UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: PROBABLITY AND RANDOM PROCESSES

Problem Set 2
Fall 2006

Issued: Thursday, September 7, 2006 **Due:** Friday, September 15, 2006

Reading: For this problem set: §1.3, §1.4. and §1.5

Problem 2.1

The weather on any given day can be sunny, cloudy, rainy, or snowy. Let us suppose a snowy day can happen only during the winter, and that each season has 90 days. Assume moreover that: (i) within each season, cloudy days and rainy days are equally likely, (ii) in the spring and the fall, sunny days and rainy days are equally likely, (iii) in the summer, sunny days are twice as likely as rainy days, and (iv) in the winter, sunny and snowy days are each one half as likely as rainy days. What is the probability that a day chosen at random from these 360 days is sunny?

Problem 2.2

A ball is in any one of n boxes. It is in the ith box with probability P_i. If the ball is in box i, a search of that box will uncover it with probability α_i. Show that the conditional probability that the ball is in box j, given that a search in box i did *not* uncover it, is

$$\mathbb{P}\left[\text{ball in box } j \mid \text{not found by search } i\right] = \begin{cases} \frac{P_j}{1-\alpha_i P_i} & \text{if } j \neq i, \\ \frac{(1-\alpha_i)P_i}{1-\alpha_i P_i} & \text{if } j = i. \end{cases}$$

Problem 2.3

Oswald likes to predict the moves of a particular stock on the stock market, but unaware of the efficient market hypothesis, Oswald bases his predictions on the movement of the stock on the previous day. He claims that the probability that this stock will move in the same direction tomorrow as it did today, is p. Let S_n be the probability that according to Oswald's model, the stock will move in the same direction as it did today, n days from now.

(a) Show that S_0=1, and $S_n = (2p-1)S_{n-1} + (1-p)$ for $n \geq 1$.

(b) Prove that $S_n = \frac{1+(2p-1)^n}{2}$. (*Hint:* A proof by mathematical induction is one option.)

Problem 2.4

One rainy afternoon, you decide to play a game using two dice. The first die has 2 green faces and 4 blue faces. The second die has 4 orange faces and 2 blue faces. You randomly select one of the two dice to roll in such a way that you select the orange/blue die with probability p and the other die with probability $1-p$. Whichever die is selected, that die will be rolled until a blue face is showing, at which point the game is over. After playing this (incredibly exciting) game many times, you observe that the probability that a game is concluded in exactly three tosses of the selected die is $\frac{7}{81}$. What is the value of p?

Problem 2.5

For each one of the following statements, indicate whether it is true or false, and provide a brief explanation.

(a) If $\mathbb{P}(A \mid B) = \mathbb{P}(A)$, then $\mathbb{P}(B \mid A^c) = \mathbb{P}(B)$.

(b) If 5 out 10 independent fair coin tosses resulted in tails, the events "first toss was tails" and "10th toss was tails" are independent.

(c) If 10 out 10 independent fair coin tosses resulted in tails, the events "first toss was tails" and "10th toss was tails" are independent.

(d) If the events $A_1, \ldots, A_n$ form a partition of the sample space, and if B, C are some other events, then

$$\mathbb{P}(B \mid C) = \sum_{i=1}^{n} \mathbb{P}(A_i \mid C)\, \mathbb{P}(B \mid A_i).$$

Problem 2.6

Consider three events A_1, A_2 and A_3. Prove that A_1, A_2, A_3 independent implies that A_1 and $A_2 \cup A_3$ are also independent.

Problem 2.7

Fischer and Spassky play a sudden death chess match. Each game ends up with either a win by Fischer, this happens with probability p, a win for Spassky, this happens with probability q, or a draw, this happens with probability $1 - p - q$. The match continues until one of the players wins a game (and the match).

(a) What is the probability that Fischer will win the last game of the match?

(b) Given that the match lasted no more than 5 games, what is the probability that Fischer won in the first game?

(c) Given that the match lasted no more than 5 games, what is the probability that Fischer won the match?

(d) Given that Fischer won the match, what is the probability that he won at or before the 5th game?