EE123 Spring 2016 Miki Lustig
Digital Signal Processing

Assignment 6

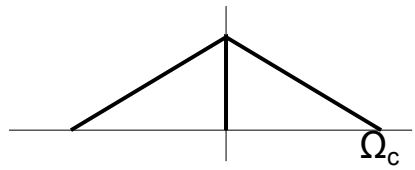
Due March 11th 2016

1. Read Chapter 4 Oppenheim and Schafer, 3rd ed.
2. Oppenheim and Schafer Exercise 4.24
3. Oppenheim and Schafer Exercise 4.32

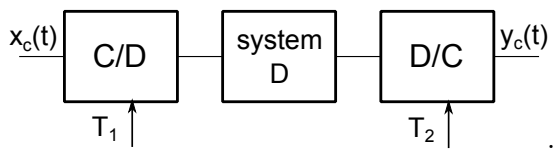
(Continued on next page)

4. *From Midterm II, sp'15: Digital Processing of Analog Signals*

You are given a continuous signal $x_c(t)$ with the following spectrum:

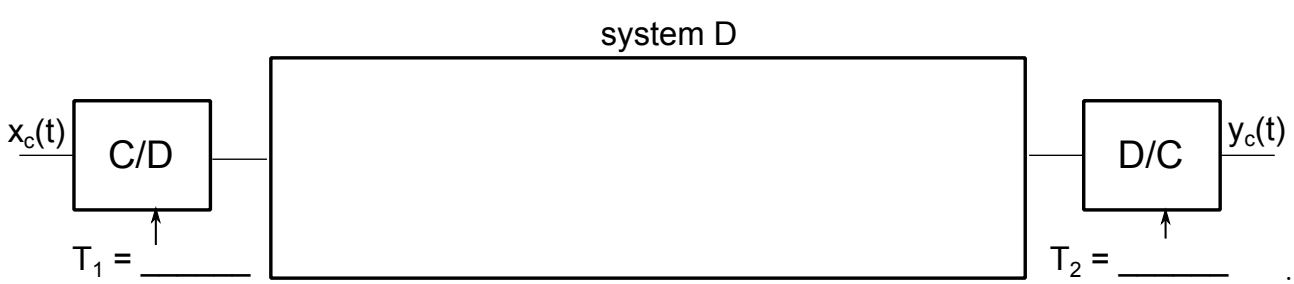


Also consider the typical digital processing system for analog signals that we covered in class where C/D and D/C are ideal continuous-to-Digital and digital-to-continuous that include the antialiasing filters. Note that C/D and D/C have no memory storage in them.



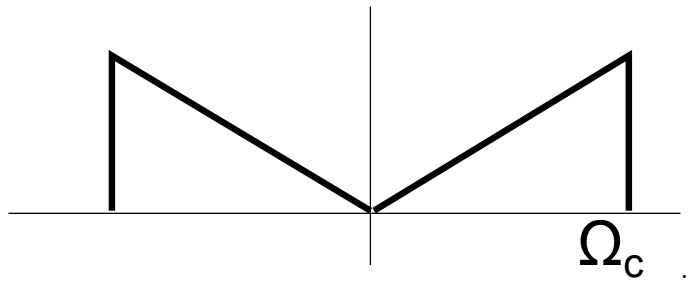
In this question you will be required to design several systems. (you can assume any system delays associated with the sampling are negligible)

a) Design and sketch the **simplest** "system D" that will result in $y_c(t) = x_c(t/2)$. Use the **maximum possible** values for T_1 and T_2.
Is "system D" memoryless? if yes explain, if not how much memory storage it needs?

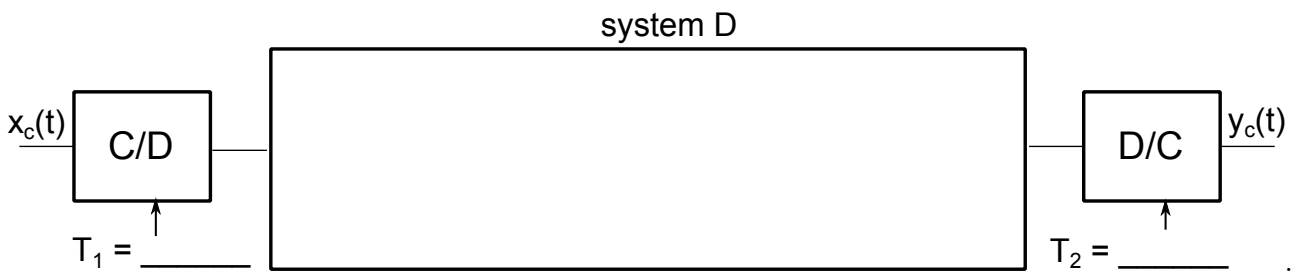


memoryless? Yes/No memory storage : _____

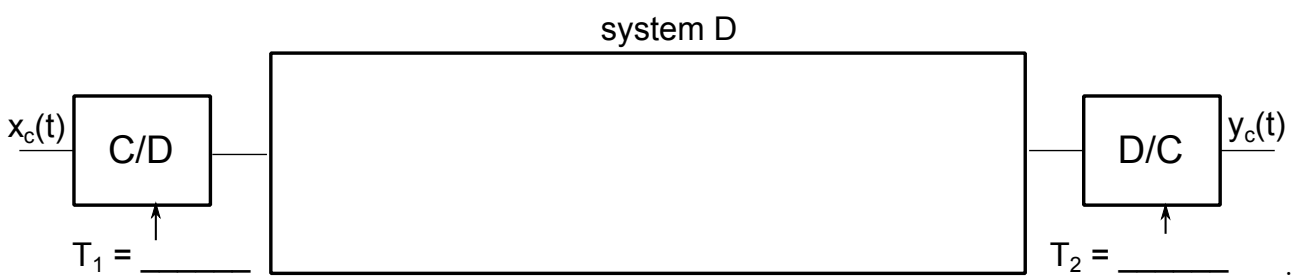
b) Design and sketch the **simplest** "system D" that given an input $x_c(t)$ above will swap the spectrum and result in $y_c(t)$ for which the frequency response is:



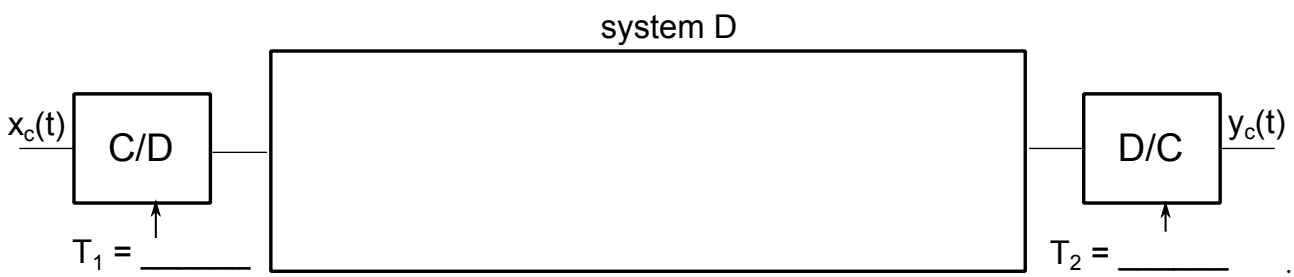
Use the **maximum possible** values for T_1 and T_2.



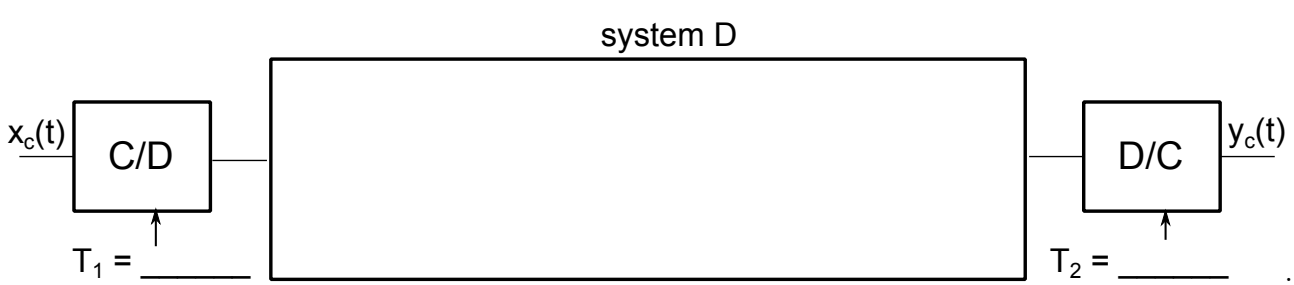
c) Is "system D" in part (b) memoryless? if so, explain. If not, change the appropriate parameters and sketch a new system in which "system D" is memoryless.



d) Design and sketch "system D" that results in the output $y_c(t) = x_c^2(t)$. Use the **maximum possible** values for T_1 and T_2.

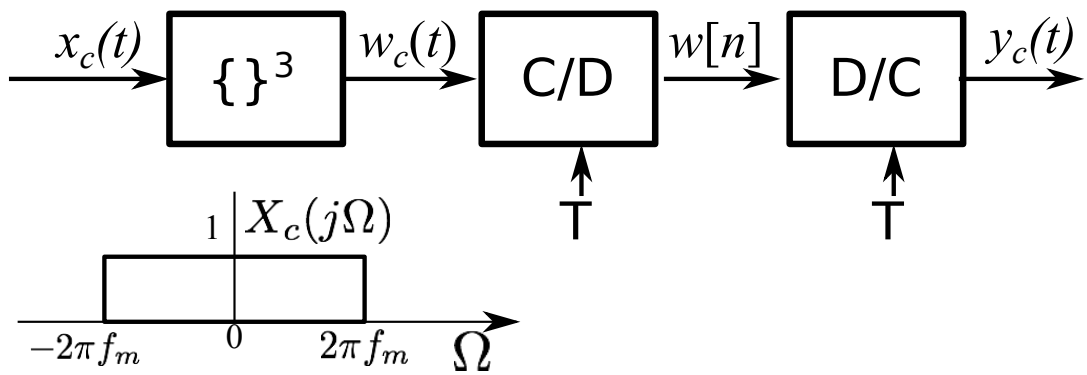


e) Is "system D" in part (d) memoryless? if so, explain. If not, change the appropriate parameters and sketch a new system in which "system D" is memoryless.



5. *From Midterm II, fall '12: Non-linearity, sampling, and reconstruction*

Consider the following system:

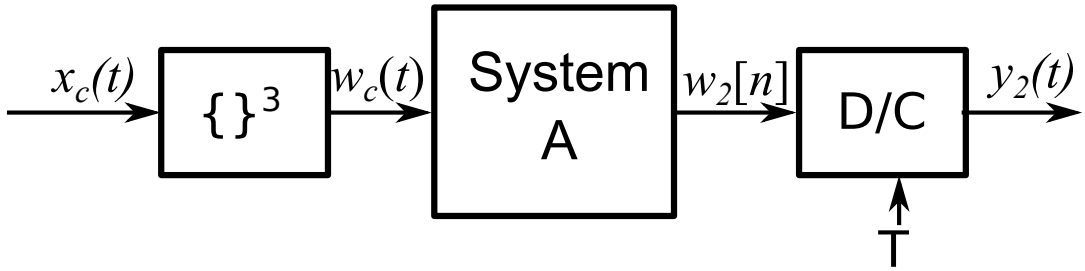


where the spectrum of $X_c(j\Omega)$ is given above, $1/T = 3f_m$ Hz and the operator $\{\}^n$ is a memoryless non-linear system that takes the nth power of its input.

a) Sketch the spectra of $w_c(t)$, $w[n]$ and $y_c(t)$. Label the axises correctly. Does $y_c(t) = x_c^3(t)$?

$W_c(j\Omega)$:
$W(e^{j\omega})$:
$Y(j\Omega)$:
$y_c(t) = x_c^3(t)$?: yes/no

b) Consider the following case:



where the signal $x_c(t)$ is the band limited signal from part (a), and $1/T = 3f_m$.

Your task is to design a continuous-time to discrete-time signal-recovery system ("system A"), for which $y_2(t) = x_c(t)$. You can use any (linear or not linear) components you need in discrete-time, but apart from a C/D, you are **not allowed to process anything in continuous time**.

sketch system A:

6. *Nonuniform Sampling*

A student had an assignment to monitor the level of the Hetch-Hetchi reservoir in Yosemite National Park. In order to meet the Nyquist rate, he has to drive and sample the water level at least once a day. Driving every day in the bay area traffic is no fun, so he decides instead to visit the reservoir every 2 days. He arrives at yosemite early in the day and records the level of the reservoir. Then, he goes on a 6 hours hike and before heading back he records the level again. The samples he collects are $g_1[n] = g(nT)$ and $g_2[n] = g(nT + 1/4)$, where $g(t)$ is the continuous water level in the lake and $T = 2$ [days]. The corresponding continuous, sampled signals are

$$\begin{aligned} \hat{g}_1(t) &= g(t)\text{Ш}_T(t) \\ \hat{g}_2(t) &= g(t)\text{Ш}_T(t - \Delta_T) \end{aligned}$$

where $\Delta_T = 1/4$ [days], and $\text{Ш}_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$ denotes an impulse train. We will show in this question that it is possible to reconstruct the continuous signal, even though the Nyquist rate appears to be violated. We will solve the problem in the frequency domain.

(a) $g(t)$ is band limited. If $T = 1$ [days] is the Nyquist rate, what is its highest frequency f_{nyq}?

(b) For the interval $0 \leq f \leq \frac{1}{2}$ [days^{-1}], express $\hat{G}_1(f)$ as a function of $G(f)$.

(c) For the interval $0 \leq f \leq \frac{1}{2}$ [days^{-1}] express $\hat{G}_2(f)$ as a function of $G(f)$.

(d) Use the result from (b)+(c) to find G(f) as a function of $\hat{G}_1(f)$ and $\hat{G}_2(f)$ for $0 \leq f \leq \frac{1}{2}$ [days^{-1}].

(e) Repeat (b)-(d) for the interval $-\frac{1}{2} \leq f \leq 0$ [days^{-1}].

(f) In order to recover $G(f)$ you had to invert a set of linear equations. Express the set of equations in (b)-(c) in matrix form. Plot the condition number (use the `numpy.linalg.cond` function in Python) for $0 < \Delta_T < \frac{1}{2}$. The condition number is the ratio between the highest and lowest eigen-values and indicates how much noise amplification occurs when inverting the matrix. From the plot, what can you conclude about the cost of nonuniform sampling?