EE123 Spring 2018 Miki Lustig
Digital Signal Processing

Assignment 4

Due February 21^{th} 2018

1. Self-grade Homework 3.
2. Read Chapter 10.1-10.2 Oppenheim and Schafer, 3rd ed.
3. Consider the time-frequency tiling of the DFT. Draw qualitatively what happens to the Heisenberg boxes when you window the signal in time domain.
4. *Adapted From Final, fall'11*

   An N-sample possibly complex signal $x[n]$ is bounded, so that $|x[n]| \leq 1$ for all n.

   a) What is the largest value possible for $|X[k]|$, the magnitude of the DFT of $x[n]$?

   b) Find an expression for all of the $x[n]$ sequences which achieve this maximum.

5. *Adapted from Midterm I fall'12:* Hadamard Transform

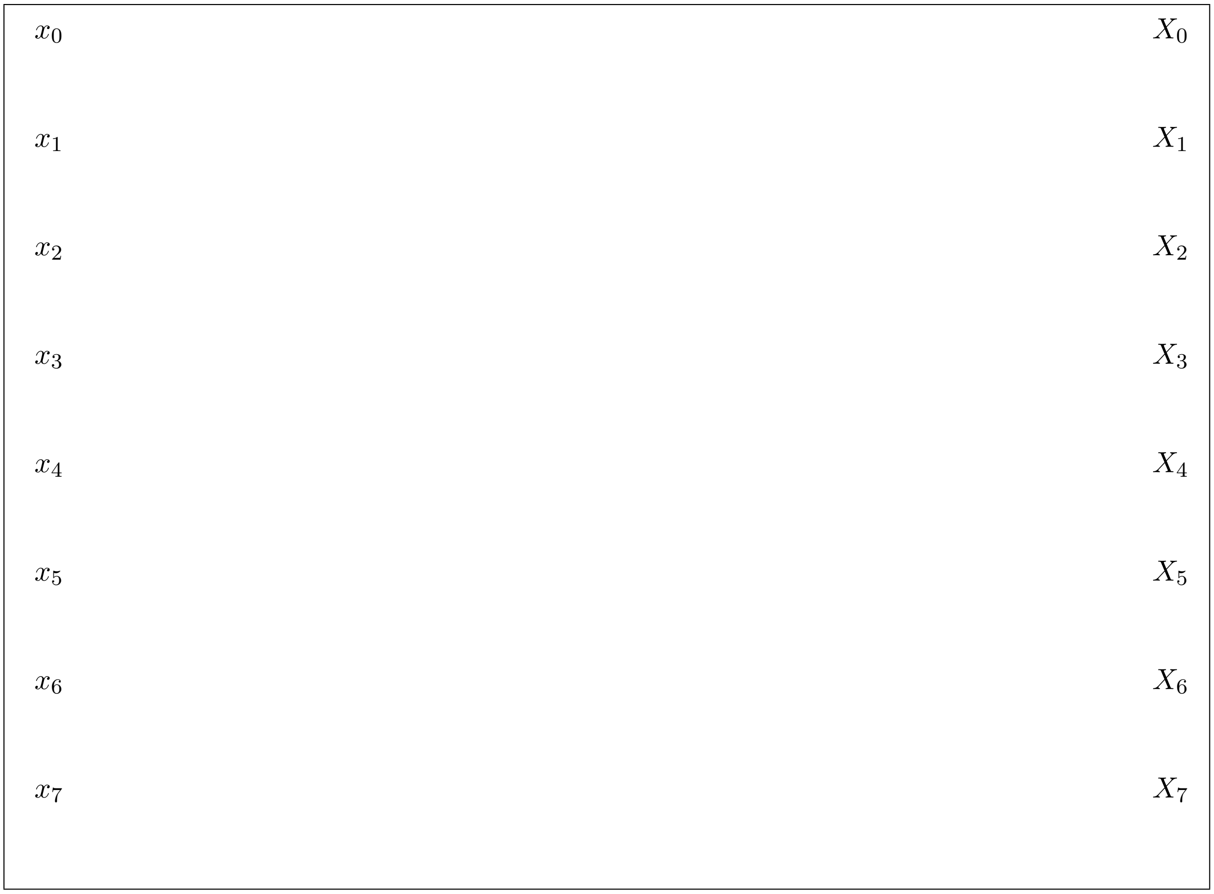
   Consider a new transform which is defined recursively in matrix form as:

$$H_m = \begin{bmatrix} H_{m-1} & H_{m-1} \\ H_{m-1} & -H_{m-1} \end{bmatrix},$$

   where $H_0 = 1$. (This transform is called the Hadamard Transform)

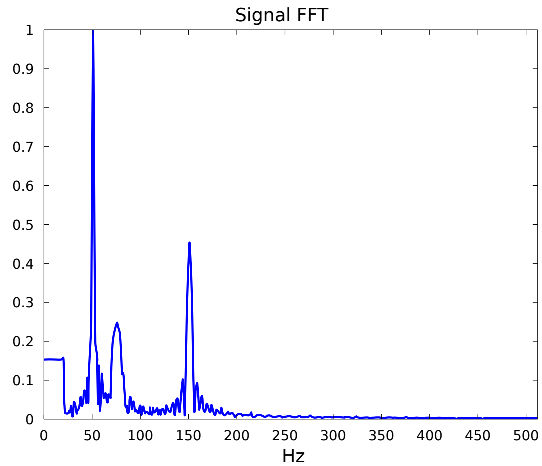
   a) This transform can be used to perform (somewhat OK) frequency analysis. Compute H_3 which is an 8×8 matrix. What is the order of the basis functions that represent functions with increasing frequency content?

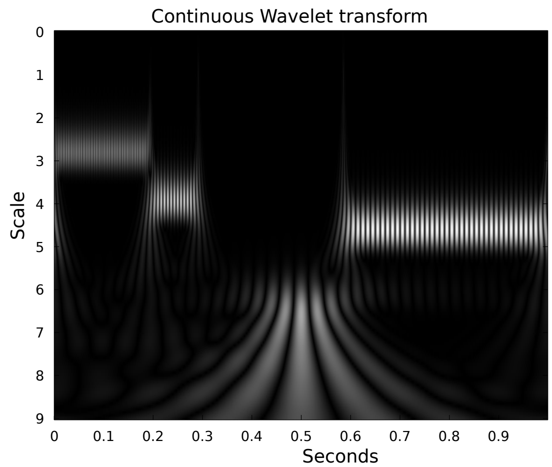
   b) Much like the DFT, the Hadamard matrix has structure which can be exploited for rapid computation. Draw a flow diagram (similarly to the FFT) for calculating the 8×8 Hadamard transform (H_3) in $O(N\text{Log}_2 N)$. Note, that there are several options, much like the decimation-in-time and decimation-in-frequency FFT's.


x_0 X_0
x_1 X_1
x_2 X_2
x_3 X_3
x_4 X_4
x_5 X_5
x_6 X_6
x_7 X_7

6. *Adapted from Midterm I fall'12:* Time-Frequency Analysis
We saw in class (and we will discuss more on Monday) that there are multiple ways to analyze the temporal-spectral components of signals. Here you will *qualitatively* determine a signal in the time domain based on these transforms.

You are given two graphs showing transforms of the signal s(t) and its samples. The left is the magnitude spectrum of a signal, the right is the continuous wavelet transform.





Draw qualitatively the signal s(t). Provide explanation on how you got to the result based on the FFT and CWT. In particular explain the difference in width of each spectral component in the FFT plot. Label each portion of the plot!

7. *From Midterm I, sp'13 (Optional)*

Consider the sequence $h[n] = \{1, -1\}$

a) Compute $H_4[k]$, the 4-point DFT of $h[n]$ and sketch its magnitude.
Is $H_4[k]$ even, odd, conjugate symmetric, or none of the above? Is it low pass, high pass, or neither?

b) We are given that $y[n]$ is the circular convolution of $h[n]$ with an unknown length-4 **real** sequence $x[n]$

$$y[n] = x[n] \; ④ \; h[n]$$

Given $y[n]$ can you uniquely determine $X[k]$ (and hence $x[n]$)?
Write an expression for $X[k]$ or for the class of signals $X[k]$ could be.

Example of a possible answer for a class of signals: $X[k] = C \cdot (-1)^k Y[k]$, where C is unknown.

c) Now, in addition to $y[n]$, you are given that $\sum_{n=0}^{3} |x[n]|^2 = D$, with the same unknown **real** sequence $x[n]$
Is it true that we can now uniquely determine $x[n]$?
Write an expression for $X[k]$ or for the class of signals $X[k]$ could be.

d) For this part, consider a complex modulated version of $h[n]$,

$$\tilde{h}[n] = e^{-j\frac{2\pi}{4}n} h[n]$$

We are given that $\tilde{y}[n]$ is the circular convolution of $\tilde{h}[n]$ with the same unknown **real** sequence $x[n]$

$$\tilde{y}[n] = x[n] \; ④ \; \tilde{h}[n]$$

Amazingly, it is now possible to uniquely determine $x[n]$ from $\tilde{y}[n]$. Show that this is the case.