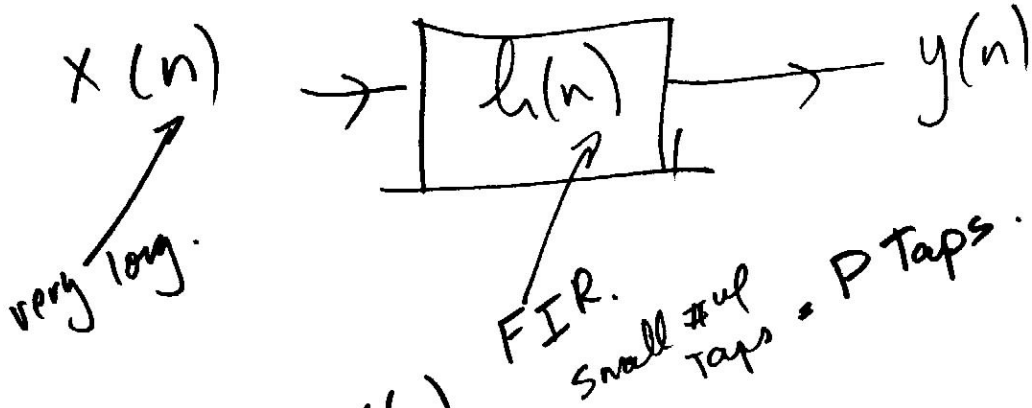
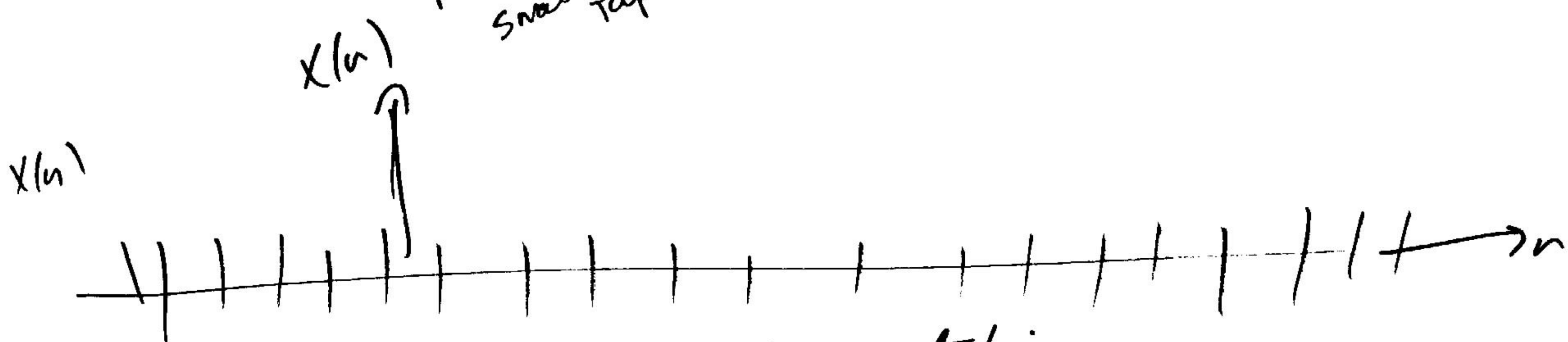
Using DFT for filtering ∞ly long Sequences

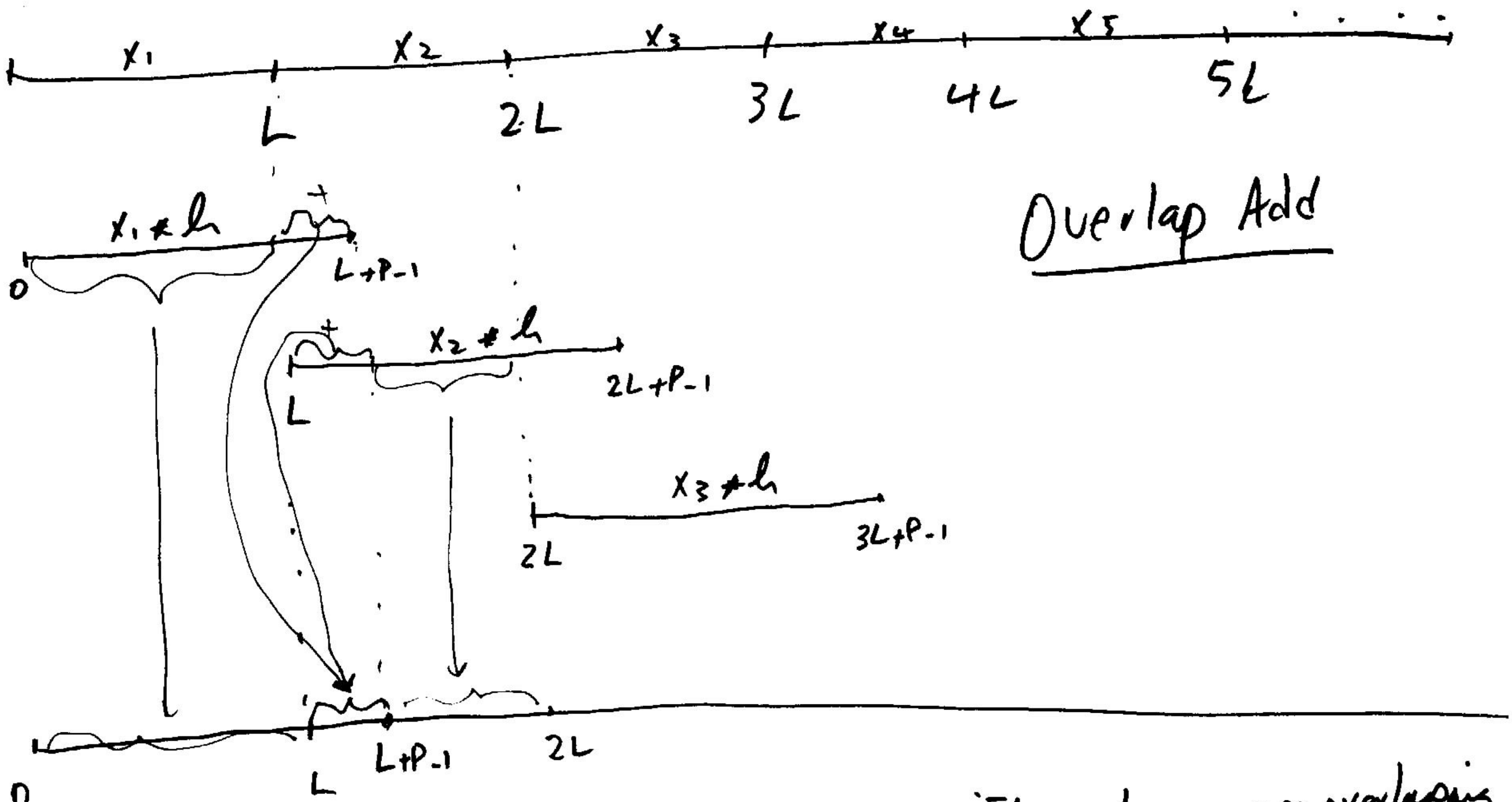




Exploit linearity property of convolution

$$[x_1(n) + x_2(n)] * h = (x_1 * h) + (x_2 * h)$$

Overlap Add



① * segment ∞ly long sequence into L, nonoverlapping chuncks.

(2) Convolve each chunk with h
$(L+P-1)$ new point.

(3) add up all the convolutions.

Overlap Save. $L >> P$

Thoughtexperiment

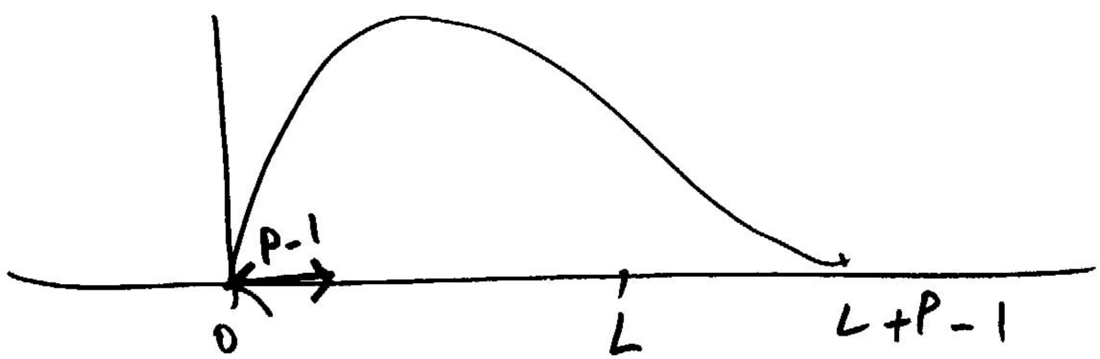
$x_1 \longrightarrow L$ pt seq.
$x_2 \longrightarrow P$ pt seq.

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \; x_1 * x_2 \longrightarrow L+P-1$ pt seq.

- correct:
  - pad with zeros to get $L+P-1$ pt sequence
  - multiply 2, $L+P-1$ point DFTS.
  - Take $L+P-1$ point IDFT.

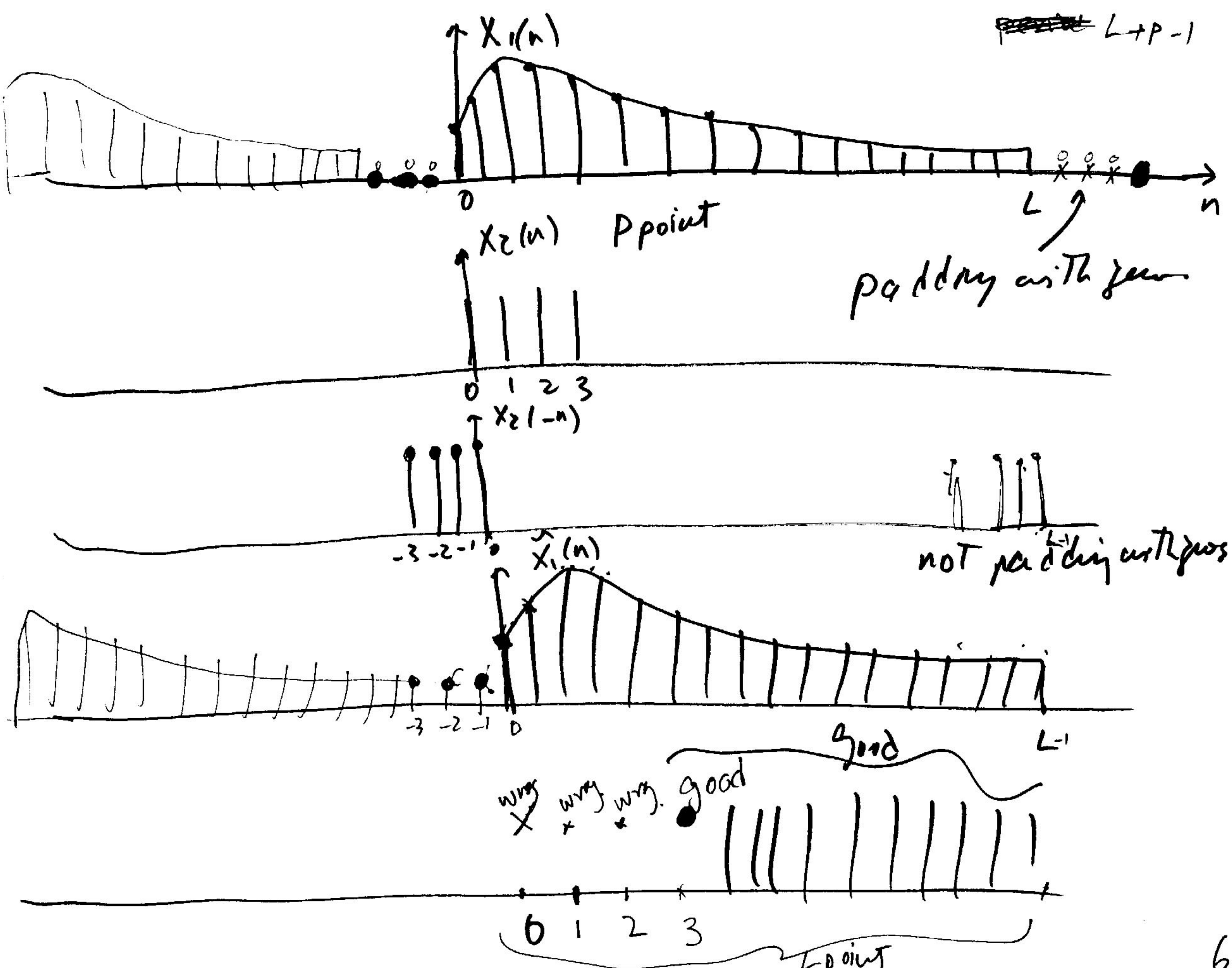
- Suppose I Take L pT DFT of x_1 & x_2.
  multiply Two, Lpt, DFTs.

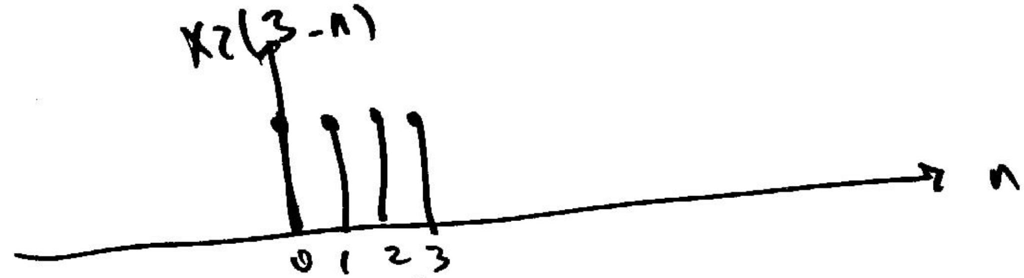
Take L point IDFT.

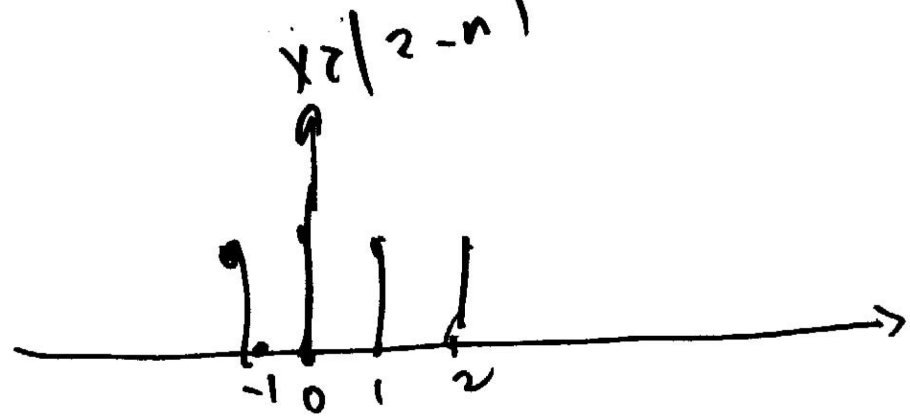


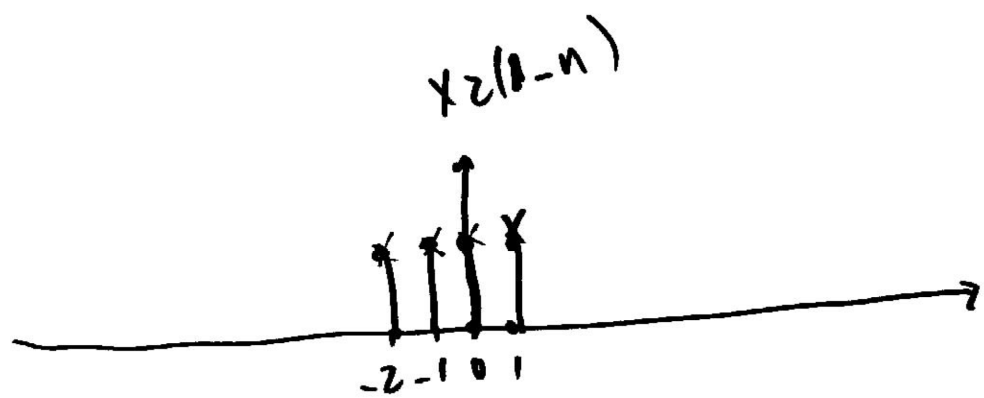
What goes wrong? Can show only The first P-1 points are wrong. Rest are correct.

- why? sketchy illustration


~~[illegible]~~ L+p-1
$x_1(n)$
0
L
n
P point
$x_2(n)$
padding with zero
0 1 2 3
$x_2(-n)$
-3 -2 -1 0
L-1
not padding with zeros
$\tilde{x}_1(n)$
-3 -2 -1 0
L-1
good
wrong wrong wrong. good
0 1 2 3
L point

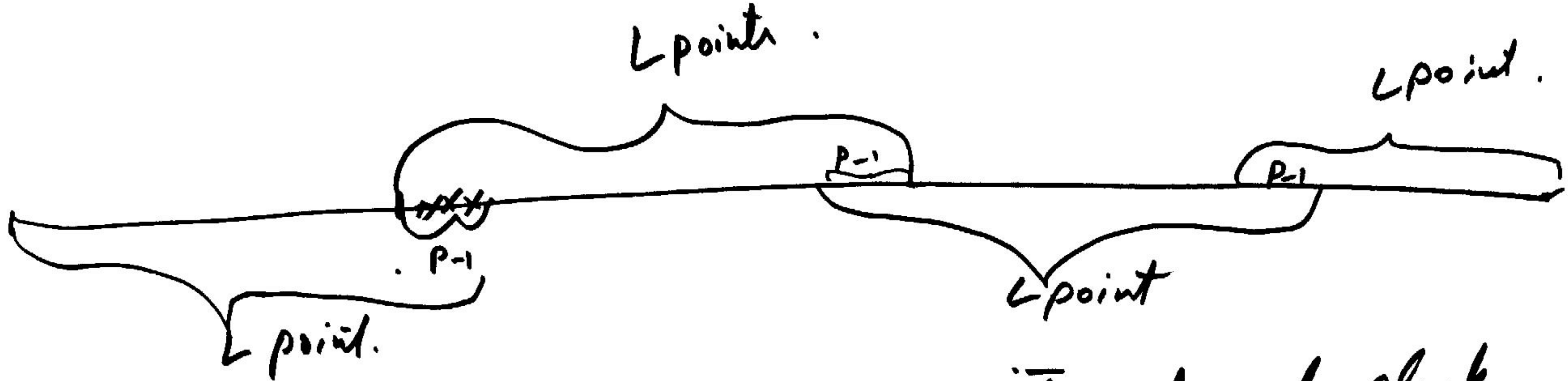

x2(3-n)
n
0 1 2 3


x2(2-n)
-1 0 1 2


x2(1-n)
-2 -1 0 1

Overlap Save.

① Segment sequence into L point chuncks. overlapp. with each other by P-1 points.



② L point circular convolution of each chunk.

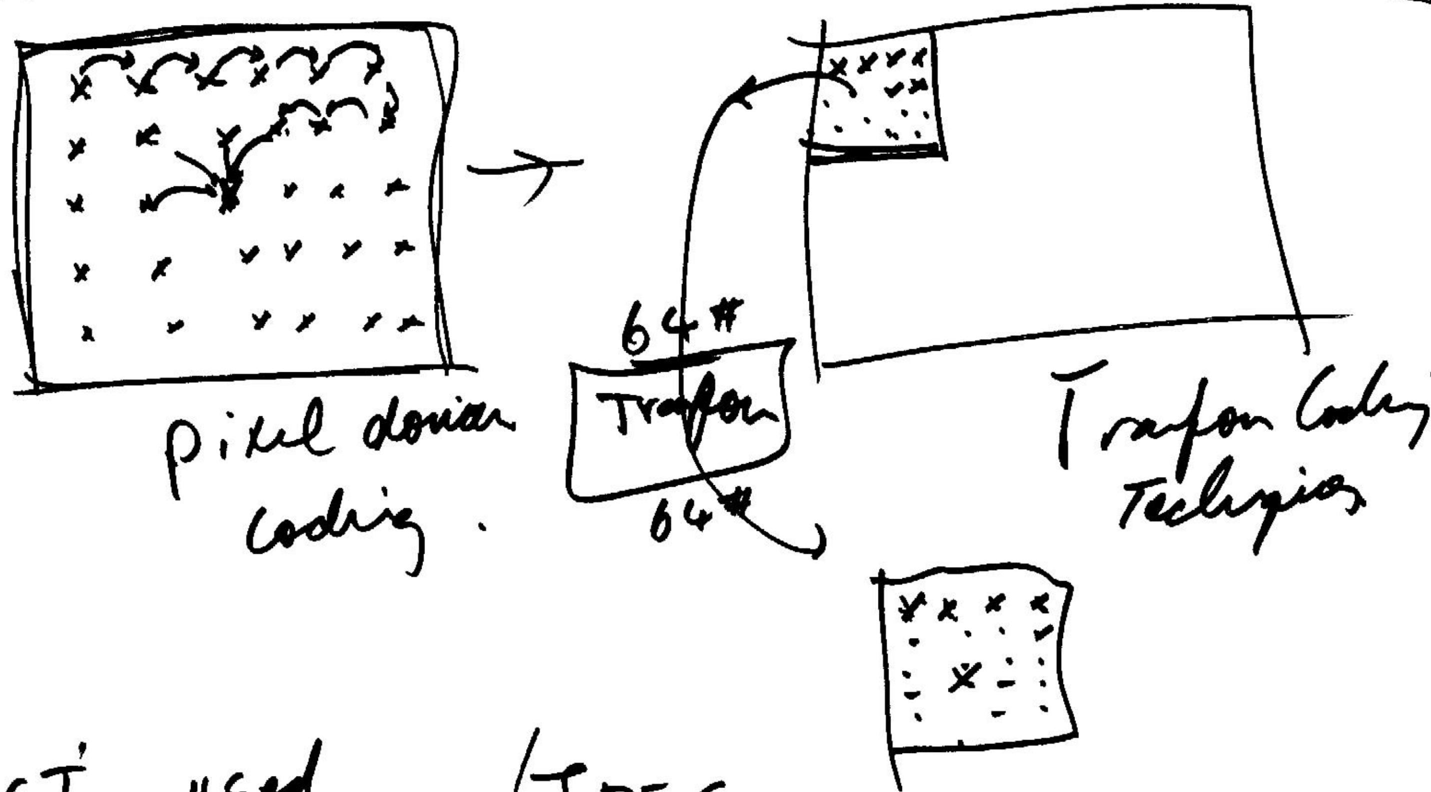
Multiply L pt DFT of chunk
L pt DFT of X2 } ⟶ IDFT L pt.

③ Throw away first P-1 points of the answer in part 2. replace with previous segment.

Fig 8.23 of O&S
8.24 of O&S.

DCT = Discrete Cosine Transform.

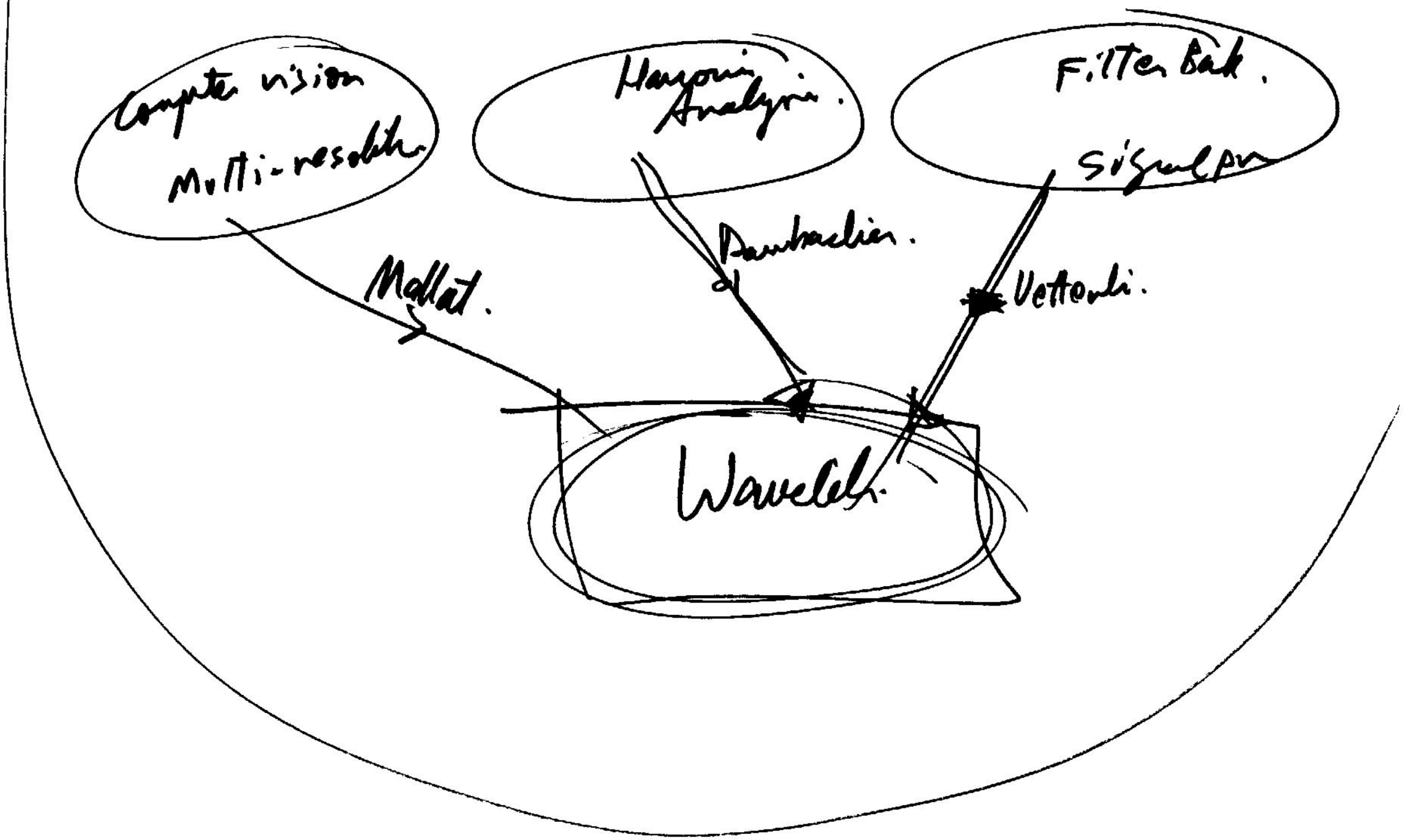
* Compaction property → coeff. stat. indep. of each other



* DCT used standard.

JPEG,
MPEG 1
2
4
H.261, H.263, H.263+
H.263++, H.264, ...

Other than JPEG 2000, uses wavelets.
every other image / video stand uses DCT.



+ Haar, Hartley, Haddamard, Walsh . — ....

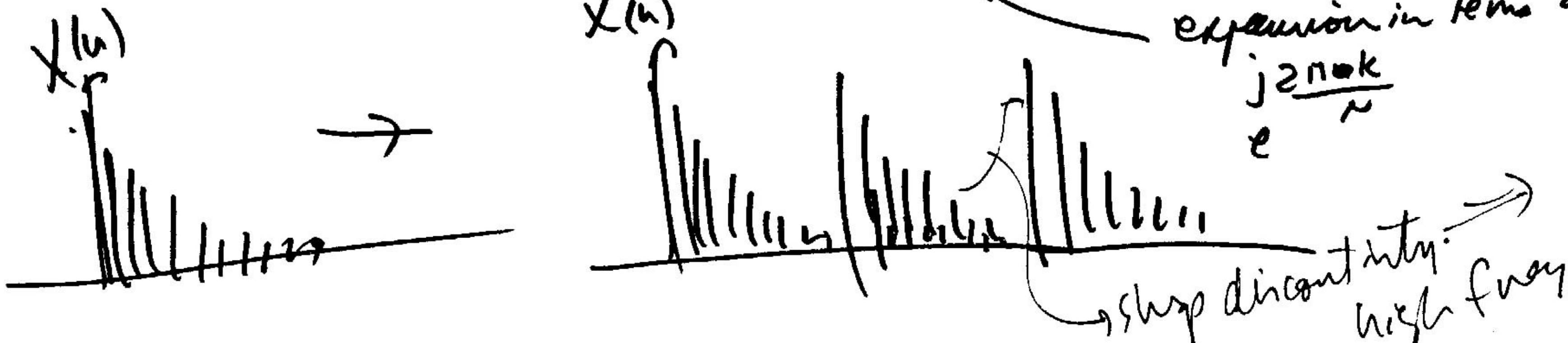
$x(n) \rightarrow$ [Real] $\rightarrow$ Real.

$x(n)$ real $\rightarrow$ [DFT] $\rightarrow$ $X(k)$ complex.

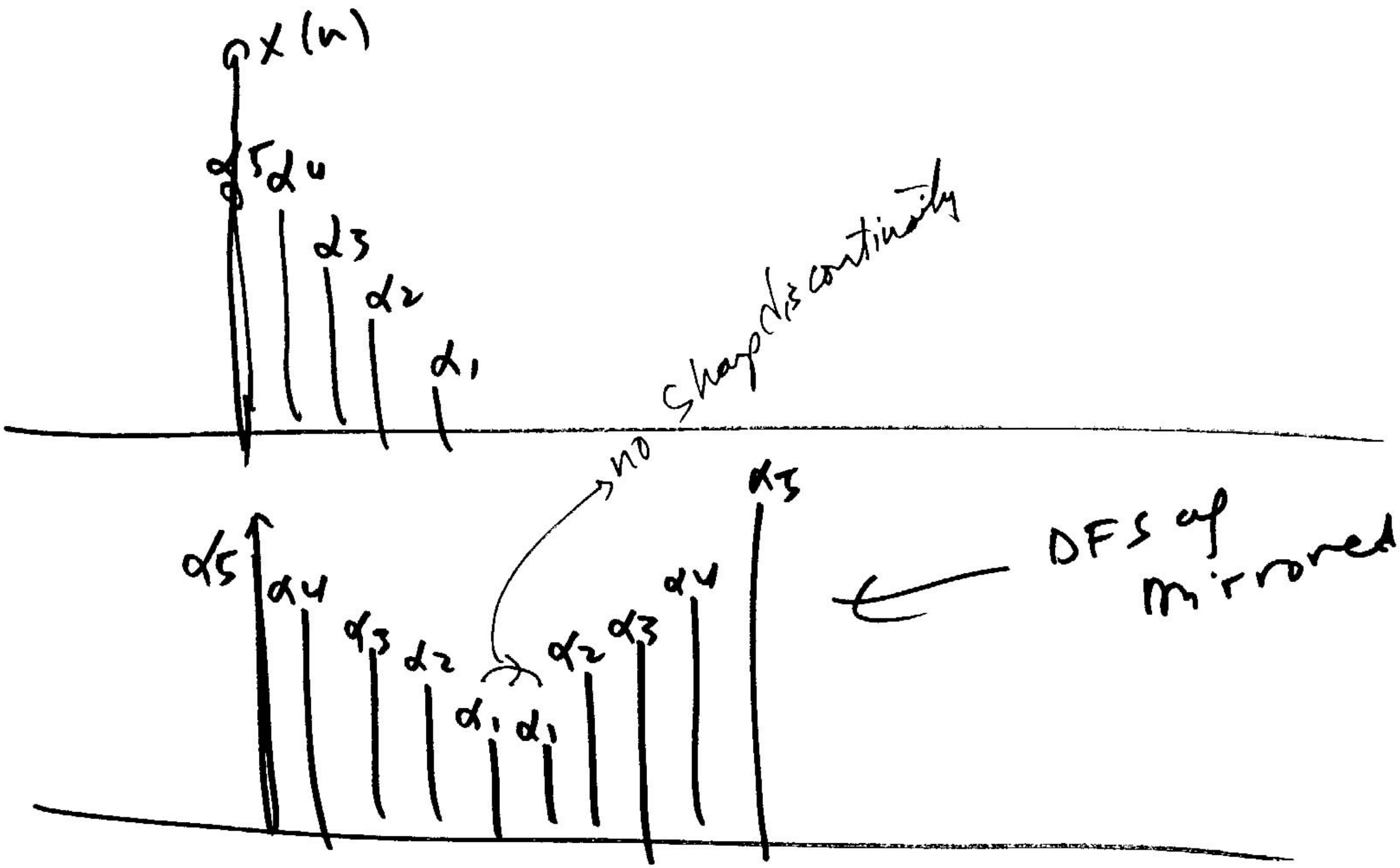
Why DCT?

DFT: $x(n) \rightarrow \tilde{x}(n) \xrightarrow{DFS} \tilde{X}(k) \xrightarrow{\text{one period}} X(k)$ DFT

expansion in terms of $e^{j2\pi nk/N}$



Basic idea behind DCT is to replicate signal in a "better way" so that there is no sharp discontinuity.



4 Kinds of DCT

We'll focus DCT_2 .

$$X^{C_2}(k) = 2 \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi k(2n+1)}{2N}\right)$$

analysis

Synthesis.

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} \beta(k) X^{C_2}(k) \cos\left(\frac{\pi k(2n+1)}{2N}\right)$$

$$\beta(k) = \begin{cases} \frac{1}{2} & k = 0 \\ 1 & 1 \le k < N \end{cases}$$

Relation DFT of $x(n)$ to its DCT-2.

proposal ①

① $x(n)$. N pt seq. Real.

② Take 2N pt DFT $\longrightarrow X(k)$

③ $2\,\mathrm{Re}\left\{ X(k)\, e^{-j\frac{2\pi k}{2N}} \right\}$

Proposal ②

① start with N pt real seq $x(n)$

② Pad it with N zeros $\rightarrow x_{2N}(n)$

③ form a periodic seq:

$$\tilde{x}_2(n) = \tilde{x}_{2N}(n) + \tilde{x}_{2N}(-n-1)$$

④ Take 2N pt DFT of one period of $\tilde{x}_2(n) \longrightarrow X_2(k)$