NAME SOLUTION

EE/BIOE 125 Midterm 1

Open book; Closed notes
Use additional paper if necessary; please put your name on extra sheets
Each problem is worth 25 points.

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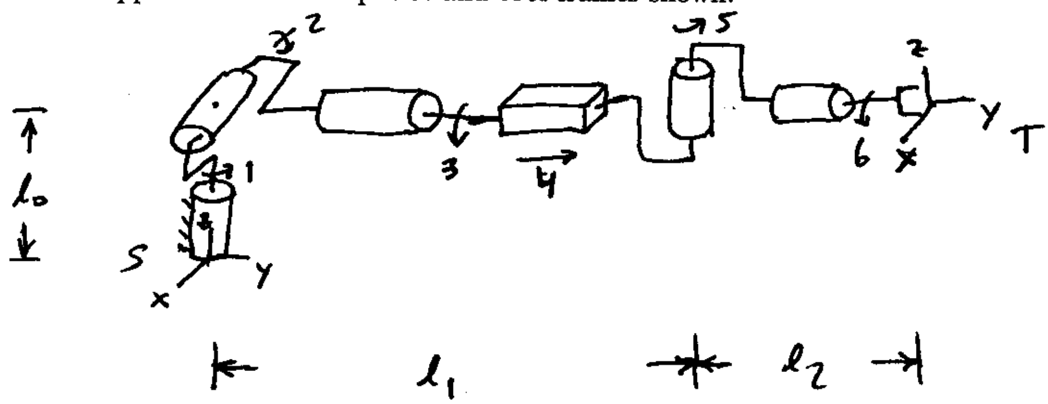
1. For the manipulator shown, find the twist coordinates ξ_i and $g_{st}(0)$ using the product of exponentials approach. Use the spatial and tool frames shown.



$$\xi_1 = \begin{bmatrix} -\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}0\\0\\0\end{bmatrix} \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix} \qquad \xi_2 = \begin{bmatrix} -\begin{bmatrix}-1\\0\\0\end{bmatrix} \times \begin{bmatrix}0\\0\\l_0\end{bmatrix} \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix}0\\-l_0\\0\\-1\\0\\0\end{bmatrix}$$

$$\xi_3 = \begin{bmatrix} -\begin{bmatrix}0\\1\\0\end{bmatrix} \times \begin{bmatrix}0\\0\\l_0\end{bmatrix} \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix}-l_0\\0\\0\\0\\1\\0\end{bmatrix} \qquad \xi_4 = \begin{bmatrix}0\\1\\0\\0\\0\\0\end{bmatrix}$$

$$\xi_5 = \begin{bmatrix} -\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}0\\l_1\\0\end{bmatrix} \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix}l_1\\0\\0\\0\\0\\1\end{bmatrix} \qquad \xi_6 = \xi_3 = \begin{bmatrix}-l_0\\0\\0\\0\\1\\0\end{bmatrix}$$

$$g_{st}(0) = \begin{bmatrix} [\,I\,] & \begin{matrix}0\\l_1+l_2\\l_0\end{matrix} \\ 0\;0\;0 & 1 \end{bmatrix}$$

2. Show how you would solve for the inverse kinematics of the manipulator shown in problem 1, given a desired g_d. Show which subproblems you would solve and which points you would use to solve them. You may use subproblems 1, 2, 3, or 5. How many solutions are possible?

NOTE THAT ξ_3 ALWAYS INTERSECTS ξ_5 AND ξ_6, INDEPENDENT OF ξ_4. GEOMETRICALLY, IT DOESN'T MATTER IF YOU ROTATE FIRST THEN TRANSLATE ALONG THE SAME AXIS, OR VICE VERSA. SO FOR <u>THIS</u> PROBLEM,

$$e^{\hat{\xi}_3 \theta_3} e^{\hat{\xi}_4 \theta_4} = e^{\hat{\xi}_4 \theta_4} e^{\hat{\xi}_3 \theta_3}$$

ONCE YOU DO THIS, THE LAST 3 AXES INTERSECT AND YOU CAN USE THE SAME SEQUENCE OF SUBPROBLEMS AS THE STANFORD MANIPULATOR!

<u>OR</u> INVERT EVERYTHING:

$$e^1 e^2 e^3 e^4 e^5 e^6 g_{st}(\theta) = g_d$$

$$e^{-6} e^{-5} \cdots e^{-1} = \cancel{g_d} \, g_{st}(0) \, g_d^{-1} := g_1^{-1}$$

CHOOSE p_{123} AT INTERSECTION OF ξ_1, ξ_2, ξ_3

$$p_{123} = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix} \qquad p_{56} = \begin{bmatrix} 0 \\ l_1 \\ l_0 \end{bmatrix}$$

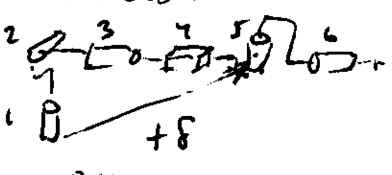
$$e^{-6} \cdots e^{-1} p_{123} = e^{-6} e^{-5} e^{-4} p_{123} = g_1^{-1} p_{123}$$

$$\| e^{-6} e^{-5} e^{-4} p_{123} - p_{56} \| = \| g_1^{-1} p_{123} - p_{56} \|$$

$$\| e^{-4} p_{123} - p_{56} \| = \| \quad " \quad \|$$

ξ_4 IS PRISMATIC: USE <u>SUBPROBLEM 5</u>

* NOTE: ALGEBRAICALLY, S.P. 5 HAS 2 SOLUTIONS, BUT ONLY ONE IS PHYSICALLY FEASIBLE:



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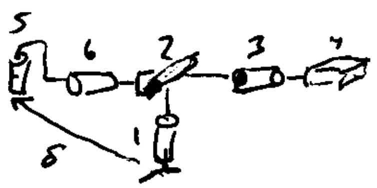

NEGATIVE DISPLACEMENT

$$e^{-6} e^{-5} (e^{-4} p_{123}) = g_1^{-1} p_{123}$$

USE <u>SUBPROBLEM 2</u> TO FIND $-\theta_5, -\theta_6$

2 SOLUTIONS

$$e^1 e^2 e^3 = g_d \, g_{st}^{-1}(0) \, e^{-6} e^{-5} e^{-4}$$

PICK POINT ON ξ_3, NOT ON ξ_1 OR ξ_2: $\quad p_3 = \begin{bmatrix} 0 \\ y \neq 0 \\ l_0 \end{bmatrix}$

$$e^1 e^2 e^3 p_3 = e^1 e^2 p_3 = g_d \cdots e^{-4} p_3$$

<u>SUBPROBLEM 2</u> GIVES θ_1, θ_2

PICK POINT <u>NOT</u> ON ξ_3. SAY $p_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$e^3 p_0 = e^{-2} e^{-1} g_d \, g_{st}^{-1}(0) \, e^{-6} e^{-5} e^{-4} p_0$$

<u>SUBPROBLEM 1</u> GIVES θ_3

TOTAL SOLUTIONS

$$1 \times 2 \times 2 \times 1 = 4$$

(I ALSO ACCEPTED 8 IF YOU USED 2 SOLUTIONS FOR S.P. 5)

3. You have a telescope mounted on a set of gimbals on the Y, Z, and X axes. You must choose the rotations α, β, and γ, respectively, about these axes to produce the same orientation as would result from a 180° rotation about an equivalent axis of $(0, 1/\sqrt{2}, 1/\sqrt{2})$. Show your work. Hint: this gimbal sequence is the same as an Y-Z-X Euler angle sequence about changing axes.

$$R = R_y(\alpha) R_z(\beta) R_x(\gamma)$$

$$= \begin{bmatrix} c\alpha & 0 & s\alpha \\ 0 & 1 & 0 \\ -s\alpha & 0 & c\alpha \end{bmatrix} \begin{bmatrix} c\beta & -s\beta & 0 \\ s\beta & c\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix}$$

$$= \begin{bmatrix} c\alpha\, c\beta & \text{UGLY} & \text{UGLY} \\ s\beta & c\beta\, c\gamma & -c\beta\, s\gamma \\ -s\alpha\, c\beta & \text{UGLY} & \text{UGLY} \end{bmatrix}$$

NO NEED TO SOLVE THE UGLY TERMS: YOU DON'T NEED THEM

$$\hat{\omega} = \begin{bmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

$$R = e^{\hat{\omega}\theta} = I + \hat{\omega} \overset{0}{\cancel{\sin\theta}} + \hat{\omega}^2 \underbrace{(1-\cos\theta)}_{2} = I + 2\hat{\omega}^2$$

$$\hat{\omega}^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1/2 & 1/2 \\ 0 & 1/2 & -1/2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$r_{21} = \sin\beta = 0 \Rightarrow \beta = 0 \text{ OR } \pi$$

$$\gamma = \text{ATAN2}(-r_{23}/\cos\beta,\ r_{22}/\cos\beta)$$

$$= \text{ATAN2}(-1/1,\ 0/1) = 3\pi/2 \quad \text{WHEN } \beta = 0$$

$$= \text{ATAN2}(-1/(-1),\ 0/(-1)) = \pi/2 \quad \text{WHEN } \beta = \pi$$

$$\alpha = \text{ATAN2}(-r_{31}/\cos\beta,\ r_{11}/\cos\beta)$$

$$= \text{ATAN2}(0,\ -1/1) = \pi \quad \text{WHEN } \beta = 0$$

$$= \text{ATAN2}(0,\ -1/(-1)) = 0 \quad \text{WHEN } \beta = \pi$$

4. Show the equivalent homogeneous transformation g for a screw motion of angle $\theta = 90°$ about a twist $\xi = [h, 0, m, 1, 0, 0]^T$.

$$g = e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})\omega \times v + \omega\omega^T v \theta \\ 0\,0\,0 & 1 \end{bmatrix}$$

(BETTER TO USE EQ. 2.36 IN TEXT THAN 2.40, SINCE YOU'RE GIVEN v, NOT q)

$$\omega = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad e^{\hat{\omega}\theta} = R_x(\theta) \text{ ON P. 31!} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} h \\ 0 \\ m \end{bmatrix} \qquad \omega \times v = \begin{bmatrix} 0 \\ -m \\ 0 \end{bmatrix} \qquad (I - e^{\hat{\omega}\theta})\omega \times v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & +1 & +1 \\ 0 & -1 & +1 \end{bmatrix}\begin{bmatrix} 0 \\ -m \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -m \\ m \end{bmatrix}$$

$$\omega^T v = \omega \cdot v = h \qquad \omega(\omega^T v)\theta = \omega(h\pi/2) = \begin{bmatrix} h\pi/2 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad g = \begin{bmatrix} 1 & 0 & 0 & h\pi/2 \\ 0 & 0 & -1 & -m \\ 0 & 1 & 0 & m \\ 0 & 0 & 0 & 1 \end{bmatrix}$$