UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Problem Set 7
Fall 2006

Issued: Thursday, October 19, 2006 **Due:** Friday, October 27, 2006

Reading: For this problem set: §4.1—4.4

Problem 7.1
Xavier and Yolanda enter a frisbee-throwing contest. The distance (in meters) that Xavier throws is uniformly distributed between 0 and 100, and the distance that Yolanda throws (in meters) is exponential with $\lambda = 1/60$.

(a) What is the probability that Xavier's frisbee lands at 75m?

(b) What is the probability that Yolanda throws further than 100m?

(c) What is the expected distance of each competitor's throw?

(d) Which of the two competitors is more likely to throw further?

(e) Given that Xavier's frisbee lands at 75m, find PDF for the distance of Yolanda's throw.

(f) Let W be the distance Yolanda's frisbee lands past Xavier's. Find the PDF of W.

Problem 7.2
An annoying professor is known for his arbitrary grading policies. Each paper receives a grade from the set $\{A, B, C\}$, with equal probability, independently of other papers. What is the PMF of the number of papers that you hand in before you receive each possible grade at least once?

Problem 7.3
Alice and Bob flip biased coins independently. On each toss,Alice's coin comes up heads with probability 1/4, while Bob's coin comes up heads with probability 3/4. Successive tosses are independent of one another. Alice and Bob each stop as soon as they get a head; that is, Alice stops when she gets a head while Bob stops when he gets a head. What is the PMF of the total amount of flips until both stop? (That is, what is the PMF of the combined total amount of flips for both Alice and Bob until they stop?)

Problem 7.4
Consider random variable Z with transform:

$$M_Z(s) = \frac{8 - 3s}{s^2 - 6s + 8}$$

(a) Find $\mathbb{P}(Z \geq 0.5)$.

(b) Find $\mathbb{E}[Z]$ by using the probability distribution of Z.

(c) Find $\mathbb{E}[Z]$ by using the tranform of Z and without explicity using the probability distribution of Z.

(d) Find $\text{var}(Z)$ by using the probability distribution of Z.

(e) Find $\text{var}(Z)$ by using the tranform of Z and without explicity using the probability distribution of Z.

Problem 7.5

Four fair 6-sided dice are rolled independently of each other. Let X_1 be the sum of the numbers on the first and second dice, and X_2 be the sum of the numbers on the third and fourth dice. Convolve the PMFs of the random variables X_1 and X_2 to find the probability that the outcomes of the four dice rolls sum to 8.

Problem 7.6

A coin is tossed repeatedly, heads appearing with probability q on each toss. Let random variable T denote the number of tosses when a run of n consecutive heads has appeared for the first time.

(a) Show that the PMF for T can be expressed as

$$p_T(k) = \begin{cases} 0 & , \quad k < n \\ q^n & , \quad k = n \\ \left(\sum_{i=k-n}^{\infty} p_T(i) \right) (1-q)q^n & , \quad k \geq n+1 \end{cases} .$$

(b) Determine the transform $M_T(s)$ associated with random variable T.

(c) Compute the expectation $\mathbb{E}[T]$.

Problem 7.7

Suppose you are playing roulette with a biased wheel, such that your chance of winning on each spin is $p > 1/2$. Suppose that you start out with x dollars, and that on each play, you bet $2p - 1$ of what you have. Assuming that when you win a round you win what you bet, and that otherwise you lose what you bet, find your expected holdings after n rounds of play.