UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Problem Set 1
Fall 2006

Issued: Thursday, August 31, 2006 **Due:** Friday, September 8, 2006

Reading: For this problem set: §1.1, §1.2. and §1.3

Problem 1.1
Express each of the following events in terms of the events A, B and C as well as the operations of complementation, union and intersection:

(a) at least one of the events A, B, C occurs;

(b) at most one of the events A, B, C occurs;

(c) none of the events A, B, C occurs;

(d) all three events A, B, C occur;

(e) exactly one of the events A, B, C occurs;

(f) events A and B occur, but not C;

(g) either event A occurs or, if not, then B also does not occur.

In each case draw the corresponding Venn diagrams.

Problem 1.2
Compute $P\left(A \cup (B^c \cup C^c)^c\right)$ in each of the following cases:

(a) A, B, C are mutually exclusive events and $P(A) = 3/7$.

(b) $P(A) = 1/2$, $P(B \cap C) = 1/3$, $P(A \cap C) = 0$.

(c) $P\left(A^c \cap (B^c \cup C^c)\right) = 0.65$.

Problem 1.3
Let A and B be two events. Use the axioms of probability to prove the following:

(a) $P(A^c) = 1 - P(A)$.

(b) If $A \cap B = \emptyset$, then $P(A) \leq P(B^c)$.

Problem 1.4
Suppose that each of two people choose at random a number x and y, respectively, between zero and one. (We take "at random" to mean "according to the uniform probability law" introduced in lecture.) Consider the following events:

$$\begin{aligned} A &= \{\text{The magnitude of the difference of the two numbers is at most } 1/3.\} \\ B &= \{\text{None of the numbers exceeds } 2/3.\} \\ C &= \{\text{The two numbers are equal.}\} \end{aligned}$$

Find the following probabilities: $\mathbb{P}(A)$, $\mathbb{P}(B)$, $\mathbb{P}(A \cap B)$, and $\mathbb{P}(C)$.

Problem 1.5
Let A and B be two events. Show the following inequalities. A mathematical derivation is required, but you can use Venn diagrams to guide your thinking.

(a) $\mathbb{P}(A) + \mathbb{P}(B) \geq \mathbb{P}(A \cup B) \geq \max\{\mathbb{P}(A), \mathbb{P}(B)\}$.

(b) $\min\{\mathbb{P}(A), \mathbb{P}(B)\} \geq \mathbb{P}(A \cap B) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1$.

Under what conditions on A and B are these inequalities satisfied as equalities?

Problem 1.6

Mike and John are playing a friendly game of darts where the dart board is a disk of a radius of 10 inches. Whenever a dart falls within 1 inch of the center 50 points are scored. If the point of impact is between 1 and 3 inches from the center 30 points are scored, if it is at a distance of 3 to 5 inches then 20 points are scored, and if it is further than 5 inches then 10 points are scored. Assume that both players are skilled enough to be able to throw the dart within the boundaries of the board.

Mike can place the dart uniformly on the board (i.e. the probability of the dart falling in a given region is proportional to its area.)

(a) What is the probability that Mike scores 50 points on one throw?

(b) What is the probability of him scoring 30 points?

(c) John is right handed and is twice more likely to throw in the right half of the board than in the left half. Across each half, the dart falls uniformly in that region. Answer the previous questions for John's throw.