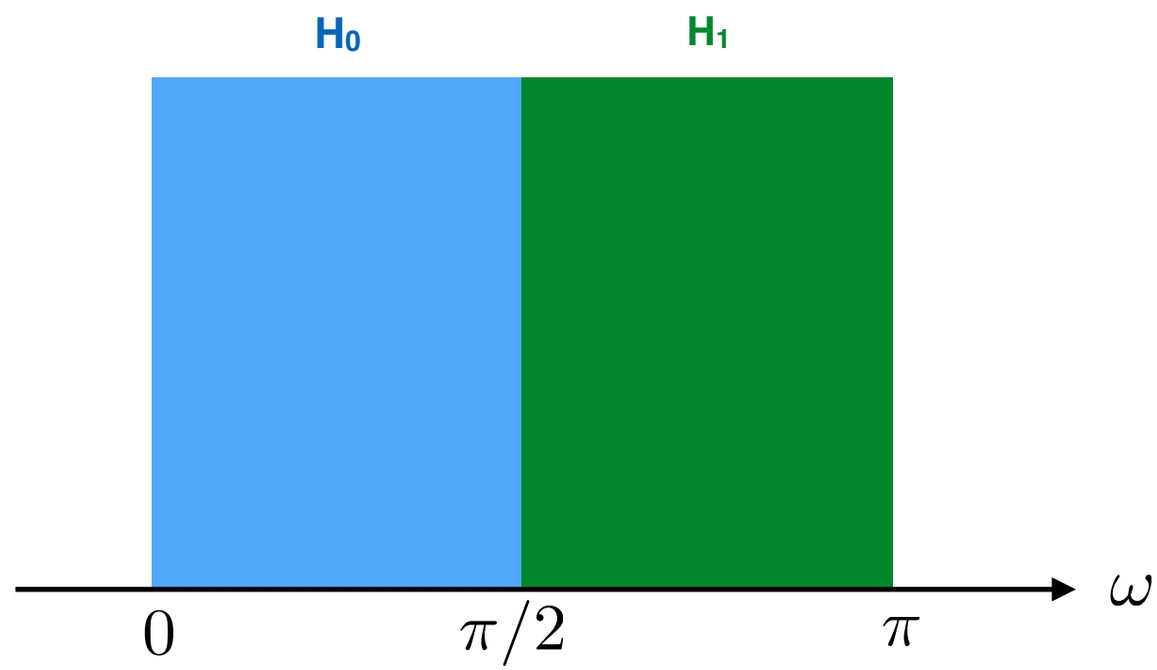




- Two downsamplers and upsamplers with crappy anti-aliasing and reconstruction filters

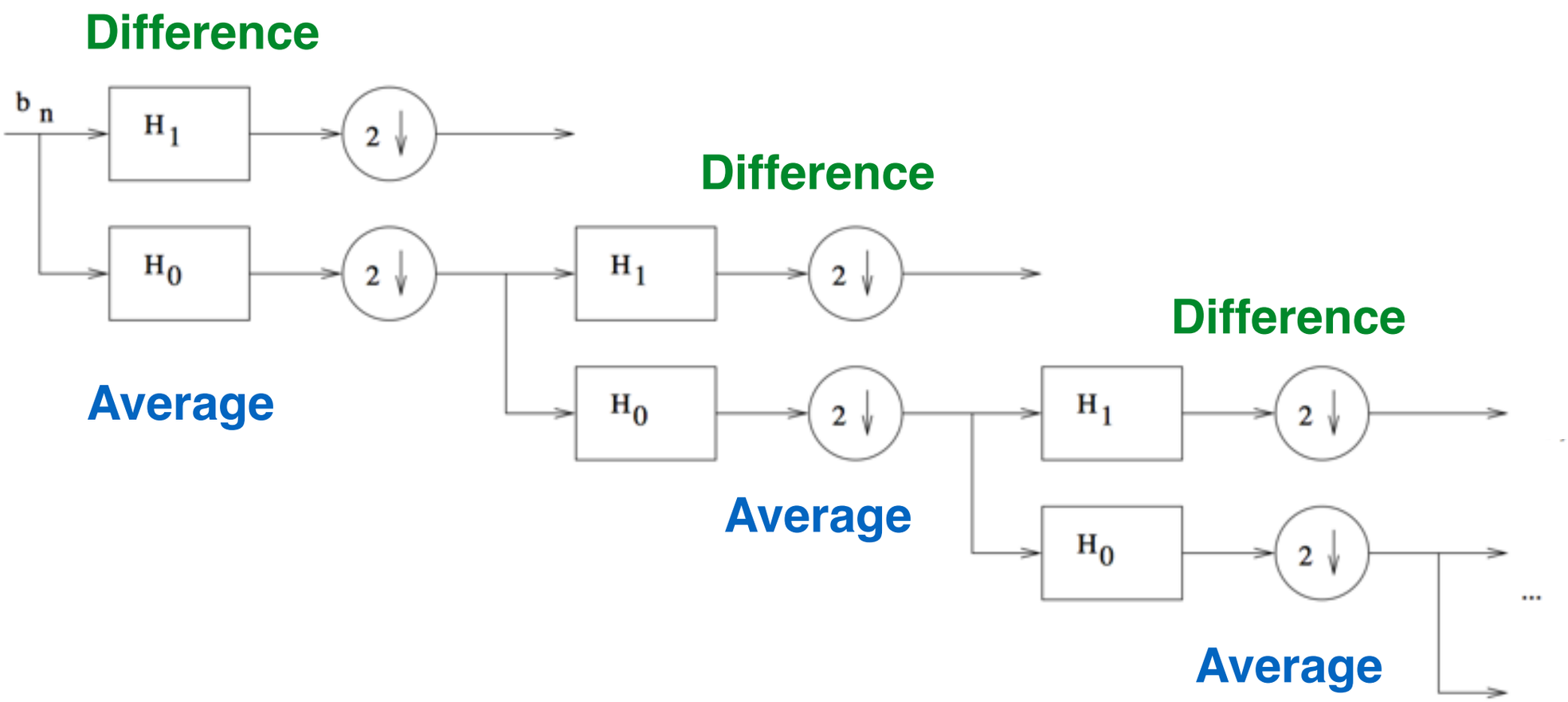
Sinc / Shannon Filters

- Ideal low pass/high pass filters also work



- Essentially the ideal downsamplers and upsamplers

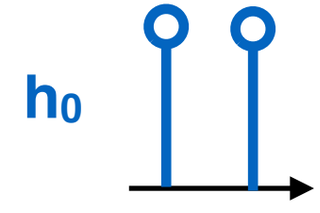
Fast Wavelet Transform

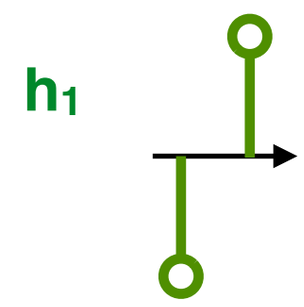


- Once you have the 2-channel filter bank
- Simply iterate on the average coefficients to obtain the wavelet transform

Haar decomposition example (ignoring scaling)

- x[n] = (1, 1, –2, –2, 1, 1, –1, 2)
  - **xd1 = (0, 0, 0, -3)**
- xa1 = (2, -4, 2, 1)
  - **xd2 = (-6, 1)**
  - xa2 = (-2, 3)
    - **xd1 = (-5)**
    - **xa1 = (1)**





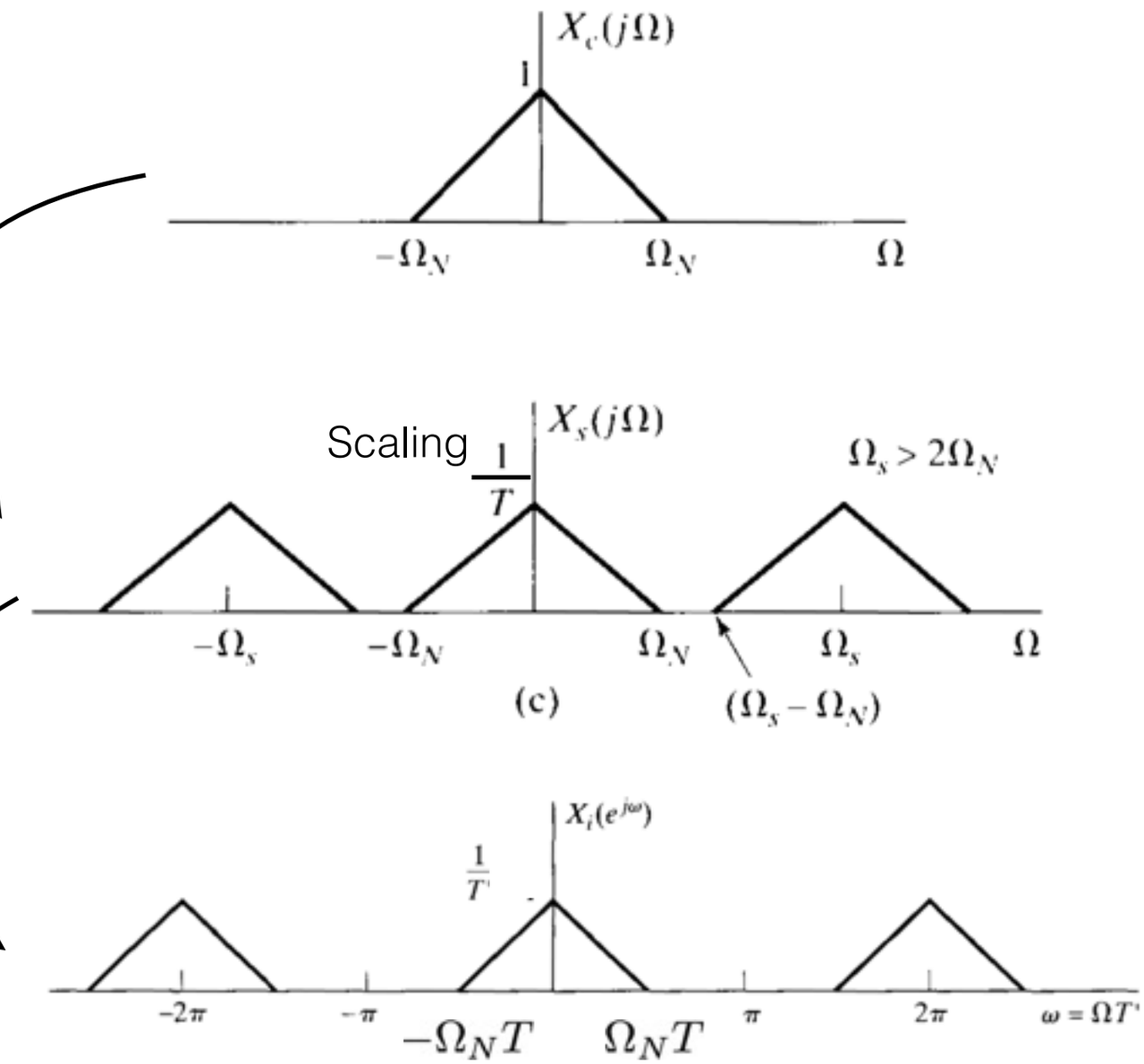
Plan

- Review of Wavelet Transform
- **Sampling**

Two steps to sample continuous signals

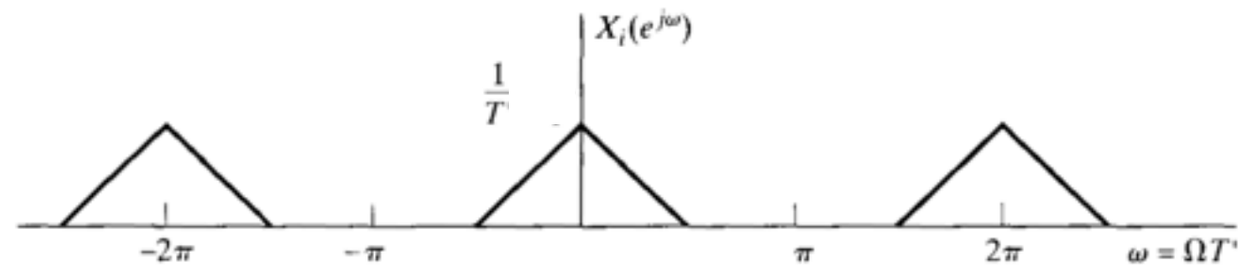
Replicate every Ω_s

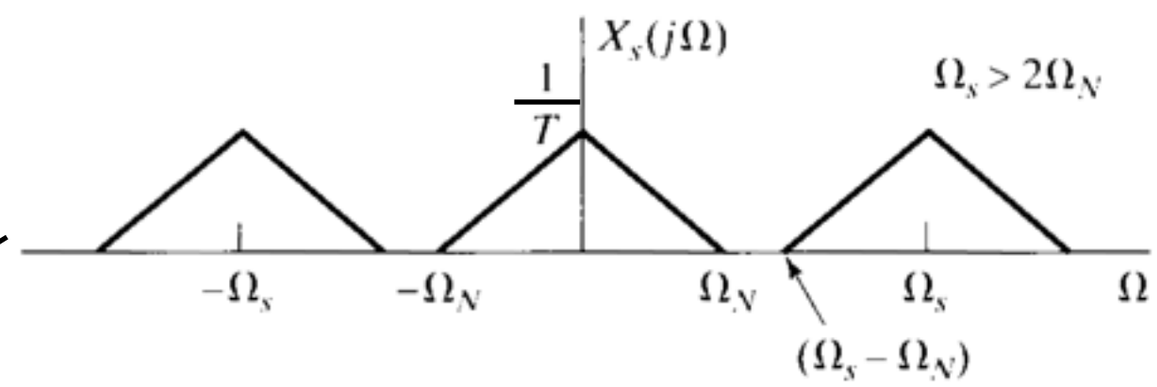
Set $\Omega_s \Rightarrow 2\pi$



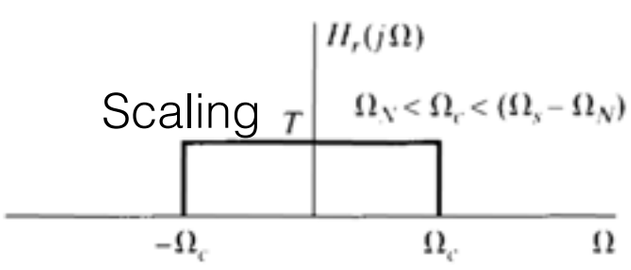
Two steps to reconstruct continuous signal

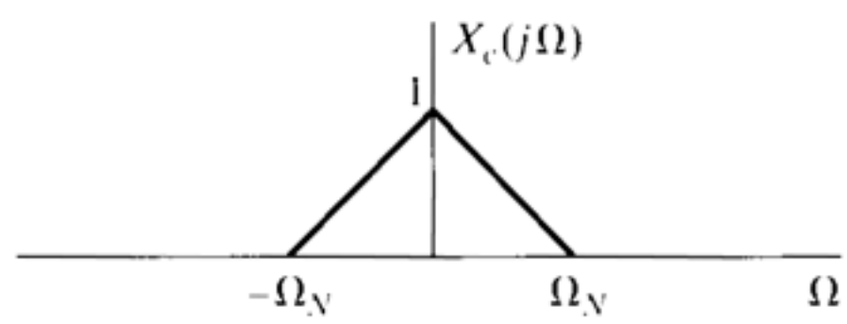
Set
$2\pi \Rightarrow \Omega_s$





Multiply by



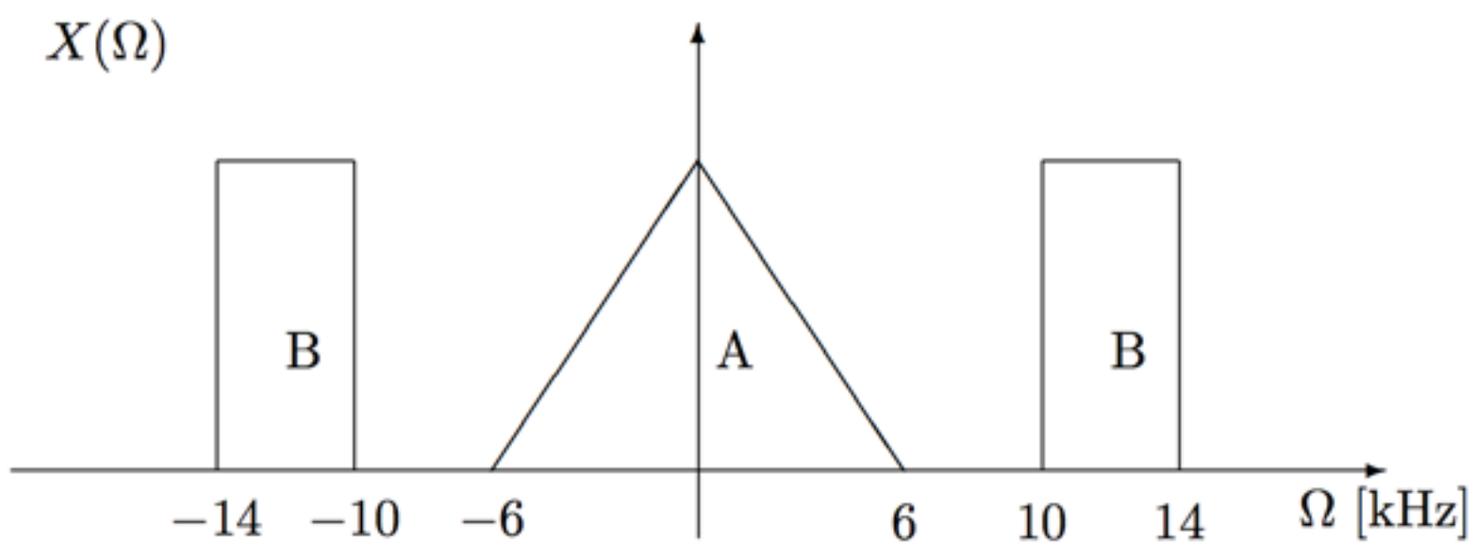


Problem 1

4. An analog signal, whose spectrum is shown below, is to be processed with a digital filter using ideal C/D and D/C converters (with no analog anti-aliasing filters).

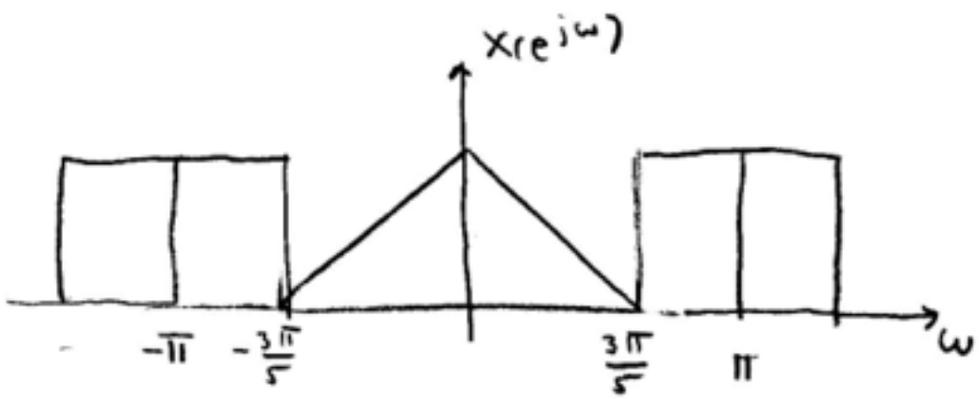
a) (8 points) What is the minimum sufficient sampling rate to extract portion A of the signal? Sketch the magnitude of the digital filter that would be used at this sampling rate.

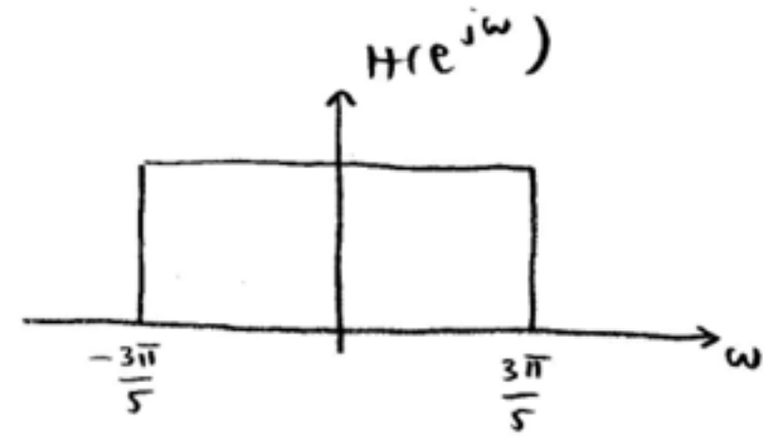
b) (7 points) Repeat for portion B of the signal.



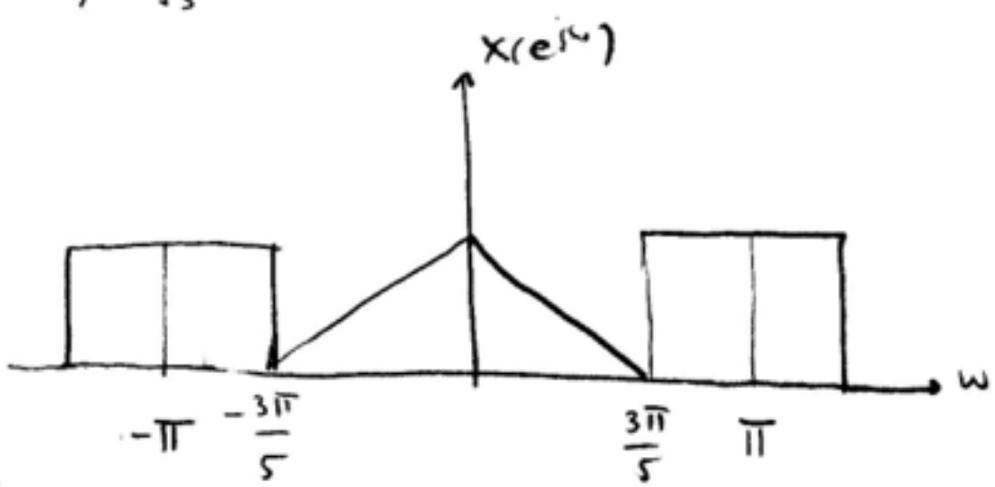
Solution 1

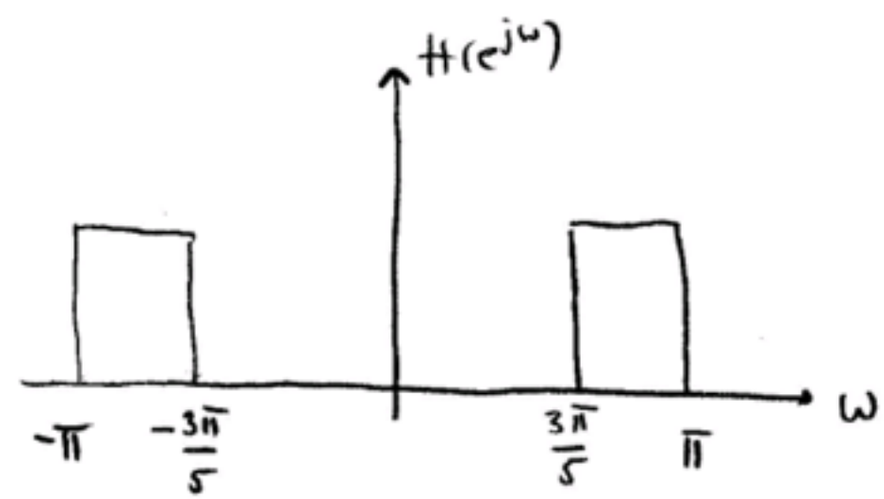
a) $f_s = 20\,kHz$





b) $f_s = 20\,kHz$





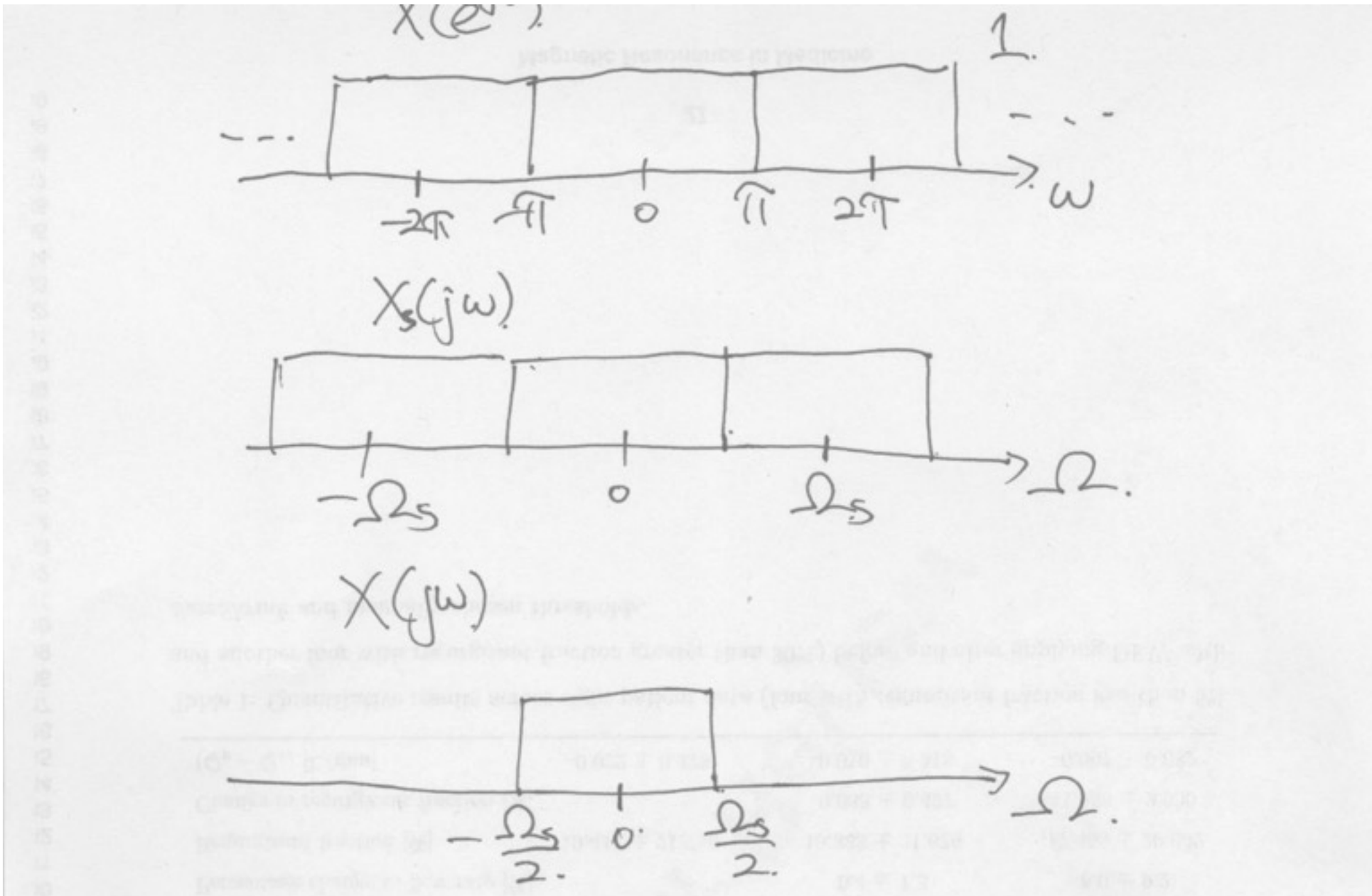
Question 2

- Suppose you start with a discrete delta function,

$$x[n] = \delta[n]$$

- What is the reconstructed continuous-time signal when you use the ideal reconstruction filter with sampling period T?

Solution 2



$\Rightarrow x(t)$ is a sinc.

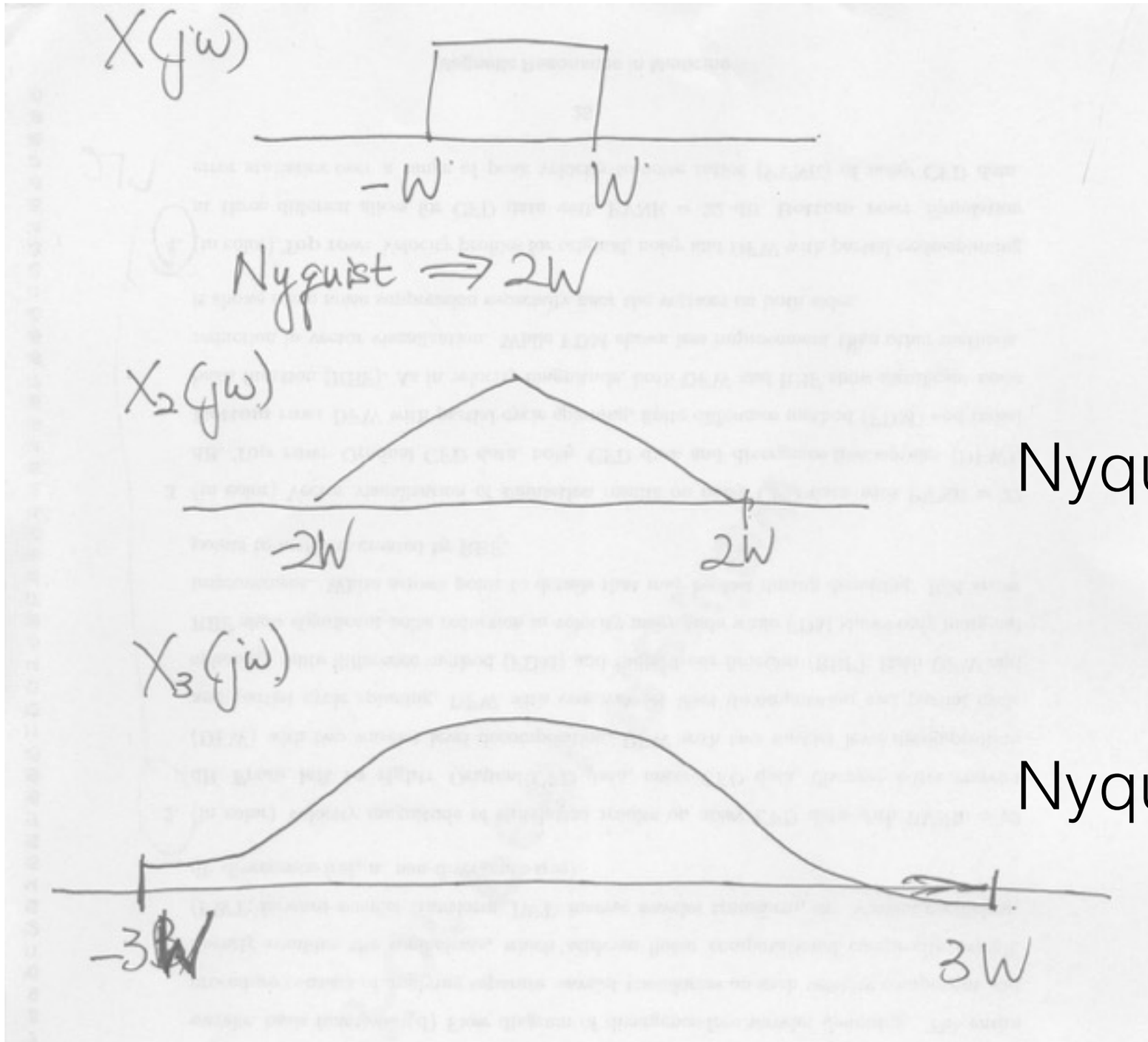
Question 3

- Consider a signal x(t) band-limited from -W to W.

Suppose you sample $x^2(t)$ and wish to reconstruct $x^2(t)$. At what rate must $x^2(t)$ be sampled?

Suppose you sample $x^3(t)$ and wish to reconstruct $x^3(t)$. At what rate must $x^3(t)$ be sampled?

Solution 3



Nyquist = 4W

Nyquist = 6W