Lecture 25

EE 123 Discussion Section 7

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Based on notes from Giulia Fanti, Frank Ong, Michael Lustig
Announcements

• HW8 – to be released later today, due Friday

• HW7 self grade – due today

• Lab 4 – due Thursday March 31

• Midterm 2 – Monday, April 4
  • Same policies as those for Midterm 1

• HAM radios passed out during lab sessions

• Questions?
Plan

• Poles and zeros

• Midterm 2 topics
## Poles and zeros

<table>
<thead>
<tr>
<th></th>
<th>Magnitude</th>
<th>Phase</th>
<th>Group delay</th>
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</thead>
<tbody>
<tr>
<td>Poles</td>
<td>push up</td>
<td>go down</td>
<td>delay (positive)</td>
</tr>
<tr>
<td>Zeros</td>
<td>push down</td>
<td>go up</td>
<td>advance (negative)</td>
</tr>
</tbody>
</table>
Problem 1

Consider

\[ h_1[n] = \begin{cases} 
1 & |n| \leq 1 \\
0 & \text{else}
\end{cases} \]

- Compute its z-transform and plot the poles and zeros
- Sketch its magnitude response

What about the pole zero diagram for

\[ h_1[n] = \begin{cases} 
1 & |n| \leq 3 \\
0 & \text{else}
\end{cases} \]
Solution 1
Problem 2

Match the following magnitude response to their pole-zero plot
Solution 2

1. Butterworth
2. Chebyshev Type II
3. Chebyshev Type I
4. Elliptic
Problem 3

b) (10 points) Match the frequency responses below with the transfer functions:

\[ H_1(z) = 0.25 \frac{1-z^{-2}}{1-0.75z^{-1}+0.5z^{-2}} \]
\[ H_2(z) = 0.75 \frac{1+z^{-2}}{1+0.75z^{-2}} \]
\[ H_3(z) = \frac{1+z^{-1}+z^{-2}}{1+\frac{2}{3}z^{-1}+\frac{1}{3}z^{-2}} \]
\[ H_4(z) = 0.2(1+z^{-1}+z^{-2}+z^{-3}+z^{-4}). \]
Solution 3

b) (10 points) Match the frequency responses below with the transfer functions:

\[ H_1(z) = 0.25 \frac{1 - 0.5z^{-2}}{1 - 0.75z^{-1} + 0.5z^{-2}} \quad H_2(z) = 0.75 \frac{1 + z^{-2}}{1 + 0.75z^{-2}} \]

\[ H_3(z) = \frac{\frac{1}{3} + \frac{2}{3}z^{-1} + z^{-2}}{1 + \frac{2}{3}z^{-1} + \frac{5}{3}z^{-2}} \quad H_4(z) = 0.2(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}) \]
Problem 4

Consider the problem of reconstructing the signal from only its Fourier magnitude $|H(e^{jw})|^2$

a  How does the pole-zero plot of $|H(e^{jw})|^2$ look like compared to $H(e^{jw})$?
   Hint: the z-transform of $|H(e^{jw})|^2$ is $H(z)H^*(1/z)$

b  If the system $h[n]$ is causal and stable, can you uniquely recover $h[n]$ from $|H(e^{jw})|^2$?

C  Assume that $h[n]$ is causal and stable and that, in addition, you know that the system function has the form

$$H(z) = \frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

for some finite $a_k$. Can you uniquely recover $h[n]$ from $|H(e^{jw})|^2$?
Solution 4

b) $h[n]$ causal and stable does not help
   The above example is a counterexample.

c) $h[n]$ causal and stable $+$ all pole system
   can uniquely recover the signal.
   because

   Only solution $H(z)$
Problem 5

Consider the linear time-invariant discrete-time system represented by the rational system function

\[
H(z) = \frac{1}{1 - \alpha z^{-N}}
\]

a) How many poles are there in this system?

b) Write the corresponding difference equation and sketch the system (i.e., adders, multipliers, and delays).

c) If the sampling rate is 16 kHz, \( N = 16000 \), \( \alpha = .5 \), and the input is a short speech segment of someone saying “ba”, what would the output sound like (assuming all the necessary machinery like antialiasing filters, A/D, D/A, etc.)?

d) For \( N=4 \), \( \alpha \) slightly less than 1, sketch the frequency response.
a) \( H(z) = \frac{1}{1-\alpha z^{-N}} \)

Poles: \( z_k = \alpha e^{j\frac{2\pi}{N}} \)  \( k = 0, \ldots, N-1 \)

b) \( y[n] - \alpha y[n-N] = x[n] \)
Solution 5

c) Assuming "ba" lasts less than 1 second. That means it takes less than 16,000 sample time. Since 
\[ y[n] = x[n] + ax[y[n-1]] \]
\[ \Rightarrow \text{ba ba ba ba ba ba} \ldots \]

d) Magnitude \( \frac{1}{1-x} \)
Midterm 2 topics

- Time-frequency tiling
- Time-frequency decomposition
- Sampling
- Resampling
- Multi-rate
- Polyphase decomposition
- Non-ideal sampling
- Filter design, TBW
- Phase response
- Min-phase, all-pass, linear-phase systems

HW8 and previous midterms
4.53. Consider the analysis–synthesis system shown in Figure P4.53-1. The lowpass filter $h_0[n]$ is identical in the analyzer and synthesizer, and the highpass filter $h_1[n]$ is identical in the analyzer and synthesizer. The Fourier transforms of $h_0[n]$ and $h_1[n]$ are related by

$$H_1(e^{j\omega}) = H_0(e^{j(\omega+\pi)}).$$

(a) If $X(e^{j\omega})$ and $H_0(e^{j\omega})$ are as shown in Figure P4.53-2, sketch (to within a scale factor) $X_0(e^{j\omega})$, $G_0(e^{j\omega})$, and $Y_0(e^{j\omega})$. 
(a) First, $X(e^{j\omega})$ is plotted.

\[ X(e^{j\omega}) \]

The lowpass filter cuts off at $\frac{\pi}{2}$.

\[ R_0(e^{j\omega}) \]

The downsampler expands the frequency axis. Since $R_0(e^{j\omega})$ is bandlimited to $\frac{\pi}{2}$, no aliasing occurs.

\[ X_0(e^{j\omega}) \]

The upsampler compresses the frequency axis by a factor of 2.

\[ G_0(e^{j\omega}) \]

The lowpass filter cuts off at $\frac{\pi}{2} \Rightarrow Y_0(e^{j\omega}) = R_0(e^{j\omega})$ as sketched above.
(b) Write a general expression for $G_0(e^{j\omega})$ in terms of $X(e^{j\omega})$ and $H_0(e^{j\omega})$. Do not assume that $X(e^{j\omega})$ and $H_0(e^{j\omega})$ are as shown in Figure 4.53-2.

\[
(b) \quad G_0(e^{j\omega}) = \frac{1}{2} (X(e^{j\omega}) H_0(e^{j\omega}) + X(e^{j(\omega+\pi)}) H_0(e^{j(\omega+\pi)}) )
\]
(c) Determine a set of conditions on \( H_0(e^{j\omega}) \) that is as general as possible and that will guarantee that \( y[n] \) is proportional to \( x[n - n_d] \) for any stable input \( x[n] \).

Note: Analyzer-synthesizer filter banks of the form developed in this problem are very similar to quadrature mirror filter banks. For further reading, see Crochier and Rabiner (1983), pp. 378–392.

\[
H_1(e^{j\omega}) = H_0(e^{j(\omega + \pi)}).
\]

\[
\begin{align*}
Y_0(e^{j\omega}) &= \frac{1}{2} H_0(e^{j\omega}) \left( X(e^{j\omega})H_0(e^{j\omega}) + X(e^{j(\omega+\pi)})H_0(e^{j(\omega+\pi)}) \right) \\
Y_1(e^{j\omega}) &= \frac{1}{2} H_1(e^{j\omega}) \left( X(e^{j\omega})H_1(e^{j\omega}) + X(e^{j(\omega+\pi)})H_1(e^{j(\omega+\pi)}) \right) \\
Y(e^{j\omega}) &= Y_0(e^{j\omega}) - Y_1(e^{j\omega}) \\
&= \frac{1}{2} X(e^{j\omega}) \left[ H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) \right] \\
&\quad + \frac{1}{2} X(e^{j(\omega+\pi)}) \left[ H_0(e^{j\omega})H_0(e^{j(\omega+\pi)}) - H_1(e^{j\omega})H_1(e^{j(\omega+\pi)}) \right]_{\omega=0}
\end{align*}
\]

The aliasing terms always cancel. \( Y(e^{j\omega}) \) is proportional to \( X(e^{j\omega}) \) if \( H_0^2(e^{j\omega}) - H_1^2(e^{j\omega}) \) is a constant.