UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS126: Probability in EECS

Problem Set 11
Fall 2013

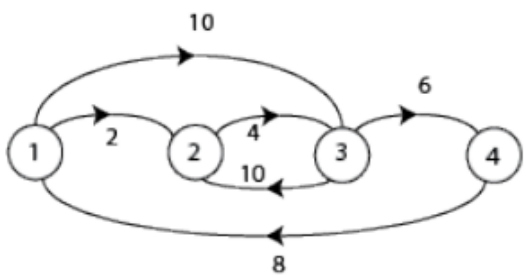
Issued: Tuesday, November 19, 2013 **Due:** In class Tuesday, November 26, 2013

Problem 1. Consider the continuous time Markov chain in Figure 1.

(a) Find the invariant distribution.

(b) Simulate the MC and see that the fraction of time spent in state 1 converges to $\pi(1)$.

Figure 1: CTMC



Problem 2. Consider a discrete-time queuing system. The arrivals are Bernoulli with rate λ. The service times are i.i.d. and independent of the arrival times. Each service time Z takes values in $\{1, 2, \ldots, K\}$ such that $E(Z) = 1/\mu$ and $\lambda < \mu$.

(a) Construct the Markov chain that models the queue. What are the states and transition probabilities? [Hint: Suppose the head of the line task of the queue still requires z units of service. Include z in the state description of the MC.]

(b) Use Lyapunov-Foster argument to show the queue is stable or equivalently the MC is positive recurrent.

Problem 3. Suppose that random variable X takes value in the set $\{1, 2, \ldots, K\}$ such that $\Pr(X_1 = k) = p_k > 0$, and $\sum_{k=1}^{K} p_k = 1$. Suppose $X_1, X_2, \ldots, X_n$ is a sequence of n i.i.d. samples of X.

(a) How many possible sequences exist?

(b) How many typical sequences exist when n is large?

(c) Find a condition that answers to parts (a) and (b) are the same.

Problem 4. Let $\{N_t, t \geq 0\}$ be a Poisson process with rate λ. Let S_n denote the time of the n-th event. Find

(a) the pdf of S_n.

(b) $E[S_5]$.

(c) $E[S_4|N(1) = 2]$.

(d) $E[N(4) - N(2)|N(1) = 3]$.

Problem 5. A queue has Poisson arrivals with rate λ. It has two servers that work in parallel. Where there are at least two customers in the queue, two are being served. When there is only one customer, only one server is active. The service times are i.i.d. $Exp(\mu)$.

(a) Argue that the queue length is a Markov Chain.

(b) Draw the state transition diagram.

(c) Find the minimum value of μ so that the queue is positive recurrent and solve the balance equations.