UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Problem Set 10
Fall 2006

Issued: Wednesday, November 22, 2006 **Due:** Friday, December 1, 2006

Reading: For this problem set: Chapter 5

Problem 10.1

You are visiting the rainforest, but unfortunately your insect repellent has run out. As a result, at each second, a mosquito lands on your neck with probability 0.5. If one lands, with probability 0.2 it bites you, and with probability 0.8 it never bothers you, independently of other mosquitoes. What is the expected time between successive bites? What is the variance of the time between successive bites?

Problem 10.2

On each trial of a game, both Don and Greg simultaneously, but independently, flip biased coins. On each trial, the probability that Don's flip results in a head is p_D, while the probability that Greg's flip results in a head is p_G.

(a). Given that the flips on a particular trial resulted in 2 heads, find the PMF for M, the number of additional trials up to and including the next trial on which 2 heads result.

(b). Given that the flips on a particular trial resulted in *at least* one head, find the probability that Don flipped a head on the trial.

(c). Starting from a trial on which no heads result, find the probability that Don's next flip of a head will occur *before* Greg's next flip of a head.

Problem 10.3

For each night, the probability of a robbery attempt at the local warehouse is $\frac{1}{5}$. A robbery attempt is successful with probability $\frac{3}{4}$, independent of the night. After any particular SUCCESSFUL robbery, the robber celebrates by taking off either the next 2 or 4 nights (with equal probability), during which time there will be no robbery attempts. After that, the robber returns to his original routine.

(a). Let D be the number of days until (and including) the second <u>successful</u> robbery, including the days of celebration after the first robbery. Find the PMF of D, or its transform (whichever you find more convenient).

During a successful robbery, the robber steals a random number of candy bars, which is 1, 2, or 3, with equal probabilities. This number is independent for each successful robbery and independent of everything else (no candy bars are stolen in unsuccessful robberies).

(b). Let T be the number of candy bars collected in ten robbery <u>attempts</u> (whether successful or not). Find the PMF of T, or its transform, whichever is easier. Find the expectation and the variance of T.

Problem 10.4

A certain bridge crosses a river running North-South (the bridge runs East-West). This bridge is so old and narrow, that two cars cannot fit on it side by side. For this reason, when a car is on the bridge, no cars moving in the opposite direction are allowed to use the bridge. Cars traveling from East to West arrive according to a Poisson process of rate λ, and cars traveling West to East arrive according to an independent Poisson process of rate μ. Suppose that it takes one minute to cross the bridge. Starting with an empty system, i.e. no cars on the bridge, find the distribution of:

(a). N, where N is the number of the first East-West car that experiences a conflict.

(b). T_k, where T_k is the arrival time of car k.

Problem 10.5

Transmitters A and B independently send messages to a single receiver in a Poisson manner with average message arrival rates of λ_A and λ_B, respectively. All messages are so brief that we may safely assume that they occupy only single points in time.

The number of words in every message, regardless of its transmitting source, may be considered to be an independent experimental value of random variable W with PMF

$$p_W(w) = \begin{cases} 2/6 & , \quad w = 1 \\ 3/6 & , \quad w = 2 \\ 1/6 & , \quad w = 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

(a). What is the probability that, during an interval of duration t, a total of exactly nine messages will be received?

(b). Let N be the total number of <u>words</u> received during an interval of duration t. Determine the expected value for random variable N.

(c). Determine the PDF for X, the time from $t = 0$ until the receiver has received exactly eight <u>three-word messages</u> from transmitter A.

(d). Independent of what happens to all other words, a transmitter damages any particular word it sends with probability 10^{-3}. What is the probability that any particular damaged word is part of a three-word message?

(e). What is the probability that exactly eight of the next twelve messages received will be from transmitter A?

Problem 10.6

A transmitter sends out either a 1 with probability p, or a 0 with probability $(1 - p)$. If the number of transmissions within a given time interval is Poisson with rate λ, show that the number of 1's transmitted in that same time interval is also Poisson, and has rate $p\lambda$.