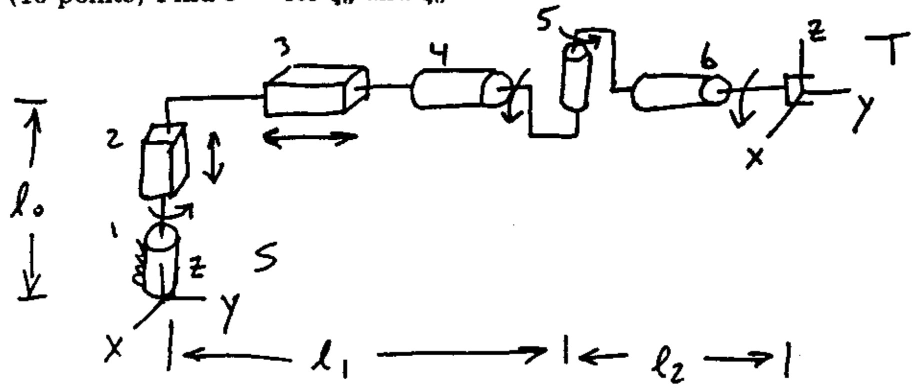
EE / BIOE 125 PRACTICE PROBLEMS 2005

SOLUTIONS

1a. (10 points) For the manipulator shown, find the twist coordinates ξ_i and $g_{st}(0)$ using the product of exponentials approach. Use the base and tool frames shown.
b. (10 points) Find $e^{\hat{\xi}_i\theta_i}$ for ξ_3 and ξ_5.



Assume $l_0 = l_1 = 0$ at zero configuration.

(a) $q_1 = \begin{bmatrix}0\\0\\0\end{bmatrix}$, $q_2 = \begin{bmatrix}0\\0\\0\end{bmatrix}$, $q_3 = \begin{bmatrix}0\\0\\0\end{bmatrix}$, $q_4 = \begin{bmatrix}0\\0\\0\end{bmatrix}$, $q_5 = \begin{bmatrix}0\\0\\0\end{bmatrix}$, $q_6 = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$\omega_1 = \begin{bmatrix}0\\0\\1\end{bmatrix}$, $v_2 = \begin{bmatrix}0\\0\\1\end{bmatrix}$, $v_3 = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\omega_4 = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\omega_5 = \begin{bmatrix}0\\0\\1\end{bmatrix}$, $\omega_6 = \begin{bmatrix}0\\1\\0\end{bmatrix}$

$\omega \times q = 0$ for all $i = 1,4,5,6$

$\xi_1 = \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}$, $\xi_2 = \begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix}$, $\xi_3 = \begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix}$, $\xi_4 = \begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix}$, $\xi_5 = \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}$, $\xi_6 = \begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix}$

$$g_{st}(0) = \begin{bmatrix} I_{3\times3} & \begin{matrix}0\\l_2\\0\end{matrix} \\ 0\;0\;0 & 1 \end{bmatrix}$$

(b) $$e^{\hat{\xi}_3\theta_3} = \begin{bmatrix} I & \begin{matrix}0\\\theta_3\\0\end{matrix} \\ 0\;0\;0 & 1 \end{bmatrix}, \quad e^{\hat{\xi}_5\theta_5} = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2a. (15 points) Show how you would solve for the inverse kinematics of the manipulator shown in Problem 1, given a desired g_d. Use subproblems (including subproblem 5) or geometric arguments as you wish. How many solutions are possible (ignoring physically impossible solutions, i.e., negative displacements)?

b. (5 points) Suppose the sequence of twists 3 and 4 were reversed, i.e., joint 3 became revolute and joint 4 became prismatic (but both still about y axis). Would this change your solution from part (a)? Why or why not?

NOTE: THIS SOLUTION ASSUMES (WITH IMPUNITY) THAT $l_0 = l_1 = 0$ IN THE ZERO CONFIGURATION, AND PRISMATIC DISPLACEMENTS θ_2 AND θ_3 WILL ACCOUNT FOR ACTUAL LINK LENGTHS. AND FWD KIN IN PROBLEM 1

a)
$$e^1 e^2 e^3 e^4 e^5 e^6 g_{st}(0) = g_d$$
$$e^1 e^2 e^3 e^4 e^5 e^6 = g_d g_{st}^{-1}(0) \triangleq g$$

let $p_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ ← intersection of ξ_4, ξ_5, ξ_6 → $e^1 e^2 e^3 e^4 e^5 e^6 p_1 = e^1 e^2 e^3 p_1 = g p_1$

$$e^1 e^2 e^3 = \left[\begin{bmatrix} \underbrace{\begin{matrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{R_z(\theta_1)} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix} \right] \left[\begin{matrix} I & \begin{matrix} 0 \\ \theta_3 \\ \theta_2 \end{matrix} \\ 0\ 0\ 0 & 1 \end{matrix} \right] p_1 = \left[\begin{matrix} R_z(\theta) & \begin{matrix} -\theta_3 \sin\theta_1 \\ \theta_3 \cos\theta_1 \\ \theta_2 \end{matrix} \\ 0\ 0\ 0 & 1 \end{matrix} \right] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\theta_3 \sin\theta_1 \\ \theta_3 \cos\theta_1 \\ \theta_2 \\ 0 \end{bmatrix}$$

Then,
$$\theta_2 = (g p_1)_z \longrightarrow \underline{1 \text{ soln.}}$$
$$\theta_1 = \operatorname{atan2}\left(-(g p_1)_x, (g p_1)_y\right) \longrightarrow \underline{1 \text{ soln.}}$$
$$\theta_3 = \left[(g p_1)_x^2 + (g p_1)_y^2 \right]^{1/2} \longrightarrow \underline{1 \text{ soln.}}$$

let $p_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \end{bmatrix}$ ← Along ξ_6 but not ξ_5 & ξ_6

$$e^4 e^5 e^6 p_2 = \underbrace{e^{-3} e^{-2} e^{-1} g}_{\triangleq g_1} p_2$$

$$e^4 e^5 p_2 = g_1 p_2 \longrightarrow \text{Sub Problem 2}$$

$\underline{2 \text{ soln.s}}$

let $p_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ ← not on ξ_6,
$$e^6 p_3 = \underbrace{e^{-5} e^{-4} g_1}_{\triangleq g_2} p_3$$

$$e^6 p_3 = g_2 p_3 \longrightarrow \text{subproblem 1}$$

$\underline{1 \text{ soln.}}$

Total: $\underline{2 \text{ solns.}}$

2. (b) This will not change the solution for part a. 7.3/6

$$e^4 e^3 = \begin{bmatrix} R_z & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 1 \end{bmatrix} \begin{bmatrix} I & \underbrace{\begin{matrix}0\\0\\\theta_3\end{matrix}}_{\to v_z\theta_3} \\ 000 & 1 \end{bmatrix}$$

v_z is invariant under R_z, i.e. $R_z v_z = v_z$

$$\Rightarrow e^4 e^3 = \begin{bmatrix} R_z & v_z\theta_3 \\ 000 & 1 \end{bmatrix}$$

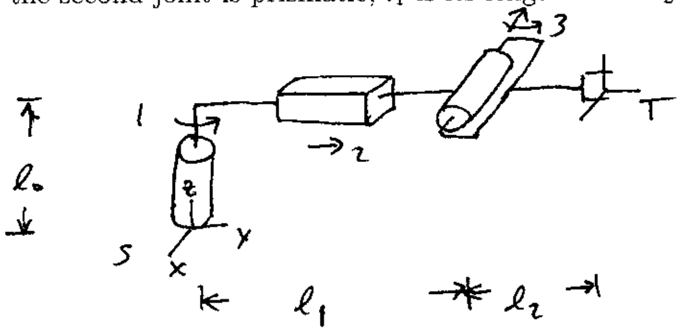
SAME

and

$$e^3 e^4 = \begin{bmatrix} I & v_z\theta_3 \\ 000 & 1 \end{bmatrix} \begin{bmatrix} R_z & \begin{matrix}0\\0\\0\end{matrix} \\ 000 & 1 \end{bmatrix} = \begin{bmatrix} R_z & v_z\theta_3 \\ 000 & 1 \end{bmatrix}$$

3. Jacobians (15 points)

For the manipulator shown, find the *body* Jacobian. Use the spatial and body frames shown. Note that the second joint is prismatic; l_1 is its length when $\theta_2 = 0$.



$$\omega_3^+ = \omega_3 = \begin{bmatrix}-1\\0\\0\end{bmatrix} \qquad q_3^+ = \begin{bmatrix}0\\-l_2\\0\end{bmatrix} \qquad \xi_3^+ = \begin{bmatrix} -\begin{bmatrix}-1\\0\\0\end{bmatrix}\times\begin{bmatrix}0\\-l_2\\0\end{bmatrix} \\ -1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\-l_2\\-1\\0\\0\end{bmatrix}$$

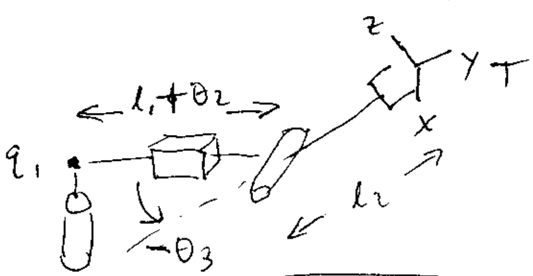
$$v_2^+ = e^{-\hat{\omega}_3\theta_3} v_2 = R_{-x}(-\theta_3) v_2 = R_x(\theta_3) v_2$$

$$= \begin{bmatrix}1 & 0 & 0\\ 0 & c3 & -s3\\ 0 & s3 & c3\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\c3\\s3\end{bmatrix}$$

$$\omega_1^+ = e^{-\hat{\omega}_3\theta_3}\omega_1 = R_x(\theta_3)\omega_1 = \begin{bmatrix}0\\-s3\\c3\end{bmatrix}$$ } CAN USE 3×3 HERE (AND ω_3^+ & v_2^+) BECAUSE IT'S A VECTOR

$$q_1^+ = g_{st}^{-1}(0)\, e^{-\hat{\xi}_3\theta_3} e^{-\hat{\xi}_2\theta_2} q_1$$ } MUST USE 4×4 TO ACCOUNT FOR TRANSLATION BECAUSE q_1 IS A POINT

= VERY LABORIOUS BY HAND! INSTEAD:



FIND $q_1^+ = q_1$ IN CURRENT T COORDINATES FROM ~~THE~~ PICTURE

$$q_1^+ = \begin{bmatrix}0\\ -l_2 - (l_1+\theta_2)c3\\ (l_1+\theta_2)\sin(-\theta_3)\end{bmatrix} = \begin{bmatrix}0\\ -l_2-(l_1+\theta_2)c3\\ -(l_1+\theta_2)s3\end{bmatrix}$$

$$v_1^+ = -\omega_1^+ \times q_1^+ = \begin{bmatrix}-(l_1+\theta_2) - l_2 c3\\ 0\\ 0\end{bmatrix}$$

RESULT:

$$J_{st}^b = \begin{bmatrix}-(l_1+\theta_2)-l_2c3 & 0 & 0\\ 0 & c3 & 0\\ 0 & s3 & -l_2\\ 0 & 0 & -1\\ -s3 & 0 & 0\\ c3 & 0 & 0\end{bmatrix}$$