Nov 10, 04

\[ \text{Inv} \left[ F_1(s) + F_2(s) \right] = \text{Inv} \left[ F_1(s) \right] + \text{Inv} \left[ F_2(s) \right] \]

- Let \( A_1, A_2, \ldots, A_n \) be a list of mutually exclusive, collectively exhaustive events.

- Assume continuous R.V. \( Y \), not independent of \( A_i \).

\[ f_y(y_0) = \sum_i \mathbb{P}(A_i) f_{y|A_i}(y_0|A_i) \]
Define...

\[ M_{y/A_i}(s) \triangleq \int_{-\infty}^{\infty} e^{sy} f_{y/A_i}(y_o|A_i) \, dy_o \]

\[ y_o = \mu \]

\[ = E(e^{-sy} | A_i) \]

\[ \Rightarrow M_{y}(s) = \sum_i P(A_i) M_{y/A_i}(s) \]

\( \text{valid S Transform} \)

\[ \ln \left[ \frac{1}{9} \right] + \frac{8}{9} \]

\[ \frac{N_q}{s + N_q} \]

\( \text{valid S Transform} \)
\[ \frac{1}{6} \cdot 5 \text{ mol} \cdot \frac{5g}{mol} + 8 \cdot \frac{6g}{mol} = 1.83 \text{ mol} \]
Covariance + Correlation

**R.V.** \( x, y \).

- \( \text{Cov}(x, y) \triangleq E[(x - \bar{x})(y - \bar{y})] \)

\( y = x \implies \text{Cov}(x, x) = \text{Var}(x) \)

- **Def.** \( \text{N.E.C.} \quad \text{iff} \quad \text{Cov}(x, y) = 0 \)

\( \implies x, y \text{ uncorrelated} \)

\[ \text{Cov}(x, y) = E[xy] - E[x\bar{y}] - E[y\bar{x}] + E[x \bar{y}] \]

\[ = E[xy] - \bar{y} \bar{x} - \bar{x} y + \bar{y} \bar{x} \]

\[ = E[xy] - \bar{xy} = \]
\[ \text{Cov}(X, Y) = E[XY] - E[X]E[Y] \]

If \( X, Y \) independent,

\[ \text{Cov}(X, Y) = E(X)E(Y) - E(X)E(Y) = 0 \]

\[ \implies \text{If } X, Y \text{ independent } \implies \text{Also uncorrelated} \]

If uncorrelated does not mean independent.
uncorrelated

\[ E(xy) = E(x) E(y) \]

Difference between indp. and uncorrelated.

Indp. deals with pdfs. Uncorrelation only deals with Expect. [5.6]
Intuitively, positive covariance means $x - ar{x}$ and $y - ar{y}$ have the same sign.
Intuitively, negative covariance means $x - ar{x}$ and $y - ar{y}$ have different signs.
Ex. 2  R.V. uncorrelated, but not indp.

Clear: \(x, y\) are not independent

\[
E(xy) = \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 0 = 0
\]

\[E(x) = 0, \quad E(y) = 0\]
Definition: Correlation Coefficient:

\[ p(x, y) = \frac{\text{cov}(x, y)}{\sqrt{\text{var}(x) \text{ var}(y)}} \]

Can show \[|p| < 1\]

- \( x - \bar{x} \) and \( y - \bar{y} \) have same sign \( \Rightarrow p > 0 \)
- \( x - \bar{x} \) and \( y - \bar{y} \) have opposite sign \( \Rightarrow p < 0 \)

\(|p|\) tells us how much these are true.
\( E[XY] = E[X]E[Y] \)

\( \text{Cov}(X, Y) = E[(X-E(X))(Y-E(Y))] \)

\( \text{Var}(Y) = E[(Y-E(Y))^2] \)

\( \text{Var}(X) = E[(X-E(X))^2] = E(X^2) - (E(X))^2 \)

\( X \) and \( Y \) are independent.

\( E(Y) = n \cdot \frac{1}{2} \) as \( n \) heads and \( n \) tails.

\( E(X) = \frac{n}{2} \) as \( n \) heads and \( n \) tails.
\[
P(x, y) = \frac{\text{COV}(x, y)}{\sqrt{\text{Var}(x) \cdot \text{Var}(y)}} \quad \Rightarrow \quad \text{Show}\]

\[
\text{Var}(x) = \text{Var}(y)
\]

\[
P(x, y) = \frac{-\text{Var}(x)}{\sqrt{\text{Var}(x) \cdot \text{Var}(x)}} = -1
\]

\[
P(x, y) = -1
\]

More generally, can show:

\[
|P| = 1 \quad \iff \quad \exists C \text{ s.t. } y - \bar{y} = c(x - \bar{x})
\]
1 \| \| = 1 \quad \iff \quad \exists c \text{ s.t. } y - \bar{y} = c(x - \bar{x})

\begin{align*}
\mathbb{E} \left[ (y - \bar{y})^2 \right] &= c^2 \mathbb{E} [x - \bar{x})^2] \\
\text{Var}(y) &= c^2 \text{Var}(x)
\end{align*}

Can show that:

\[ \text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2) + 2 \text{Cov}(X_1, X_2) \]
$X_1, X_2, \ldots, X_n \text{ (not necessarily ind.)}$

$\text{Var}(X_1 + X_2 + \ldots + X_n) = \sum_{i=1}^{n} \text{Var}(X_i) + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \text{Cov}(X_i, X_j)$