Problem 2. Suppose we have two independent Gaussian random variables $X \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $Y \sim \mathcal{N}(\mu_2, \sigma_2^2)$. Define random variables $Z$ and $W$ as mixtures of $X$ and $Y$. More specifically, let $U$ and $V$ be independent Bernoulli random variables, with $\Pr(U = 1) = p$ and $\Pr(V = 1) = q$, and $Z = X1\{U = 1\} + Y1\{U = 0\}$, $W = X1\{V = 1\} + Y1\{V = 0\}$, where $1$ denotes indicator function.

(a) Find $\mathbb{E}[Z]$ and $\text{var}(Z)$.

(b) Find $\text{cov}(Z,W)$.

Problem 3. Let $X$ and $Y$ be two independent standard normal random variables, $\mathcal{N}(0,1)$. Let $W = X^2 + Y^2$ and $Z = X/Y$. Find the marginal distributions of $Z$ and $W$, and show that they are independent.

Problem 4. Let $X$, $Y$, and $Z$ be independent random variables. $X$ is Bernoulli with $p = 1/4$. $Y$ is exponential with parameter 3. $Z$ is Poisson with parameter 5.

(a) Find the transform of $5Z + 1$.

(b) Find the transform of $X + Y$.

(c) Consider the new random variable $U = XY + (1 - X)Z$. Find the transform associated with $U$. 