Examples of Problems in Cont. R.V.

<u>Ex 1</u> : X, Y, Z. RV.

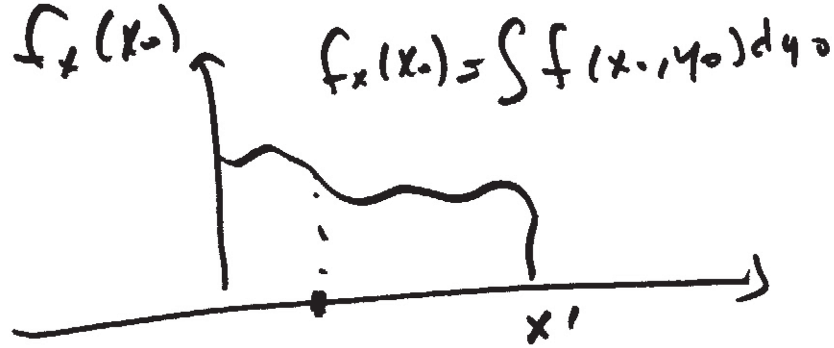
$$f_{X,Y,Z}(x_0, y_0, z_0) = \begin{cases} x_0 z_0 + 3 y_0 z_0 & \begin{matrix} 0 \le x_0 \le 1 \\ 0 \le y_0 \le 1 \\ 0 \le z_0 \le 1 \end{matrix} \\ 0 & \text{otherwise.} \end{cases}$$

<u>Q</u> $P_{X \le}(3) = 1$. range $0 \le x_0 \le 1$

<u>Q</u> Compute $f_{X,Y}(x_0, y_0)$.

$$f_{X,Y}(x_0, y_0) = \begin{cases} \int_{z_0 = 0}^{1} (x_0 z_0 + 3 y_0 z_0)\, dz_0 & 0 \le x_0, y_0 \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{x,y}(x_0, y_0) = \begin{cases} \frac{1}{2}\left(x_0 + \frac{3}{\text{[redacted]}} y_0\right) & 0 \le x_0, y_0 \le 1 \\ 0 & \text{otherwise.} \end{cases}$$



$\underline{Q}$

$$P_{x\le, y\le, z\le}(1, 2, z_0) = \begin{cases} 0 & z_0 \le 0 \\ - & 0 \le z_0 \le 1 \\ \phi & 1 \le z_0 \end{cases}$$

$$= \int_{z_0=0}^{z_0} dz_0 \int_{x_0=0}^{1} dx_0 \int_{y_0=0}^{1} dy_0 \left(x_0 z_0 + 3 y_0 z_0 \right)$$

$$= \begin{cases} 0 & z_0 \leq 0 \\ z_0^2 & 0 \leq z_0 \leq 1 \\ 1 & z_0 \geq 1 \end{cases}$$

$\underline{Q}$ $$f_x(x_0) = \begin{cases} \int_{y_0=0}^{1} f_{x,y}(x_0, y_0)\, dy_0 & 0 \leq x_0 \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$= \int_0^1 \frac{1}{2}(x_0 + 3y_0)\,dy_0 = \frac{1}{2}x_0 + \frac{3}{4}$$

$$f_X(x_0) = \begin{cases} \frac{1}{2}x_0 + 3/4 & 0 \le x_0 \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

<u>Q</u> $$E(Y|X) = \int_{y_0=0}^{1} y_0 \, f_{Y|X}(y_0|x_0)\, dy_0$$

$$f_{Y|X}(y_0|x_0) = \frac{f_{X,Y}(x_0,y_0)}{f_X(y_0)} = \frac{\frac{1}{2}(x_0+3y_0) \rightarrow 0 \le x_0, y_0 \le 1}{\frac{1}{2}x_0 + 3/4 \rightarrow 0 \le x_0 \le 1}$$

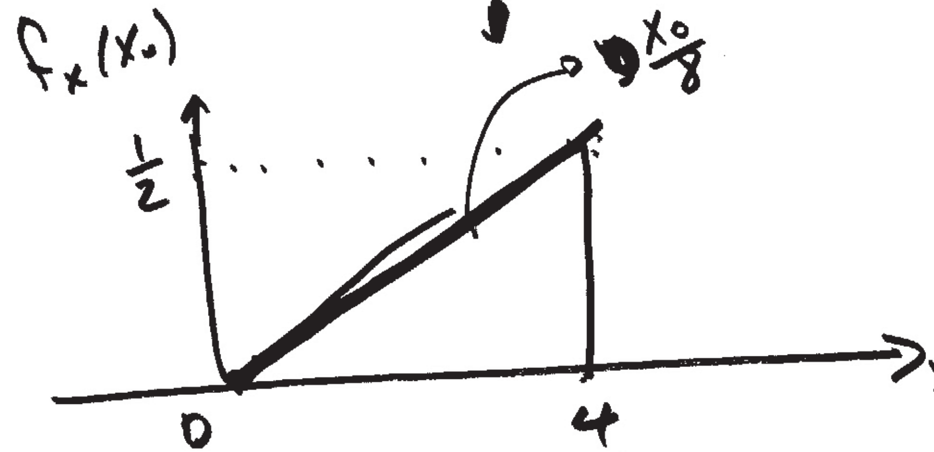
$$E(Y|X) = \int_{y_0=0}^{1} y_0 \frac{\frac{1}{2}(x_0+3y_0)}{\frac{1}{2}x_0 + 3/4}\, dy_0 = \frac{x_0+2}{2x_0+3}$$

c) $E(XY) = \iint x_0 y_0 \, f_{X,Y}(x_0, y_0)\, dx_0\, dy_0$

$$= \int_0^1 \int_0^1 \frac{1}{2}(x_0 + 3y_0)\, dx_0\, dy_0 = \frac{1}{3}$$

EX2

- Tom goes To casino each day.
- spins a biased wheel of forTune To figure out how much money he should take: → R.V. X.

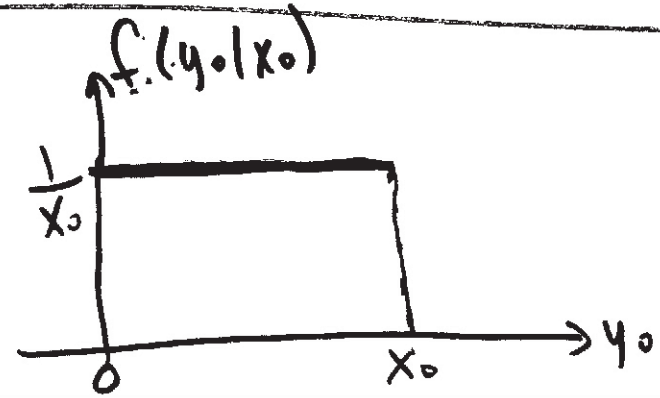


- Amount he brings home each day is uniformly distributed between zero & what he started out with.

y = R.V: brings home.

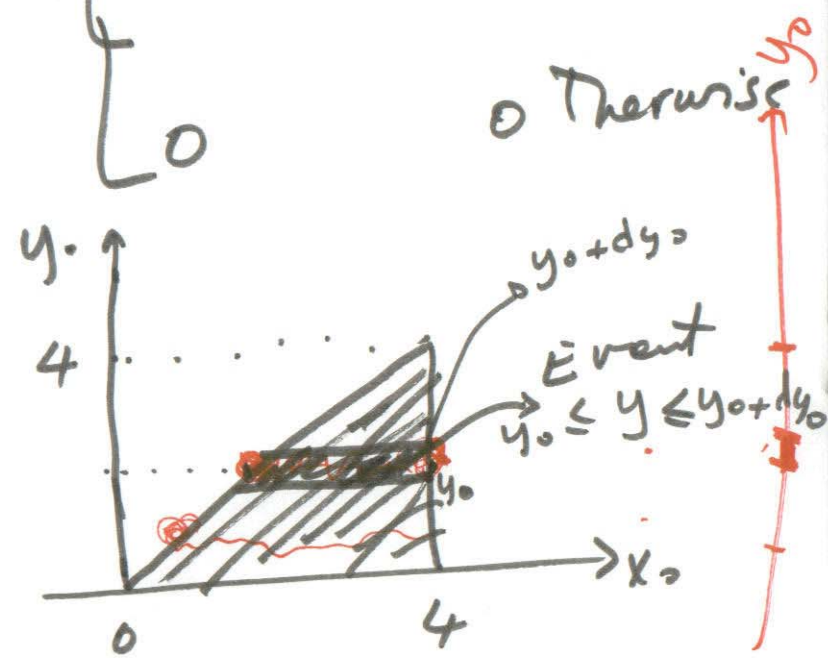
Q $f_{X,Y}(x_0, y_0)$

$f_{Y|X}(y_0|x_0) =$



$$f_{y|x}(y_0|x_0) = \begin{cases} \dfrac{1}{x_0} & \underline{0 \le y_0 \le x_0 \le 4} \\ 0 & \text{otherwise.} \end{cases}$$

$$f_{x,y}(x_0,y_0) = f_x(x_0)\, f_{y|x}(y_0|x_0) = \begin{cases} \dfrac{1}{x_0} \cdot \dfrac{x_0}{8} = \dfrac{1}{8} & 0 \le y_0 \le x_0 \le 4 \\ 0 & \text{otherwise} \end{cases}$$

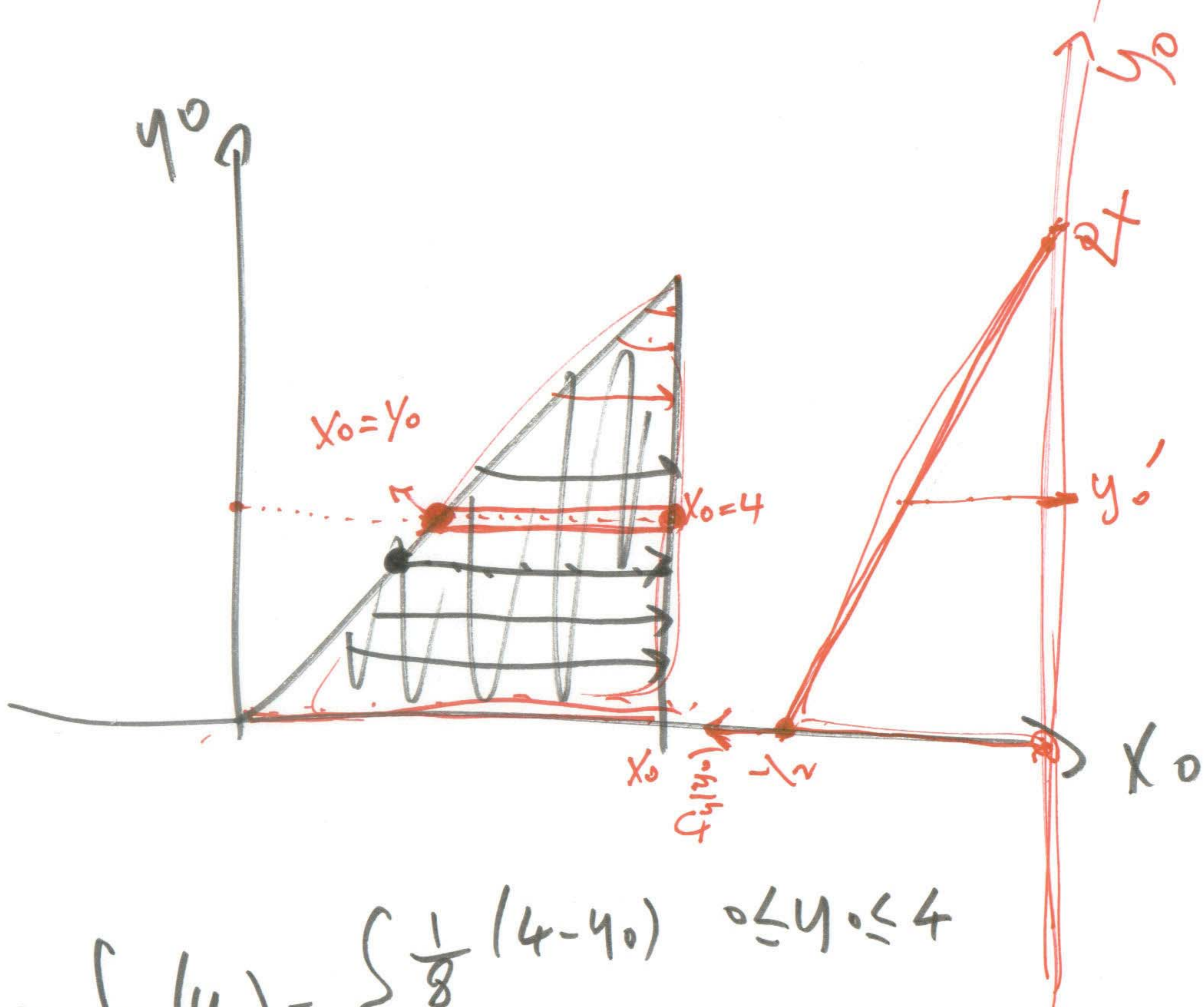


$\underline{Q}$ what is $f_y(y_0)$?

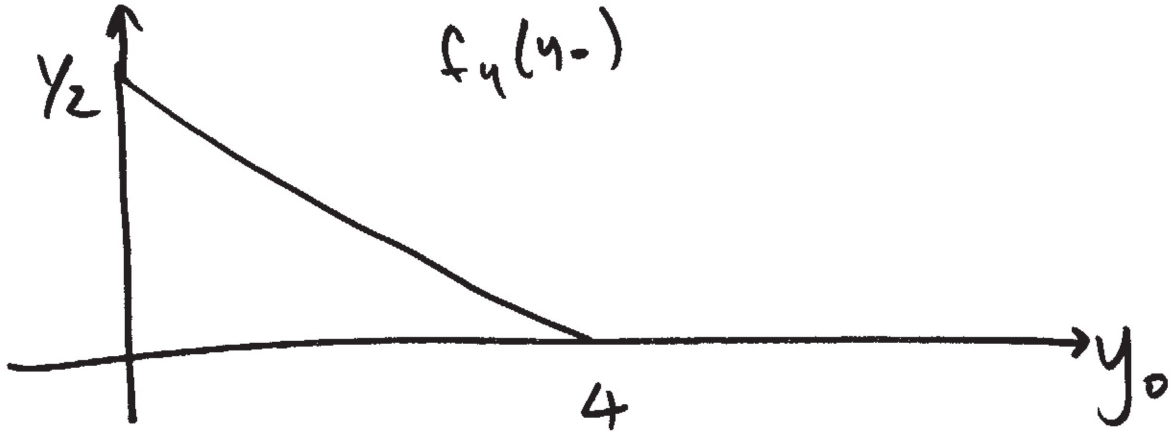
$$f_y(y_0) = \int_{-\infty}^{+\infty} f_{x,y}(x_0,y_0)\, dx_0$$

$$= \int_{x_0=y_0}^{4} dx_0\, f_{x,y}(x_0,y_0) = \int_{x_0=y_0}^{4} dx_0\, \frac{1}{8} = \phi$$

$$= \boxed{\frac{1}{8}(4-y_0)}$$



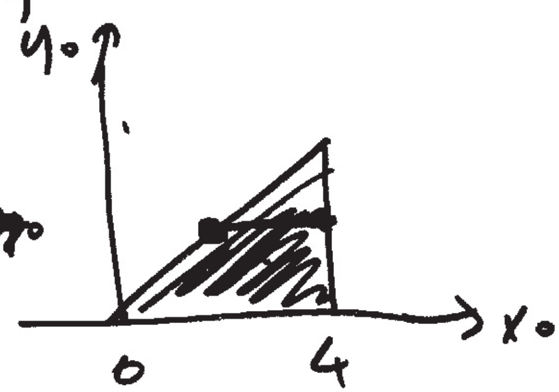
$$f_y(y_0) = \begin{cases} \frac{1}{8}(4 - y_0) & 0 \le y_0 \le 4 \\ 0 & \text{otherwise.} \end{cases}$$



Q Expected loss on any particular hight.

loss $= x - y$

$$E(X-Y) = \int_{x_0}\int_{y_0} (x_0-y_0)\, f_{x,y}(x_0,y_0)\, dx_0\, dy_0$$



$$= \int_{y_0=0}^{4} dy_0 \int_{x_0=y_0}^{4} \frac{1}{8}(x_0-y_0)\, dx_0 \cancel{dy_0}$$

$$= \$133.33.$$

$E(X-Y) = E(X) - E(Y)$

$$= \int_{x_0} x\, f_x(x_0)\, dx_0 - \int_{y_0} y_0\, f_y(y_0)\, dy_0$$

$$= \int_0^4 x_0 \cdot \frac{x_0}{8}\, dx_0 - \int_0^4 y_0\, \frac{4-y_0}{8}\, dy_0$$

$= \$133.00.$

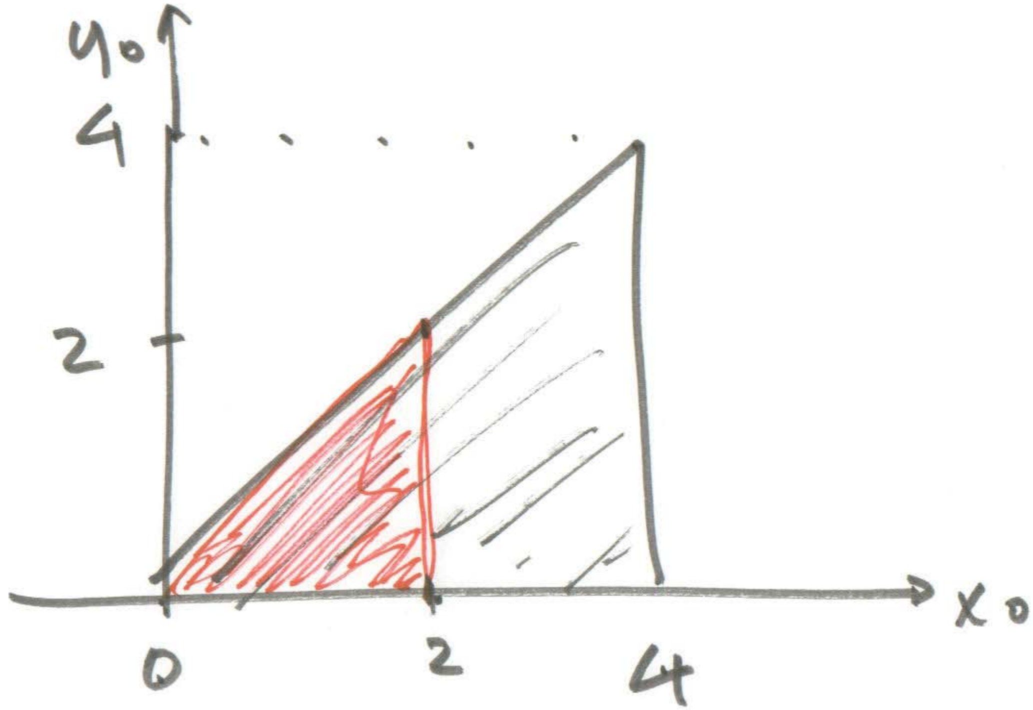
Q: returns home $\leq \$200$ = Event B

what is the conditional prob. of:

(a) started out $\leq \$210$ = Event A.

$$P(A|B) = \frac{P(AB)}{P(B)}$$

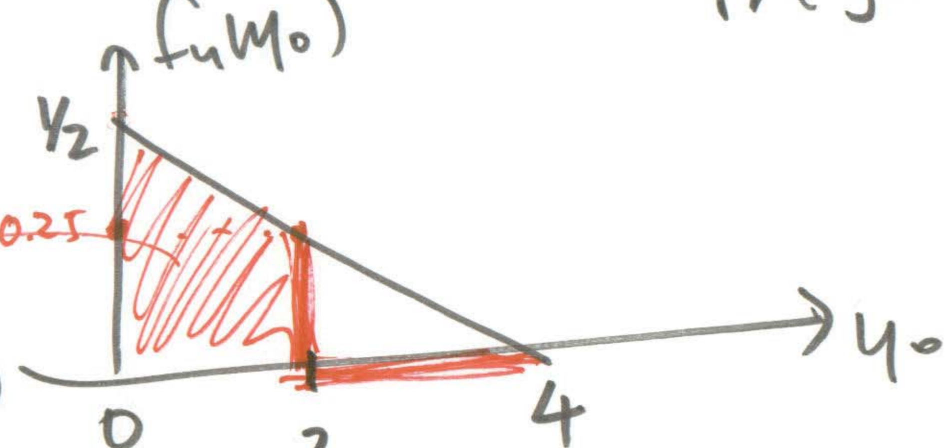
P(AB)



$$P(AB) = \frac{2 \cdot 2}{2} \cdot \frac{1}{8} = \frac{1}{4}$$

$Pr(y<2)$

P(B)



$$1 - \frac{0.25 \times 2}{2} = 3/4 = P(B)$$

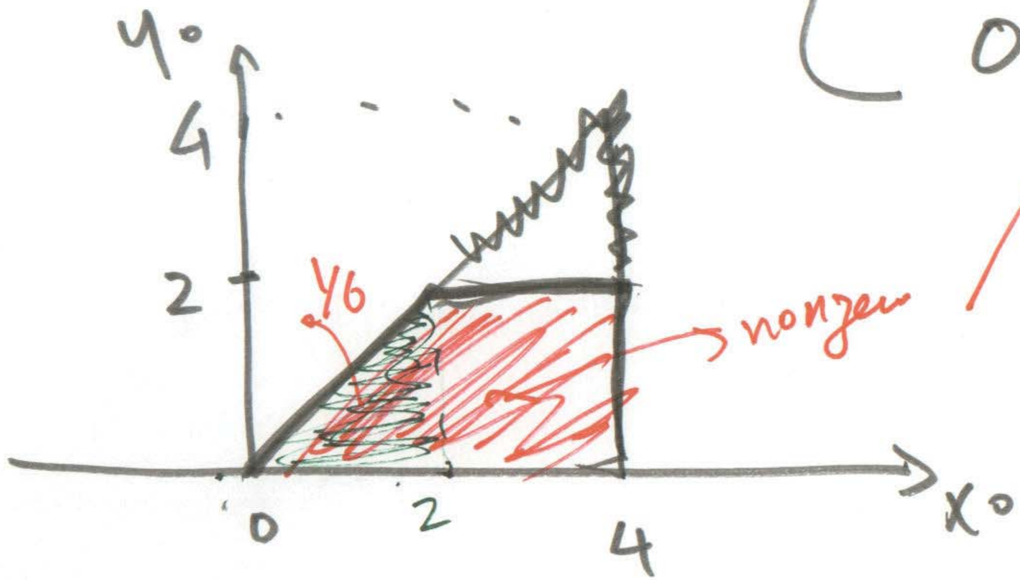
$$\Rightarrow P(A|B) = \frac{1/4}{3/4} = \frac{1}{3}$$

(a) <u>Aliter</u> Compute $f_{X,Y}(x_0, y_0 \mid B)$

$$f_{X,Y}(x_0, y_0 \mid y_0 < 2) = \begin{cases} \dfrac{f_{X,Y}(x_0, y_0)}{P(B)} & x_0, y_0 \in B \\ 0 & \text{otherwise} \end{cases}$$

$P(B) = 3/4$

$$f_{X,Y}(x_0, y_0 \mid B) = \begin{cases} \dfrac{1/8}{3/4} = \dfrac{1}{6} & \\ 0 & \text{otherwise} \end{cases}$$

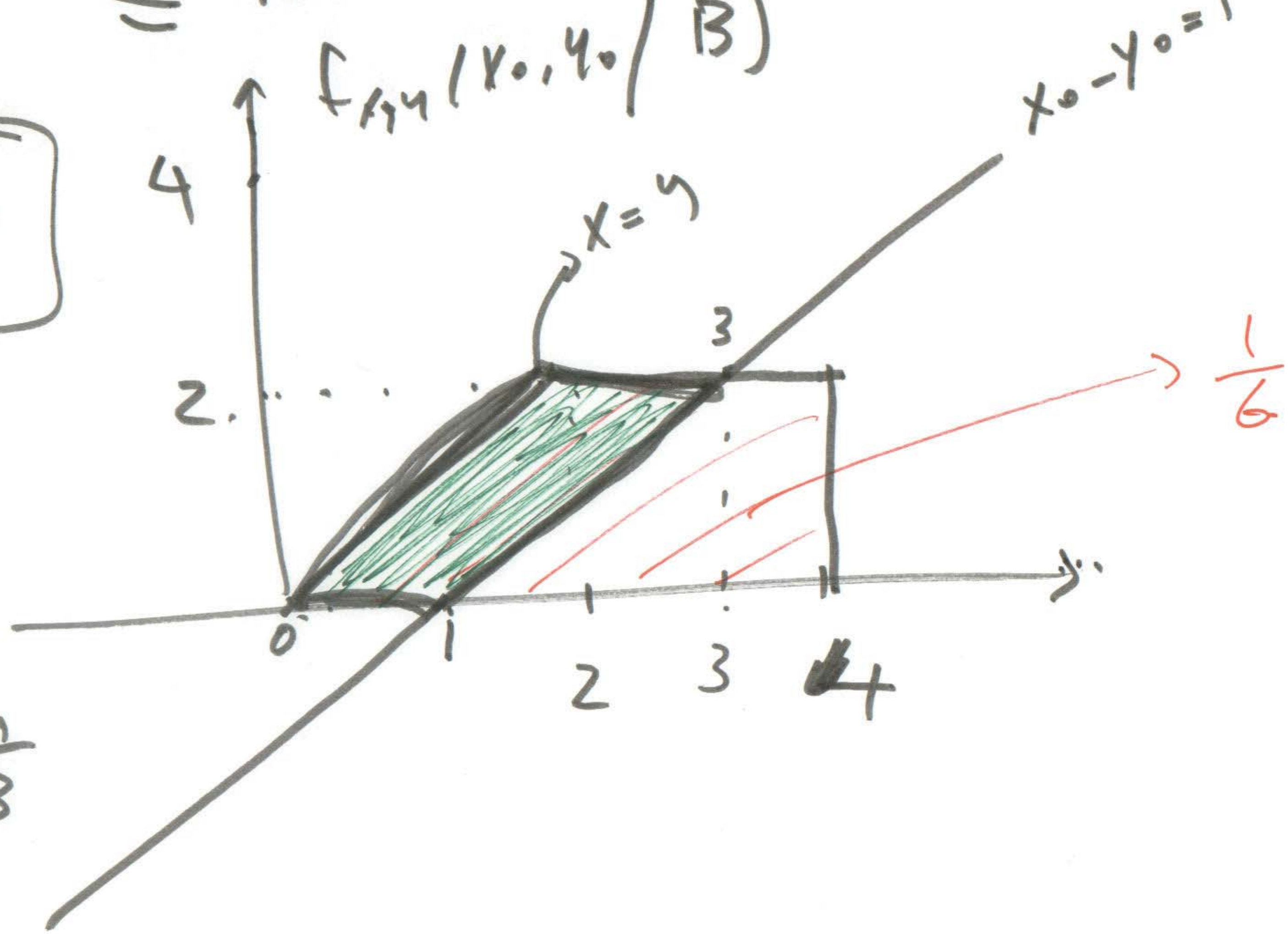


$$\frac{2 \times 2}{2} \times \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

(b) given B, what is prob his loss $\leq$ 100

$\boxed{x_0 - y_0 < 1}$

$x_0 - y_0 = 1$



$\frac{1}{6} \cdot (1 \times 2) = \frac{1}{3}$