UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: PROBABLITY AND RANDOM PROCESSES

Problem Set 2
Fall 2007

Issued: Thursday, September 6, 2007 **Due:** Friday, September 14, 2007

Reading: Bertsekas & Tsitsiklis, §1.3, §1.4. and §1.5

Problem 2.1
You keep taking your driving exam repeatedly, until the first time (if ever) that you pass it. What is the sample space for this experiment? Suppose that your test attempts are independent, and that you have probability p of passing. What is the probability of taking more than 10 attempts to pass?

Problem 2.2
Let A and B be independent events. Prove the following:

(a) A and B^c are independent.

(b) A^c and B^c are independent.

Problem 2.3
For each one of the following statements, indicate whether it is true or false, and provide a brief explanation.

(a) If $\mathbb{P}(A \mid B) = \mathbb{P}(A)$, then $\mathbb{P}(B \mid A^c) = \mathbb{P}(B)$.

(b) If 5 out 10 independent fair coin tosses resulted in tails, the events "first toss was tails" and "10th toss was tails" are independent.

(c) If 10 out 10 independent fair coin tosses resulted in tails, the events "first toss was tails" and "10th toss was tails" are independent.

(d) If the events $A_1, \ldots, A_n$ form a partition of the sample space, and if B, C are some other events, then

$$\mathbb{P}(B \mid C) = \sum_{i=1}^{n} \mathbb{P}(A_i \mid C)\mathbb{P}(B \mid A_i).$$

Problem 2.4
Anne, Betty, Chloe and Daisy were all friends in school. Subsequently each of the six subpairs meet up once; at each of the six meetings, the pair quarrels with some fixed probability p and otherwise the pair retains a firm friendship. Quarrels take place independently of each other. In the future, if any one of the four hears a rumour, she tells it to her firm friends only. Supposing that Anne hears a rumour, what is the probability that:

(a) Daisy hears it?

(b) Daisy hears it given that Anne and Betty have quarrelled?

(c) Daisy hears it given that Betty and Chloe have quarrelled?

(d) Daisy hears it given that she has quarrelled with Anne?

Problem 2.5

Imagine that you have n drawers in your filing cabinet, and that you left your term paper in drawer k with probability D_k. Furthermore, suppose that these drawers are so messy, that even if you correctly guess that the term paper is in drawer j, the probability that you find it is p_j. Suppose you search for your paper in a particular drawer, say drawer i, and the search is unsuccessful. Conditional on this event, show that the probability that your paper is in drawer j is:

(a) $\frac{D_j}{1-p_i D_i}$, if $j \neq i$, and

(b) $\frac{D_i\,(1-p_i)}{1-p_i D_i}$ if $j = i$.

Problem 2.6

Three persons roll a fair n-sided die once. Let A_{ij} be the event that person i and person j roll the same face. Show that the events A_{12}, A_{13}, and A_{23} are pairwise independent but are not independent.

Problem 2.7

Larry and Consuela love to challenge each other to coin flipping contests. Suppose that Larry brings $2n + 1$ fair coins to school, and assume that he lets Consuela play with $n + 1$ coins, and he plays with the remaining n coins. Show that the probability that after all the coins have been flipped Consuela will have gotten more heads than Larry is $\frac{1}{2}$.