Plan

- Wavelet Transform Review
- Sampling
Three Views of the Wavelet Transform

Multi-scale Time-Frequency Tiling

Wavelet Basis functions

Fast Wavelet Transform
2-Channel Filter Bank

- Corresponds to one-level of the fast wavelet transform
- Once you understand it, you understand wavelet transform
Haar filters

• For the Haar wavelet, the filters are:

Average filter $h_0$

Difference filter $h_1$

• And their magnitude responses are:

The filters have complementary frequency responses
Haar filters

- Two downsamplers and upsamplers with crappy anti-aliasing and reconstruction filters

\[ |H_{0}| = \cos(w/2) \]
\[ |H_{1}| = \sin(w/2) \]
Sinc / Shannon Filters

- Ideal low pass/high pass filters also work

- Essentially the ideal downsamplers and upsamplers
Fast Wavelet Transform

- Once you have the 2-channel filter bank
- Simply iterate on the average coefficients to obtain the wavelet transform
Haar decomposition example (ignoring scaling)

- $x[n] = (1, 1, -2, -2, 1, 1, -1, 2)$
- $x_{d1} = (0, 0, 0, -3)$
- $x_{a1} = (2, -4, 2, 1)$
  - $x_{d2} = (-6, 1)$
  - $x_{a2} = (-2, 3)$
    - $x_{d1} = (-5)$
    - $x_{a1} = (1)$
Plan

- Review of Wavelet Transform
- Sampling
Two steps to sample continuous signals

Replicate every $\Omega_s$

Set $\Omega_s \Rightarrow 2\pi$
Two steps to reconstruct continuous signal

Set

\[ 2\pi \Rightarrow \Omega_s \]

Multiply by

Scaling

\[ \Omega_N < \Omega_s < (\Omega_s - \Omega_N) \]
Problem 1

4. An analog signal, whose spectrum is shown below, is to be processed with a digital filter using ideal C/D and D/C converters (with no analog anti-aliasing filters).

   a) (8 points) What is the minimum sufficient sampling rate to extract portion A of the signal? Sketch the magnitude of the digital filter that would be used at this sampling rate.

   b) (7 points) Repeat for portion B of the signal.
Solution 1

a) $f_s = 20 \text{ kHz}$

b) $f_s = 20 \text{ kHz}$
Question 2

- Suppose you start with a discrete delta function,

\[ x[n] = \delta[n] \]

- What is the reconstructed continuous-time signal when you use the ideal reconstruction filter with sampling period T?
Solution 2

$X(e^{j\omega})$

$X(j\omega)$

$X(j\omega)$

$\Rightarrow x(t) \text{ is a sinc.}$
Question 3

- Consider a signal $x(t)$ band-limited from $-W$ to $W$.

Suppose you sample $x^2(t)$ and wish to reconstruct $x^2(t)$. At what rate must $x^2(t)$ be sampled?

Suppose you sample $x^3(t)$ and wish to reconstruct $x^3(t)$. At what rate must $x^3(t)$ be sampled?
Solution 3

Nyquist = 4W

Nyquist = 6W