Problem 1. Assumes the state of disks can be exactly modeled as a Markov chain depicted in the Figure.

(a) Alice is selling a brand-new disk for $100, and Bob is selling a 20-year-old (yet still working) disk for $80. Which one would you buy?

(b) Assume your disk is working now. You will pay $X$ dollars per repair, and $X$ is exponentially distributed with a mean of $10. Find the distribution of your repair cost until the disk dies.

(c) Find the expected life-time of the disk, $E[T]$.

(d) Find the variance of life-time of the disk, $Var(T)$.

(e) Find a reasonable upper bound of $P(T < 0.1E[T])$.

(f) Find a reasonable upper bound of $P(T > 5E[T])$.

(g) Assume your disk is being repaired now. Find the expected life-time of the disk.
Problem 2. Short problems.

(a) If 420 fair six sided dice are rolled, find an estimate of the probability that the sum is larger than 1400. Find the estimate of the probability that the sum is between 1365 and 1400.

(b) Assume $X_0 = 1$. Given the process eventually reaches state 3, what is the expected hitting time to state 3?