UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS126: Probability in EECS

Problem Set 10
Fall 2013

Issued: Tuesday, November 12, 2013 **Due:** In class Thursday, November 21, 2013

Problem 1. Suppose that the grades of students in a class are distributed as a mixture of two Gaussian distribution, $N(\mu_1, \sigma_1^2)$ with probability p and $N(\mu_2, \sigma_2^2)$ with probability $1-p$. All the parameters $\theta = (\mu_1, \sigma_1, \mu_2, \sigma_2, p)$ are unknown.

(a) You observe n i.i.d. samples, $y_1, \dots, y_n$ drawn from the mixed distribution. Find $f(y_1, \dots, y_n|\theta)$.

(b) Let random variable X_i be 0 if $Y_i \sim N(\mu_1, \sigma_1^2)$ and 1 if $Y_i \sim N(\mu_2, \sigma_2^2)$. Find $MAP[X_i|Y_i, \theta]$.

(c) Implement Hard EM algorithm to approximately find $MLE[\theta|Y_1, \dots, Y_n]$. To this end, use MATLAB to generate 1000 data points $(y_1, \dots, y_{1000})$, according to $\theta = (10, 5, 30, 10, 0.4)$. Use your data to estimate θ. How well is your algorithm working?

Problem 2. Consider a single queue with one server in discrete time. At each time, a new customer arrives to the queue with probability λ, and if the server works on the queue with full capacity, one customer is served with probability $\mu = 1$. $(\lambda < 1)$ Due to energy constraint you want your server to work on the queue with capacity as small as possible without making the queue blow up. Thus, you want your server to allocate capacity $p^* = \lambda$ to the queue. Unfortunately, you don't know the value of λ. All you can observe is the queue-length. We try to design an algorithm based on stochastic gradient to learn p^* in the following steps.

(a) Minimize the function $V(p) = \frac{1}{2}(\lambda - \mu p)^2$ over p using gradient descent. (Enough to describe the algorithm)

(b) Find $E[Q(n+1) - Q(n)|Q(n) = q > 0]$, given that server allocates capacity p_n at time slot n. $Q(n)$ is the queue-length at time n. What happens if $q = 0$?

(c) Use stochastic gradient projection and write a Matlab code based on parts (a) and (b) to learn p^*. Note that $0 \leq p \leq 1$. [Hint: to avoid the case that queue-length is 0, start with a large initial queue-length.]

Problem 3. Suppose that $y_1, \dots, y_n$ are i.i.d. samples of $N(\mu, \sigma^2)$. What is a sufficient statistic for estimating μ given $\sigma = 1$. What is a sufficient statistic for estimating σ given $\mu = 1$?

Problem 4. Customers arrive to a store according to a Poisson process with rate 4 (per hour).

(a) What is the probability that exactly 3 customers arrive during an hour?

(b) What is the probability that more than 40 minutes is required before the first customer arrives?

Problem 5. Consider two independent Poisson processes with rates λ_1 and λ_2. Those processes measure the number of customers arriving in store 1 and 2.

(a) What is the probability that a customer arrives in store 1 before any arrives in store 2?

(b) What is the probability that in the first hour exactly 6 customers arrive at the two stores? (The total for both is 6)

(c) Given exactly 6 have arrived at the two stores, what is the probability all 6 went to store 1?