EE123 Spring 2018 — Miki Lustig

Digital Signal Processing

Assignment 2

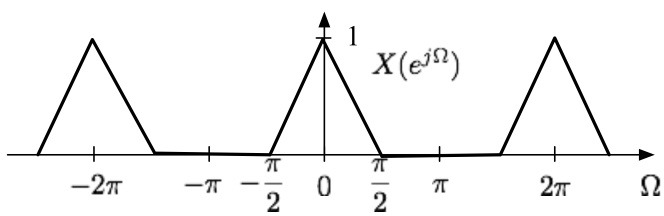
Due February 7^{th}

1. Self-grade Homework 1.
2. Read Chapter 3 Oppenheim and Schafer, 3rd ed.
3. Problem 3.45 Oppenheim and Schafer, 3rd ed.
4. The Z-transform of a right-sided sequence is given by:

$$X(z) = \frac{z^{-2}}{1 - 2.3z^{-1} + 1.6z^{-2} - 0.3z^{-3}}.$$

   Find $x[n]$ by doing partial fraction expansion with the help of Scipy's `scipy.signal.residue` function (You need to import scipy).

5. The signal $x[n]$ has the spectrum



   The signal $z[n]$ is given by

$$z[n] = x[n]y[n].$$

   Draw the DTFT $Z(e^{j\Omega})$ if $y[n]$ is:

   (a) $y[n] = \cos(\pi n)$
   (b) $y[n] = \cos(\pi n/2)$
   (c) $y[n] = \cos(\pi n/4) + \cos(3\pi n/4)$
   (d) $y[n] = \cos(\pi n + \pi/2)$

6. *From Midterm I Spring'16:* Autocorrelations and DFT Potpourri.

   a) Consider $x[n]$ a sequence of length L between $0 \leq n < L$, whos DTFT is $X(e^{j\omega})$. Let $y[n]$ be a sequence whoes DTFT is $Y(e^{j\omega}) = |X(e^{j\omega})|^2$. What can you ALWAYS say about $y[n]$ (circle all that apply and briefly explain)?

$y[n \geq L] = 0$	$y[n < 0] = 0$	Conjugate symmetric	Real	Even length	Odd length

b) We would like to compute $y[n]$ from part (a) by using the DFT. We compute the following:

$$\tilde{Y}[k] = \mathcal{DFT}\{x[n]\} = \sum_{n=0}^{L-1} x[n] W_N^{kn}$$

Then compute

$$\tilde{y}[n] = \mathcal{IDFT}\{|\tilde{Y}[k]|^2\}$$

Finally we set:

$$y[n] = \tilde{y}[m[n]]$$

What are the appropriate N and $m[n]$ that would result in the right $y[n]$? ($m[n]$ is index mapping function, for example $x[m[n]]$ where $m[n] = n - 1 \bmod L$ circularly shifts an L-length sequence $x[n]$ by one index to the right)

7. *From Midterm I fall'11:* A *stable* linear time invariant system has an impulse response

$$H(z) = \frac{3(1 - z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 - 2z^{-1})}.$$

a) Find the impulse response $h[n]$ of this system.

b) Another system has an impulse response $g[n]$ which is given by

$$g[n] = j^n h[n]$$

where $h[n]$ is the impulse response you found in part (a). Plot the poles and zeros of $G(z)$, which is the z-transform of $g[n]$. Indicate the region of convergence for $G(z)$.

8. *Adapted from Midterm I fall'10:*
The following values from the 8-point DFT of a length-8, real-valued sequence $x[n]$ are known: $X[0] = 3$, $X[2] = 0.5 - 4.5j$, $X[4] = 5$, $X[5] = 3.5 + 3.5j$, $X[7] = -2.5 - 7j$.

(a) Evaluate $x[0]$.

(b) Find the 8-point DFT of the circular convolution:

$$x[n] ⑧ \delta[n-1],$$

where $\delta[n]$ is the unit impulse.

(c) Consider a length-4 sequence $w[n]$ whose 4-point DFT is given by

$$W[k] = X[2k], \quad k = 0, 1, 2, 3.$$

Find an expression for $w[n]$ in terms of $x[n]$. What's going on here?

9. *From Midterm I spring'15:*
The FFTW package provides functions for efficiently computing DFTs. We will later see the main idea behind the implementation, which is famously known as the FFT.

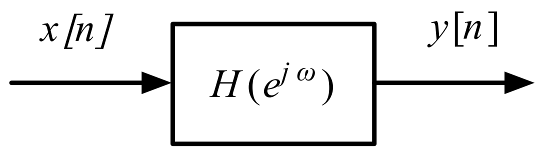
The input to the **complex-valued DFT function** in FFTW expects as an input an array in which the **real and imaginary components are interleaved**. You would like to compute the DFT_N of a N-length complex sequence $x[n] = x_r[n] + jx_i[n]$. You prepare an input array $h[n]$, $0 \leq n < 2N$ such that $h[2n] = x_r[n]$ and $h[2n+1] = x_i[n]$. Unfortunately instead of calling the complex-valued DFT function, you accidentally call the **real-valued** function which treats your input array as a $2N$-length real-valued array and returns a $2N$-length complex array $H[k]$ corresponding to the DFT_{2N} of h[n]. The question is about computing $X[k]$, the DFT_N of $x[n]$ from $H[k]$ with minimal computation.

(a) Find an expression for the first half of $H[k]$, $0 \leq k < N$ in terms of $X_r[k]$ and $X_i[k]$ the DFT_N of $x_r[n]$ and $x_i[n]$ respectively.

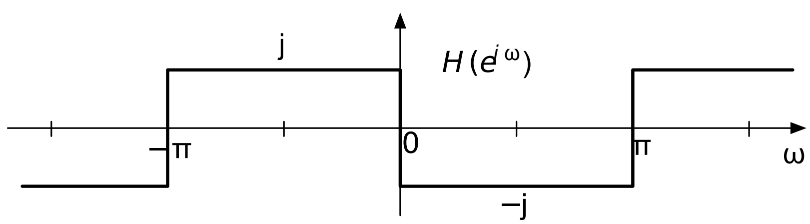
(b) Find an expression for the second half, $H[k+N]$, $0 \leq k < N$ in terms of $X_r[k]$ and $X_i[k]$ the DFT_N of $x_r[n]$ and $x_i[n]$ respectively.

(c) Find an expression to $X[k]$ in terms of $H[k]$ with minimum number of multiplications/additions that can not be precomputed. How many additions/multiplications are required?

10. *From Midterm I fall'11:* Consider this discrete-time system



The frequency response $H(e^{j\omega})$ is shown below



This is a very useful system, called a Hilbert-filter, and is often used in communication. Over the interval $-\pi < \omega < \pi$ the frequency response is given by

$$H(e^{j\omega}) = \begin{cases} j & -\pi < \omega < 0 \\ -j & 0 < \omega < \pi \\ 0 & \omega = 0 \end{cases}$$

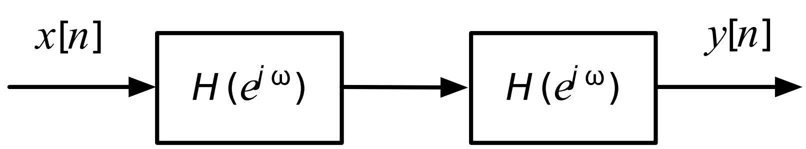
a) What is the symmetry of the impulse response of this system $h[n]$? Is it even, odd, Hermitian, or none of the above? Is it real, imaginary, or complex?

b) Assume the input to this system is

$$x[n] = \cos(\omega_0 n)$$

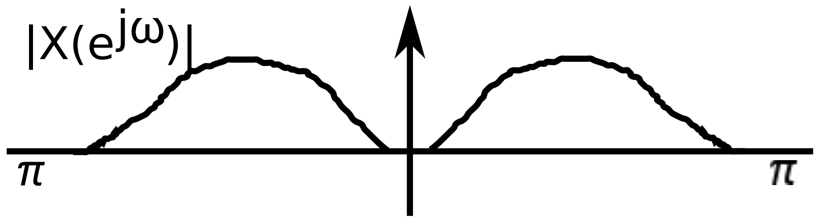
where $|\omega_0| < \pi$. Find the output $y[n]$.

c) We apply a general signal $x[n]$ to two such systems in series



Find $y[n]$.

d) Consider the samples of a speech signal $x[n]$ with the following magnitude spectrum $|X(e^{j\omega})|$:



Design and draw a system diagram that produces a baseband (around DC) Upper-Sideband signal from $x[n]$. That is, it should look like the above image, except with the lower sideband removed.