Problem 1. Let \( \{X_n, n \geq 0\} \) be a Markov chain with two states, \(-1\) and \(1\), and \( P(-1,1) = P(1,-1) = a \) for \( a \in (0,1) \). Define,
\[
Y_n = X_0 + X_1 + \ldots + X_n.
\]
Is \( \{Y_n, n \geq 0\} \) a Markov chain? Prove or disprove.

Problem 2. Problem 15 of Chapter 1, Page 23.


Problem 4. Problem 1 of Chapter 3, Page 37.

Problem 5. Problem 2 of Chapter 3, Page 37.

Problem 6. Find an approximate solution for problem 3 of Homework 1 without using Matlab when \( N = 100, p = 0.1, \) and \( k = 30 \).

Problem 7. In order to estimate the probability of head in a coin flip, \( p \), you flip a coin \( n \) times, and count the number of heads, \( S_n \). You use the estimator \( \hat{p} = S_n/n \). You choose the sample size \( n \) to have a guarantee
\[
\Pr(|S_n/n - p| \geq \epsilon) \leq \delta.
\]
Determine how the value of \( n \) suggested by Chebyshev inequality changes when \( \epsilon \) is reduced to half of its original value? How does it change when \( \delta \) is reduced to half of its original value?