# Bernouli Trial

11/12/04

$$P_x(x_0) = \begin{cases} 1-P & x_0 = 0 \rightarrow \text{failure.} \\ P & x_0 = 1 \rightarrow \text{success.} \\ 0 & \text{otherwise.} \end{cases}$$

$$M_x(s) = 1 - P + e^s P = \sum_{x_0} p_x(x_0)\, e^{s x_0}$$

$$E(x) = P \qquad \sigma_x^2 = P(1-P)$$

# Bernouli Process

Def: Series of indpendent Bernouli Trials each with the same prob of success.

- $n$ indep. Bern. trials.

$k$ = R.V. # of successes in $n$ trials.

$K$ = sum of $n$ indep. Bern. R.V.

what is $P_K(k_0)$?

method (1):

$$M_K(s) = \left[M_x(s)\right]^n = (1-P+e^s P)^n$$

Recall: $(a+b)^n = \sum_{l=0}^{n} \binom{n}{l} a^l b^{n-l}$

$$\Rightarrow M_K(s) \triangleq P_K(0) + e^s P_K(1) + P_K(2) e^{2s} + \cdots$$

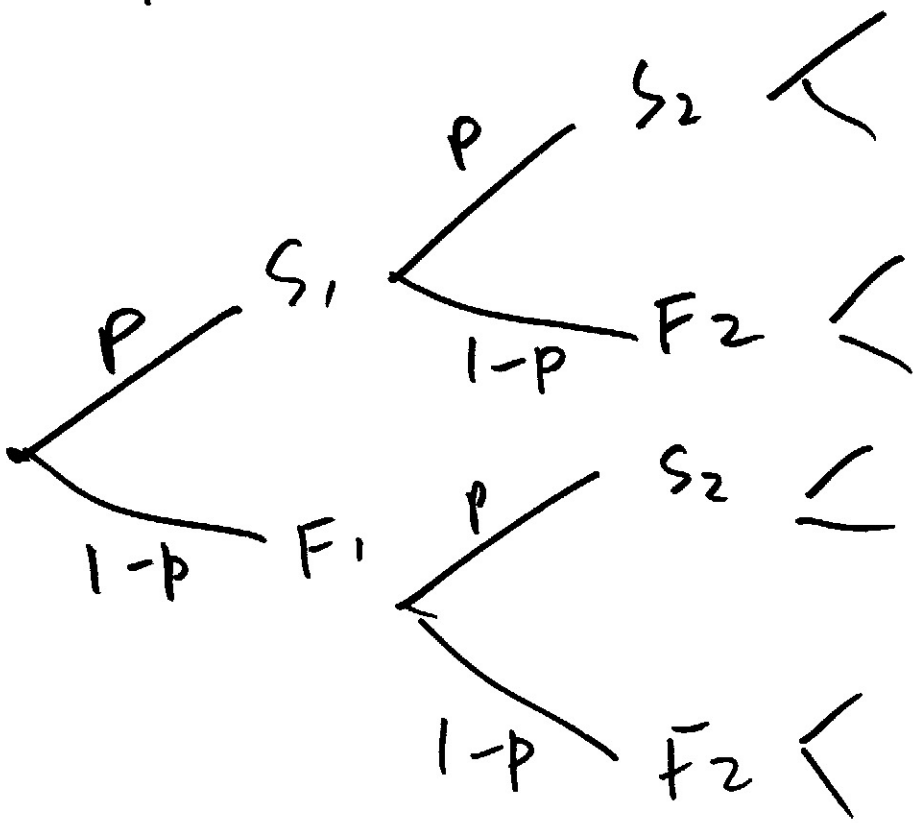
$\Rightarrow$ equate coeff of power of $e^s$

$$P_K(k_0) = \binom{n}{k_0} P^{k_0} (1-P)^{n-k_0}$$

$$\binom{n}{k_0} \triangleq \frac{n!}{(n-k_0)!\,k_0!}$$

<u>method 2</u>: look into sequential sample space.

in an experiment consisting of $n$ indep. Trials.




$$\left.\begin{matrix} S_n \\ F_n \end{matrix}\right\} = \left\{\begin{matrix} \text{success} \\ \text{failure} \end{matrix}\right\} \text{ on the } n\text{th Trial.}$$

outcome of exactly $k_0$ successes at a least of the trees $= p^{k_0}(1-p)^{n-k_0}$

- But, there are $\binom{n}{k_0}$ such leaves.

$$\Rightarrow P_K(k_0) = \binom{n}{k_0} p^{k_0}(1-p)^{n-k_0} \qquad k_0 = 0, 1, 2, \ldots$$

Binomial R.V.

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how to compute mean + Variance.

3 methods
1. directly use def.
2. Transform.
3. $K$ = sum of $n$ indep. R.V. $X_i$ $i=1,\ldots,n$

Born. Trial.
R.V.
$1$ = success $p$.
$0$ = failure $1-p$

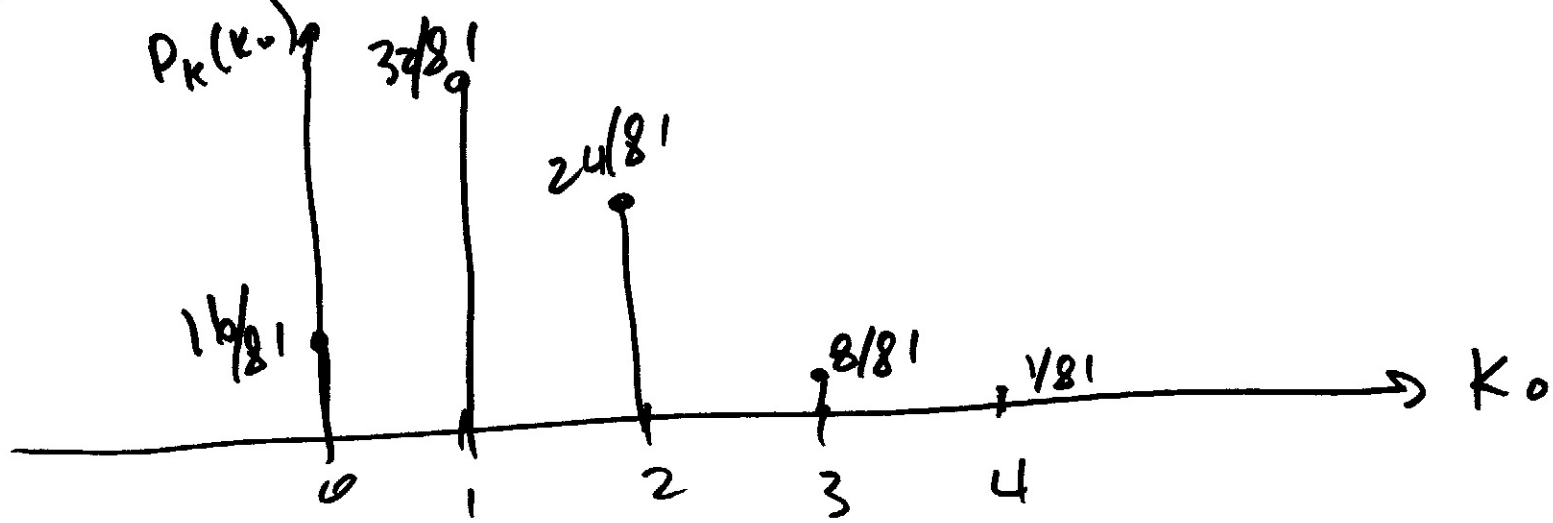
$X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5 \quad \ldots \quad X_n$

$n$ of them

$E(k) = n \quad E(x) = np.$

$Var(k) = n \quad Var(x) = n\,p(1-p)$

Ex plot Binomial. $p = 1/3$ $\quad n = 4$




$$P_k(k_0) = \binom{4}{k_0} \left(\frac{1}{3}\right)^{k_0} \left(\frac{2}{3}\right)^{4-k_0}$$

# Inter-arrival Times for Bern. Process

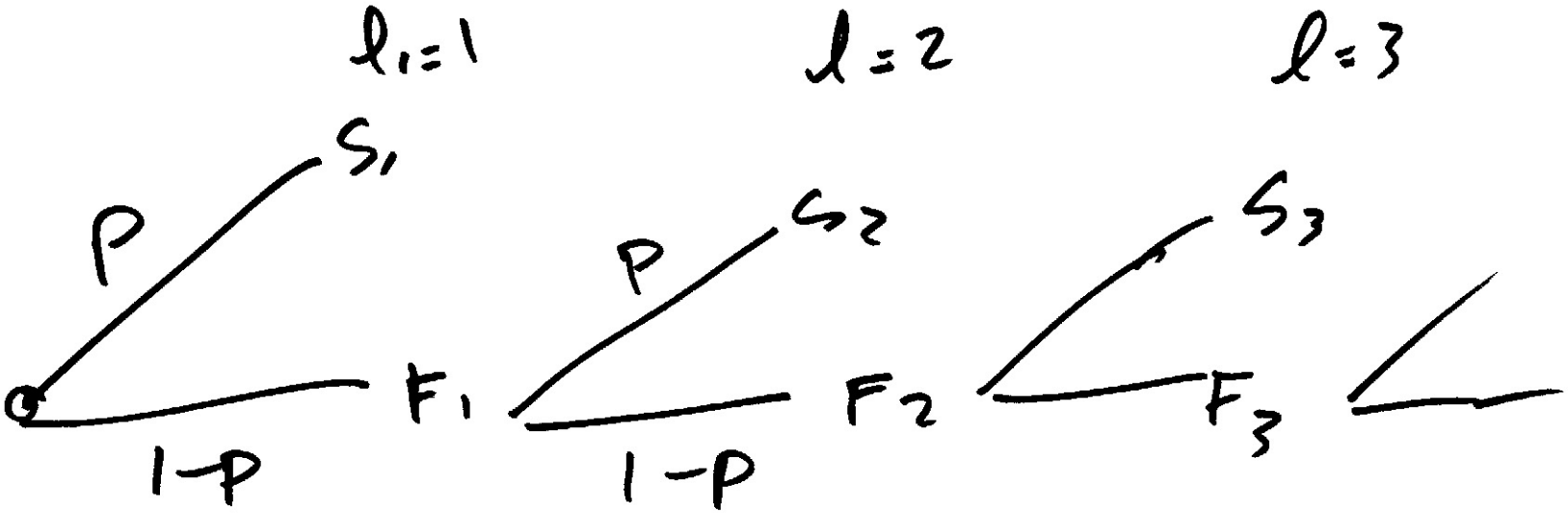
**Def** success in Bernoulli process is defined as an **Arrival**

$l_1$ = discrete R.V. # of Bernoulli trials up to and including the **first** success.

$\triangleq$ **first** order interarrival time.

taken on discrete values 1, 2, 3, . . . . .

what is pmf for $l_1$ ?




$$P_{l_1}(l) = (1-P)^{l-1} P \qquad l = 1, 2, \ldots$$

geometric PMF.

Transform: $$M_{l_1}(s) = \sum_{l=0}^{\infty} P_{l_1}(l)\, e^{sl}$$

$$= \sum_{l=1}^{\infty} P(1-P)^{l-1} e^{sl}$$

$$= \frac{e^{s} P}{1 - e^{s}(1-P)}$$

$$E(\ell_1) = \left[ \frac{d}{ds} M_{\ell_1}(s) \right]_{s=0} = \frac{1}{p}$$

$$\sigma_\ell^2 = \frac{1-p}{p^2}$$

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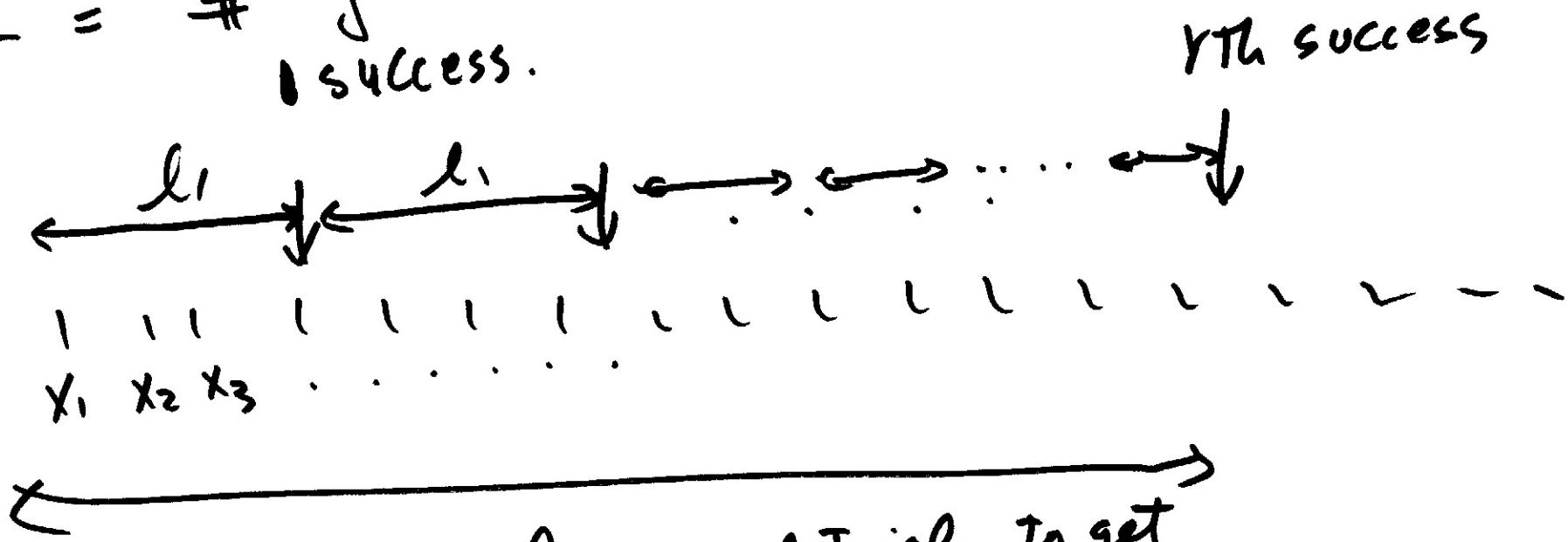
<u>Fresh start</u>: For any given time $n$, seq of R.V. $X_{n+1}, X_{n+2}, \ldots \ldots$ (The future process) is also a Bern. process and is indp. from $X_1 \ldots X_n$ (past process)

<u>Memoryless</u>: Given no success in first $m$ trials what is the conditional pmf for first success?

# rth order Interarrival Time ?

$l_r$ = # of trials up to and including rth success.




$l_r$ = # of Trials to get rth success.

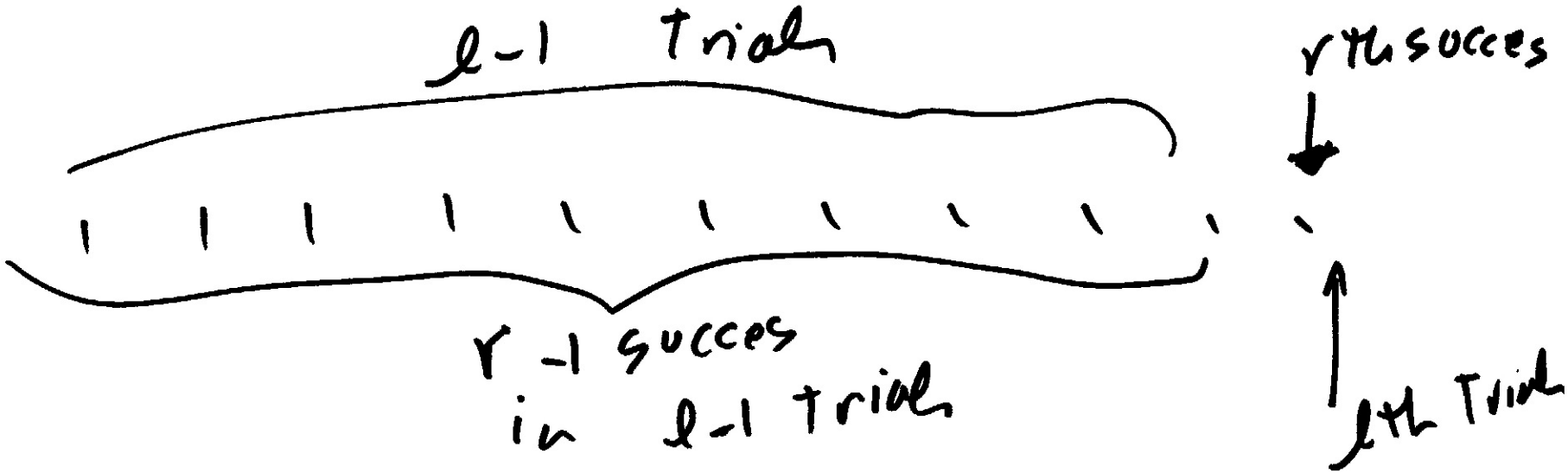
$l_r$ = sum of r indep. experimental values of R.V. $l_1$

$$M_{l_r}(s) = \left[ M_{l_1}(s) \right]^r = \left[ \frac{e^s p}{1 - e^s (1-p)} \right]^r$$

need to compute $P_{lr}(l)$ = pmf for $lr$.

= prob $r$th success is at $l$th trial.

$P_{lr}(l) =$ (Prob of having exactly $r-1$ successes in the first $l-1$ trials.) $\times$ (Given exactly $r-1$ successes in the previous $l-1$ trials, Conditional prob of having $r$th success on the $l$th trial.)




=

$$P_{\ell_r}(\ell) = \binom{\ell-1}{r-1} p^{r-1} (1-p)^{\ell-1-(r-1)} \quad p$$

$$P_{\ell_r}(\ell) = \binom{\ell-1}{r-1} p^{r} (1-p)^{\ell-r}$$

$$\ell = r, r+1, r+2, \ldots\ldots$$

$$\Longrightarrow E(\ell_r) = r\,E(\ell_1) = \frac{r}{p}$$

$$\sigma^2(\ell_r) = r\,\sigma^2_{\ell} = \frac{r(1-p)}{p^2}$$

EX - Fred gives out samples of dog food door to door.

- leave a sample if.
  - door is answered **AND**
  - There is a dog

P( door answer) = 3/4 P( dog in residence ) = 2/3

door answer + having dog are indep.

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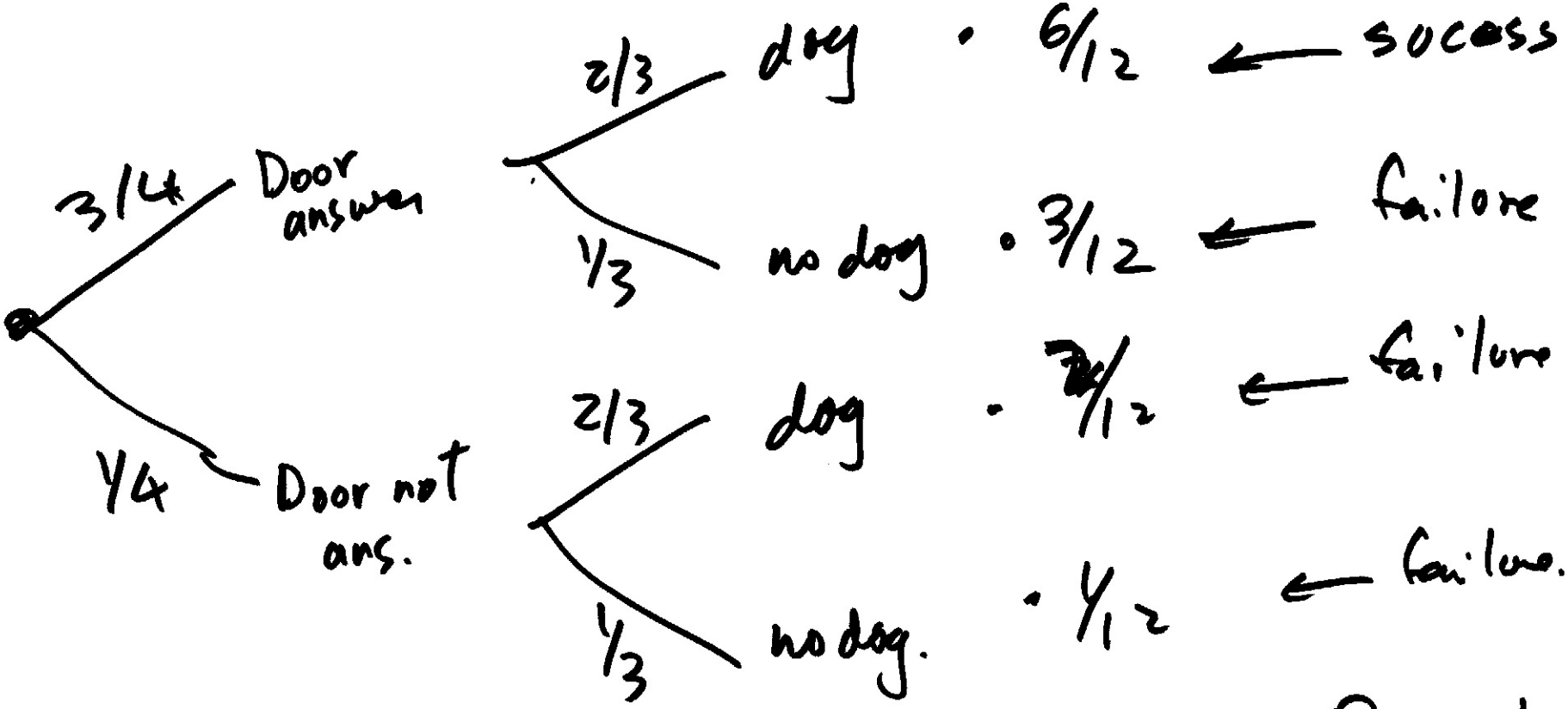
Q Prob that gives away his 1st sample in his 3rd call?

$$(1-P)^2 P = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 1/8.$$

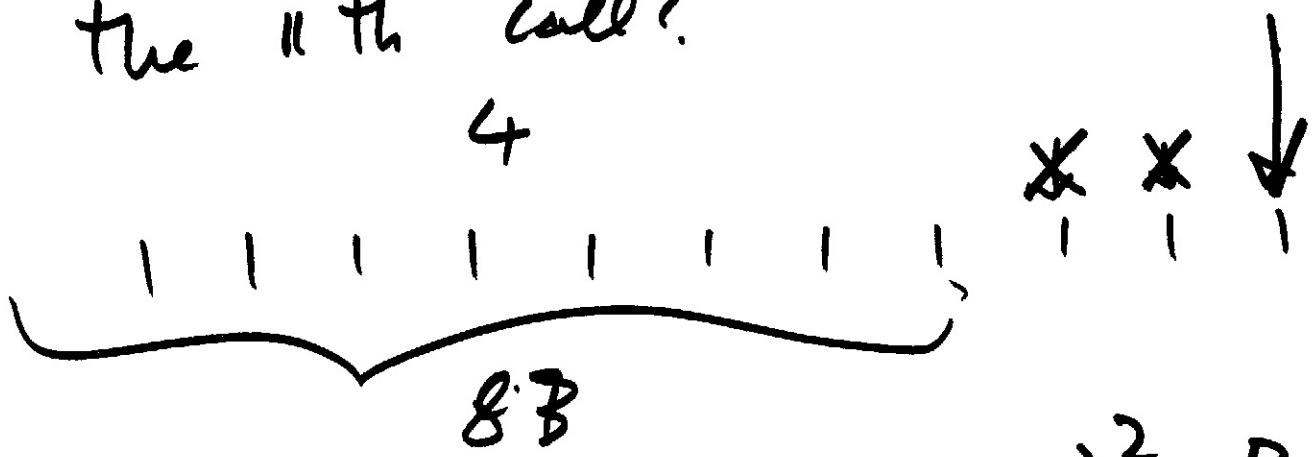



Bernoulli process = Prob success $\triangleq P = \frac{1}{2}$

prob failure $= 1-P$

success = handing in a can of dog food.

Q Given that he has given away exactly 4 samples on his 1st eight calls, what is the conditional prob that he will give away his 5th sample on the 11th call?




$$(1-p)^2 \, p$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

Q Pr( gives away his 2nd sample on 5th call) =

$$P_{\ell 2}(\ell) = \binom{\ell-1}{2-1} p^2 (1-p)^{\ell-2}$$

$$P_{\ell 2}(5) = \binom{5-1}{2-1} p^2 (1-p)^{5-2}$$

$p = 1/2$

$$= \frac{1}{8}$$

(1) Given that he didnot give away his 2nd sample on his 2nd call, what is the conditional prob. that he will leave his 2nd sample on 5th call?

$$P_{l_2 / l_2 > 2}\left( l_2 = 5 \mid l_2 > 2 \right) =$$

$$= \frac{P( l_2 = 5 \text{ and } l_2 > 2 )}{P( l_2 > 2 )}$$

$$= \frac{P( l_2 = 5 )}{1 - P_{l_2}(2)}$$

$$= \frac{\binom{5-1}{2-1} p^2 (1-p)^{5-2}}{1 - \binom{2-1}{2-1} p^2 (1-p)^0} = 1/6$$

<u>Q</u> <u>Det</u> needs a new supply immediately after giving away the last can.

starts 2 can.

Prob <u>at least</u> 5 calls before he needs new supply.

<u>A</u> $P(l_2 \geq 5) = 1 - P(l_2 \leq 4)$

$$= 1 - \sum_{\ell=2}^{4} \binom{\ell-1}{2-1} p^2 (1-p)^{\ell-2}$$

$$= 5/16$$