Using DFT for filtering very long sequences.

\[ X(n) \rightarrow h(n) \rightarrow y(n) \]

FIR: small number of taps = P taps.

Exploit linearity property of convolution:

\[ [X_1(n) + X_2(n)] * h = (X_1 * h) + (X_2 * h) \]
Overlap Add

- Segment only long sequence into L, non-overlapping chunks.
2) Convolute each chunk with \( h \).

(\( L + P - 1 \) new point).

3) add up all the convolutions.

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**Overlap Save**

\( L \gg P \)

Thought experiment:

\[ \begin{align*}
X_1 & \rightarrow L \text{ pt seq.} \\
X_2 & \rightarrow P \text{ pt seq.}
\end{align*} \]

- **Correct:** Pad with zeros to get \( L + P - 1 \) pt sequence.
  - multiply \( 2 \), \( L + P - 1 \) point DFTs.
  - Take \( L + P - 1 \) point IDFT.

- **Suppose I take** \( L \) pt DFT of \( X_1 \) & \( X_2 \).
  - multiply Two, \( L \) pt, DFTs.
Take integer point IDFT.

What goes wrong? Can show only the first \( P-1 \) points are wrong. Rest are correct.

Why 7? Sketchy illustration
$L + P - 1$

Padding with zeros

not padding with zeros

$X_1(n)$

$X_2(n)$

P point
Overlap Save:

1. Segment sequence into \( L \) point chunks, overlap with each other by \( P - 1 \) points.

2. \( L \) point circular convolution of each chunk.
   Multiply \( L \) pt DFT of chunk \( \rightarrow \) IDFT
   \( L \) pt DFT of \( X \) \( \rightarrow \) \( \frac{1}{2} \) pt.

3. Throw away first \( P - 1 \) points of the answer in part 2. replace with previous segment.
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8.24 of 085.
DCT = Discrete Cosine Transform.
+ Compression property
+ DCT used standard.

JPEG,
MPEG 1
H.261, H.263, H.263+
H.263++, H.264, ...
Other than IPEB 2000, uses wavelets. Every other image/video stand uses JPEG.
+ Haar, Hartley, Hadamard, Walsh -

\[ x(n) \xrightarrow{\text{Real}} \text{Real} \]

\[ x(n) \xrightarrow{\text{Real}} \text{DFT} \xrightarrow{\text{Complex}} X(k) \]

\[ \text{Why DCT?} \]

\[ \text{DFT: } x(n) \xrightarrow{\text{DFS}} \hat{x}(n) \xrightarrow{\text{X(k)}} \text{over periodic DFT} \]

\[ \text{expansion in terms of } \sum_{k=-n/2}^{n/2-1} \frac{e^{j2\pi nk}}{2} \]

\[ \text{sharp discontinuity} \rightarrow \text{high freq.} \]
Basic idea behind DCT is to replicate signal in a "better way" so that there is no sharp discontinuity.

\[ x(n) \]

\[ d_1, d_2, d_3 \]

\[ d_1, d_2, d_3 \]

\[ DFTs of mirrored \]
4 Kinds of DCT

We'll focus on DCT-2.

\[ X(k) = 2 \sum_{n=0}^{N-1} x(n) \cos \left( \frac{Nk}{2N} (2n+1) \right) \]

Synthesis:
\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} \beta(k) X(k) \cos \left( \frac{Nk}{2N} (2n+1) \right) \]

\[ \beta(k) = \begin{cases} \frac{1}{2} & k = 0 \\ 1 & 1 \leq k < N \end{cases} \]
A relation DFT of $x(n)$ to its DCT.

**Proposal:**

1. $X(n)$, N pt seq. Real.
2. Take 2N pt DFT $\rightarrow X(k)$
3. $2 \text{Re}\{\sum X(k) e^{-\frac{j2\pi kn}{2N}}\}$

**Proposal 2:**

1. Start with N pt real seq $x(n)$
2. Pad it with N zeros $\rightarrow X_{2N}(n)$
3. Form a periodic seq.
   $\hat{x}(n) = X_{2N}(n) + X_{2N}(-n-1)$
4. Take 2N pt DFT of one period of $\hat{x}(n)$ $\rightarrow X_2(k)$