Nov 10,04

$$\mathrm{Inv}\left[F_1(s) + F_2(s)\right] \neq \mathrm{Inv}\left[F_1(s)\right] + \mathrm{Inv}\left[F_2(s)\right]$$

- Let $A_1\ A_2 \ldots A_k$ list of mutually exclusive, collectively exhaustive events
- Assume continuous R.V. y, not indp. of A_i

$$\boxed{f_y(y_0) = \sum_i P(A_i)\, f_{y|A_i}(y_0/A_i)}$$

Define..

$$M_{Y|A_i}(s) \triangleq \int_{y_0=-\infty}^{+\infty} e^{-sy_0} f_{Y|A_i}(y_0|A_i)\, dy_0$$

$$= E(e^{-sy} | A_i)$$

$$\Longrightarrow \boxed{M_Y(s) = \sum_i P(A_i)\, M_{Y|A_i}(s)}$$

valid s Transform

$$\mathrm{Inv}\left[\frac{1}{9}\,1 + \frac{8}{9}\,\frac{1/9}{s+1/9}\right]$$

valid s transform

$P(A_1)$ $P(A_2)$

$$\cancel{\frac{1}{9}\,\mathrm{Inv}[\ ]} = \frac{1}{9}\,\mathrm{Inv}[1] + \frac{8}{9}\,\mathrm{Inv}\left[\frac{\lambda/9}{s+\lambda/9}\right]$$

$$= \frac{1}{9}\,\delta(\tau_0) + \frac{8}{9}\,e^{-\lambda\tau_0/9}\,u(\tau_0)$$

step fn

Covariance + Correlation

R.V. X, Y.

- $\mathrm{Cov}(X,Y) \triangleq E[(X-\bar{X})(Y-\bar{Y})]$

  $Y = X \implies \mathrm{Cov}(X,X) = \mathrm{Var}(X)$

- <u>Def</u> If $\mathrm{Cov}(X,Y) = 0$

  $\implies X, Y$ uncorrelated)

$$\mathrm{Cov}(X,Y) = E[XY] - E[X\bar{Y}] - E[Y\bar{X}] + E[\bar{X}\bar{Y}]$$

$$= E[XY] - \bar{Y}\bar{X} - \bar{X}\bar{Y} + \bar{X}\bar{Y}$$

$$= E[XY] - \bar{X}\bar{Y} =$$

$$\boxed{\mathrm{Cov}(X,Y) = E[XY] - E[X]\,E[Y]}$$

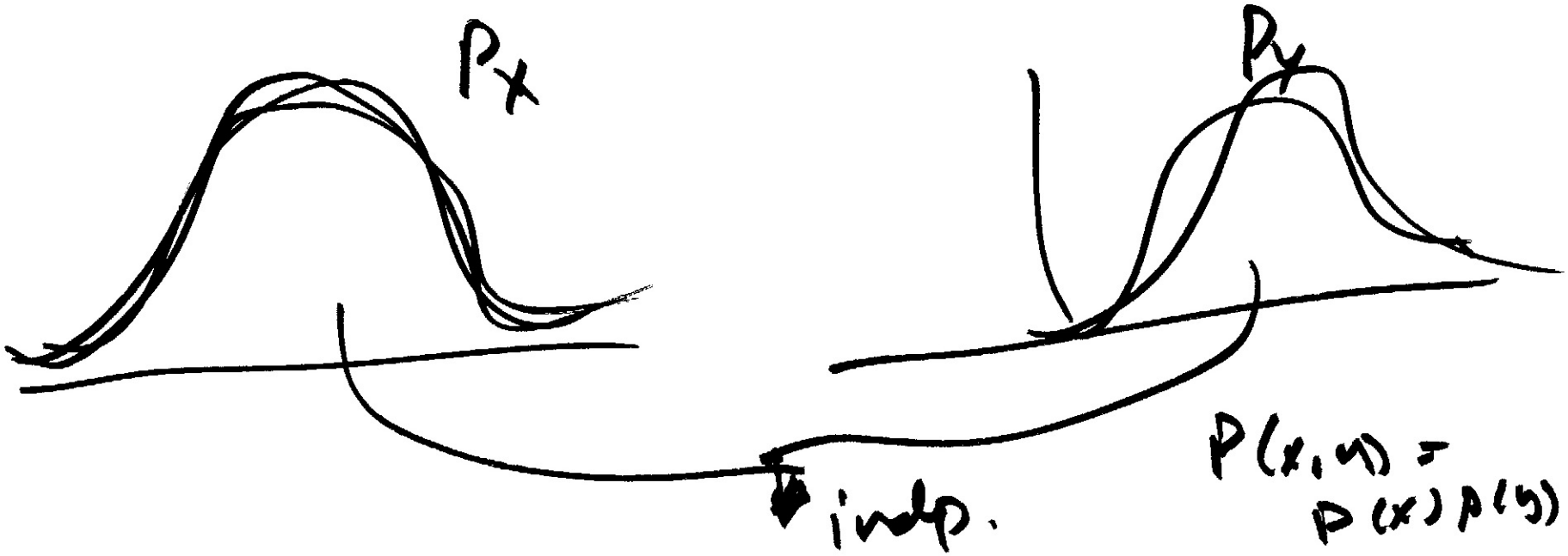
If X, Y independent

$$\mathrm{Cov}(X,Y) = E(X)E(Y) - E(X)E(Y)$$
$$= 0$$

$\Longrightarrow$ If X, Y indp $\Longrightarrow$ Also uncorrelated

$\not\Longleftarrow$

if uncorrelated does <u>not</u> mean indp.



uncorr $\quad E(xy) = E(x)\,E(y)$

Difference between indp. and uncorrelated.

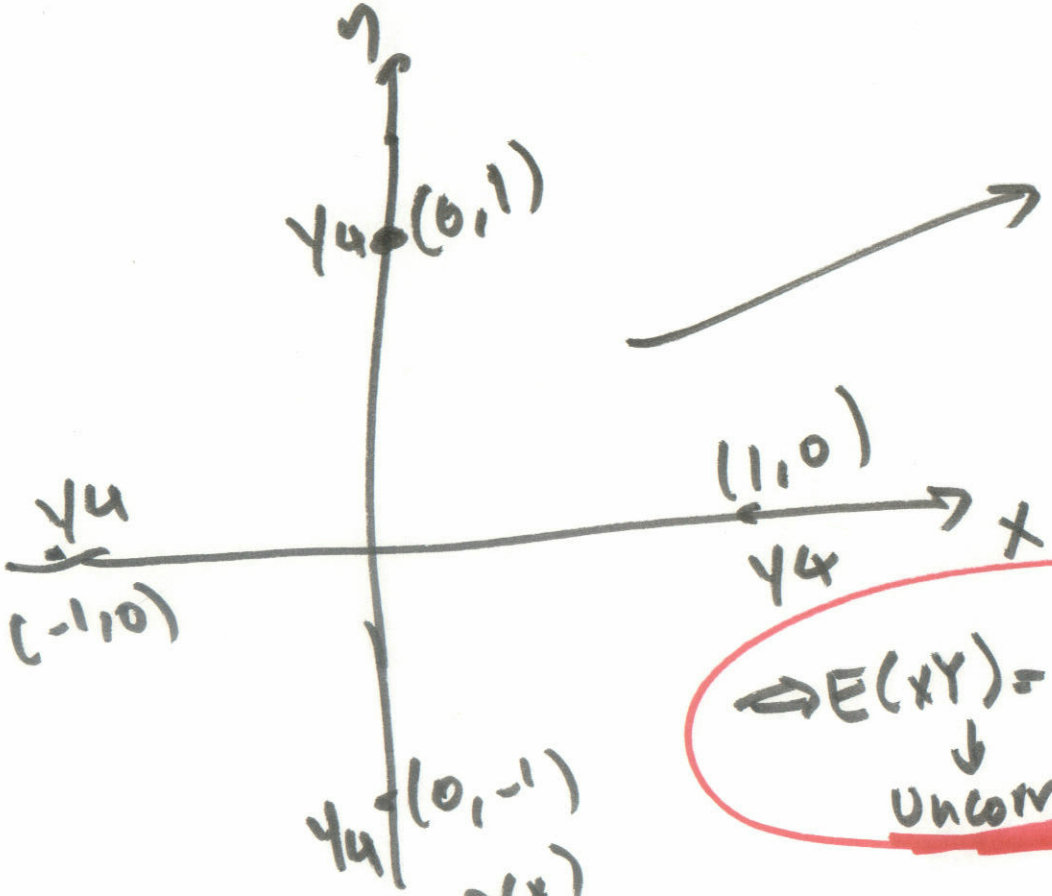
Indp $\longrightarrow$ deals with pdfs. $\qquad$ uncorrelation only deals with Expect. 5.5

$\Rightarrow$ Intuitively

positive covariance $\Longrightarrow$ $x - \bar{x}$ and $y - \bar{y}$ have same sign.

negative covariance $\Longrightarrow$ $x - \bar{x}$, $y - \bar{y}$ have different sign.

Ex. 2 R.V. uncorrelated, But not indp.



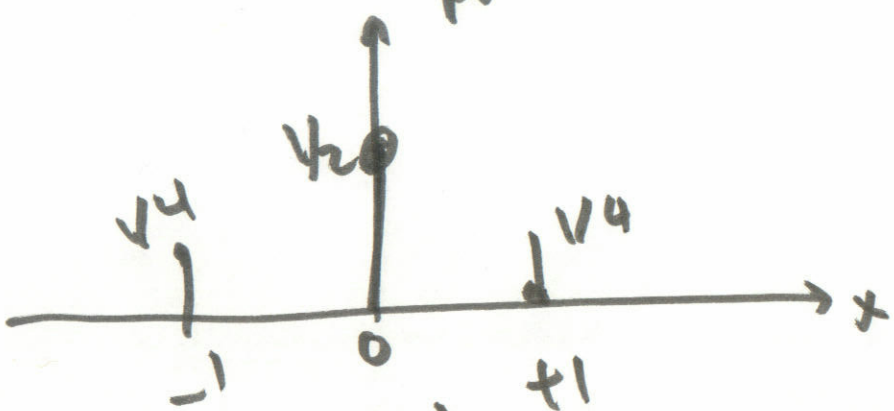
clear: x, y are not independent

$E(XY) = 0 = \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 + \frac{1}{4}\cdot 0 = 0$

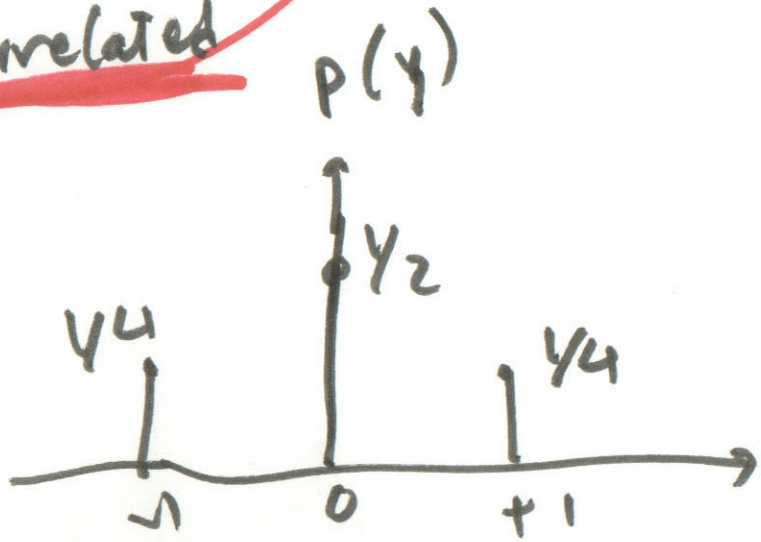
$\Rightarrow E(XY) = E(X)E(Y)$

$\downarrow$

uncorrelated



$E(X) = 0$



$E(Y) = 0$

Def <u>Correlation coefficient</u>:

$$\rho(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}$$

can show $|\rho| < 1$

- $X-\bar{X}$ and $Y-\bar{Y}$ have same sign $\xRightarrow{\text{on average}} \rho > 0$
- $X-\bar{X}$ and $Y-\bar{Y}$ have opposite sign $\Rightarrow \rho < 0$

$|\rho|$ tells us how much these are true

Ex

n indp. tosses of a coin.

$p(\text{head}) = p$.

$X = \#$ of heads $\qquad Y = \#$ of Tails.

$$X + Y = n \implies E(X) + E(Y) = n$$

$$X + Y = E(X) + E(Y)$$

$$\implies \boxed{X - E(X) = -(Y - E(Y))}$$

$$\mathrm{Cov}(X, Y) = E\left[(X - \bar{X})(Y - \bar{Y})\right]$$

$$= E\left(-(X - \bar{X})^2\right)$$

$$= -E\left((X - \bar{X})^2\right)$$

$$= -\mathrm{Var}\, X$$

$$\rho(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}$$

Show $\mathrm{Var}(X) = \mathrm{Var}(Y)$ $\Longrightarrow$

$$\rho(X,Y) = \frac{-\mathrm{Var}(X)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(X)}} = -1$$

$$\boxed{\rho(X,Y) = -1}$$

More generally can show:

$$|\rho| = 1 \underset{\text{iff}}{\Longleftrightarrow} \exists c \text{ s.t. } Y - \bar{Y} = c(X - \bar{X})$$

$|\rho| = 1 \iff \exists c$ s.th. $Y - \bar{Y} = c(X - \bar{X})$

iff

$$E\left[(Y-\bar{Y})^2\right] = c^2 E\left[(X-\bar{X})^2\right]$$

$$\text{var}\, Y = c^2 \,\text{var}\,(X)$$

Can show that:

X_1, X_2.

$$\text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2) + 2\,\text{Cov}(X_1, X_2)$$

$X_1, X_2, \ldots, X_n$. (not necessarily indp).

$$\mathrm{Var}(X_1 + X_2 + \cdots + X_n) = \sum_{i=1}^{n} \mathrm{Var}(X_i)$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \mathrm{Cov}(X_i, X_j)$$