

EE/BIOE 125 Problem Set 8  
Assigned 11/21/05; Due 12/8/05

1. Answer the questions from the Adept robot lab. You do not need to implement a full pick-and-place program, but please outline the steps.
2. Show that the control law

$$\tau = M(\theta)(\ddot{\theta}_d - \lambda\dot{e}) + C(\theta, \dot{\theta})(\dot{\theta}_d - \lambda e) + N(\theta, \dot{\theta}) - K_v\dot{e} - K_p e$$

is uniformly asymptotically stable when  $\lambda$  is a positive scalar,  $K_p, K_v$  are positive definite, and  $e = \theta - \theta_d$ . *Hint:* You can find a Lyapunov function that gives asymptotic stability without using any fancy tricks like Lasalle's principle (which wouldn't apply anyway because this is a time-varying system). It might take a little trial and error, though.

For background on why this is an interesting control law, you are welcome to look up: *J.E. Slotine and W. Li. On the adaptive control of robot manipulators. International Journal of Robotics Research, 6:49-59, 1987.*

3. A slightly different 2-port parameterization from that used in the Hannaford papers treats the environment effort and flow as independent variables and the human side as dependent:

$$\begin{bmatrix} F_h(s) \\ V_h(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} V_e(s) \\ -F_e(s) \end{bmatrix}$$

where  $F$  signifies force and  $V$  velocity. Using this parameterization, it is easy to see that the human side variables are related by

$$F_h = \left( \frac{P_{11} - P_{12}Z_e}{P_{21} - P_{22}Z_e} \right) V_h$$

where  $Z_e$  is the environment impedance  $F_e(s)/V_e(s)$ , and

$$Z_t = \left( \frac{P_{11} - P_{12}Z_e}{P_{21} - P_{22}Z_e} \right)$$

is the impedance that the human perceives through the teleoperator. Ideally this would match the environment impedance exactly.

- a. Verify that this parameterization can be related to the hybrid parameters used by Hannaford by:

$$\begin{bmatrix} P_{11} = \frac{h_{11}}{h_{21}} & P_{12} = \frac{h_{11}h_{22}}{h_{21}} - h_{12} \\ P_{21} = \frac{1}{h_{21}} & P_{22} = \frac{h_{22}}{h_{21}} \end{bmatrix}$$

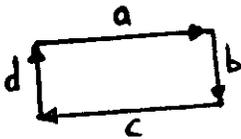
- b. Consider the two extremes where  $Z_e \rightarrow 0$  and  $Z_e \rightarrow \infty$ . From the  $h$  parameters given on page 429 of "A Design Framework..." and the relationships in part (a), find  $Z_t$  at these two extremes for both the "classical" and "forward flow" architectures. Does the human feel the environment accurately at these extremes? If not, what characteristic of the teleoperator architecture limits his/her perception in each extreme and each environment? If you do not understand these questions, be sure to ask for enlightenment in discussion or office hours.

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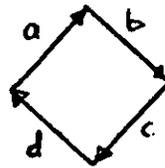
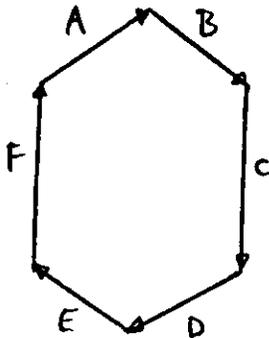
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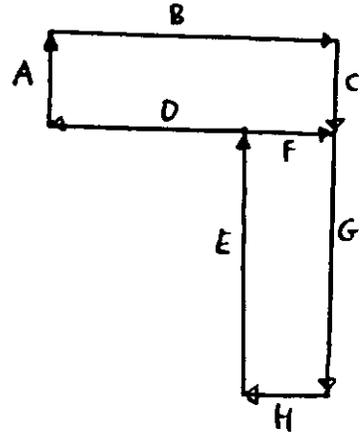
1. Sketch the configuration space obstacles for the following two robot-obstacle pairs, assuming the robots can translate but not rotate. Your sketch doesn't need to be geometrically precise, but each CO edge should be labeled to show which edge is in contact.



(i)



(ii)



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2. The graph below is the visibility graph of a certain environment. The numbers are the lengths of paths between the two vertices. Use Dijkstra's algorithm to label every vertex with the length of the shortest path from the source S to that vertex. Also label each vertex with a number indicating the order in which you labeled them. Also identify the parent of each vertex along the shortest path.

