P[ A + CD + B(A+C|D) ]

Real *

Preparation of Prob laws

- If $A \subset B$, $P(A) < P(B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
\[
P(A \cup B) = P(A) + P(B) - P(AB)
\]

\[
P(A + B) \leq P(A) + P(B)
\]

- Conditional prob.
- Trees
- Total Prob. Then
- Bayes Rule.

& Topics
Conditional Prob
provide a way to reason about outcome of
an exp. based on partial informa.

Ex 2 successive rolls of a die.

Given Sum = 9 → event B. event A

what is Prob first roll was 6?

\[(3, 6), (4, 5), (5, 4), (6, 3)\]

\[\frac{1}{4} = P(A|B)\]
\[ P(\text{A|B}) = \frac{\text{area of blue}}{\text{area of B}} = \frac{\text{area of } \text{AB}}{\text{area of } \text{B}} \]

\[ = \frac{\text{area of } \text{AB}}{\text{P(AB)}} \cdot \frac{\text{P(AB)}}{\text{P(B)}} = \frac{\text{P(AB)}}{\text{P(B)}} \]

\[ \text{B} = \{(3,6), (4,5), (5,4), (6,3)\} \]
\[ P(A | B) = \frac{P(AB)}{P(B)} \]

\[ P(AB) = \frac{1}{36} = \text{prob (sum is 9 and first roll is 6)} \]

\[ P(B) = \frac{4}{36} \]

\[ P(A/B) = \frac{1/36}{4/36} = 1/4 \]

Reminder:
\[ AB = A \cap B \]
\[ A + B = A \cup B \]
\[ \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{P(\text{even})}{P(\text{2 and even})} = \frac{P(\text{even})}{P(\text{2 and even})} \]

\[ \frac{\frac{2}{4}}{\frac{1}{6}} = \frac{P(\text{2})}{P(\text{2})} = \frac{3}{2} \]

Given outcome in even odds in \( P(\text{2}) \).
Ex. Fair coin twice.

Given: observed: at least one toss was a head. $\rightarrow B$

What is the prob both tosses were heads $A$

$\Pr(A|B)$

Two ways: 1) intuitively:
- H, H: $\frac{1}{4}$
- H, T: $\frac{1}{4}$
- T, H: $\frac{1}{4}$
- T, T: $\frac{1}{4}$

2) Equation: $\Pr$ $A$: $\frac{1}{4}$ $\checkmark$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$
Prob Trees for Sequential Exps

- For Exp. seq character, can use conditional prob to determine unconditional prob.

\[ P(A_1 B_1) = P(A_1) P(B_1 | A_1) \]
Ex. If aircraft present, prob radar registers is 0.99.

- If aircraft is NOT present, prob radar registers something is 0.1
- Aircraft is present ≤ x Time.

Compute prob false alarm and prob miss

\[ A = \{ \text{aircraft present} \} \quad A^c = \{ \text{aircraft absent} \} \]
\[ B = \{ \text{radar says, "yes"} \} \quad B^c = \{ \text{radar says, "no"} \} \]

\[ P(\text{false alarm}) = P( A^c B) \]
\[ P(\text{miss}) = P( A B^c) \]
Multiplication Rule

\[
P(B|A) = \frac{P(AB)}{P(A)}
\]

\[
P(AB) = P(A) \cdot P(B|A)
\]

or any set of events

\[
P\left( A_1, A_2, \ldots, A_n \right) = P(A_1) P(A_2|A_1) P\left( A_3 \bigg| A_1, A_2 \right) \cdots P\left( A_n \bigg| A_1, \ldots, A_{n-1} \right)
\]
Ex: 52 card deck.

Draw 3 cards without replacement.

\[ P(\text{none of cards in heart}) \]

\[ A_i = \text{ith card is not a heart} \]

\[ P( A_1 \text{ and } A_2 \text{ and } A_3 ) = \]

\[ P( A_1 ) = \frac{39}{52} \]

\[ P( A_2 \mid A_1 ) = \frac{38}{51} \]

\[ P( A_3 \mid A_1 A_2 ) = \frac{37}{50} \]

\[ P( A_1 A_2 A_3 ) = P( A_1 ) P( A_2 \mid A_1 ) P( A_3 \mid A_1 A_2 ) \]
\[ = \frac{39}{52} \times \frac{38}{51} \times \frac{37}{50} \]

**Total Prob. Then**

- An disjoint events that form partition of sample space.

For any event \( B \),

\[
P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)
\]

\[
= P(A_1) P(B | A_1) + P(A_2) P(B | A_2) + P(A_3) P(B | A_3)
\]

\[ p(A_i) \neq 0 \]
\[ P(B) = P(\text{A}_1 \cap B) + P(\text{A}_2 \cap B) + P(\text{A}_3 \cap B) \]

\[ = P(\text{A}_1) P(B | \text{A}_1) + P(\text{A}_2) P(B | \text{A}_2) + P(\text{A}_3) P(B | \text{A}_3) \]

\[ = P(B | \text{A}_1) P(\text{A}_1) + P(B | \text{A}_2) P(\text{A}_2) + P(B | \text{A}_3) P(\text{A}_3) \]

\[ = P(B | \text{A}_1) P(\text{A}_1) + P(B | \text{A}_2) P(\text{A}_2) + P(B | \text{A}_3) P(\text{A}_3) \]
Ex chess.

Type 1 = 50\%

Type 2 = \frac{1}{4}

Type 3 = \frac{1}{4}

P(\text{winning against type 1}) = 30\%.

P(\text{winning against type 2}) = 40\%.

P(\text{winning against type 3}) = 50\%.

B = \text{event of winning}.

A_i = \text{event of playing against type } i.

P(\text{winning}) = P(B) = 30\%.

P(A_1) = 25\%.

P(A_2) = 25\%.

P(A_3) = 25\%.

P(B) = P(\text{type 1})P(\text{win}/1) + P(2)P(\text{win}/2) + P(3)P(\text{win}/3)

= 50\% \times 30\% + 25\% \times 40\% + 25\% \times 50\%

= 0.375