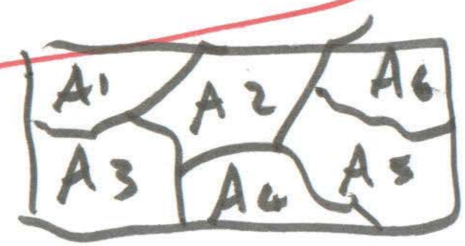
$$\mathrm{Inv}\left[F_1(s) + F_2(s) \right] \neq \mathrm{Inv}\left[F_1(s)\right] + \mathrm{Inv}\left[F_2(s)\right]$$

- Let $A_1\ A_2 \dots A_K$ list of mutually exclusive, collectively exhaustive events
- Assume continuous R.V. y, not indp. of A_i

$$\boxed{f_y(y_0) = \sum_i P(A_i)\, f_{y/A_i}(y_0/A_i)}$$



Define..

$$M_{Y|A_i}(s) \triangleq \int_{y_0=-\infty}^{+\infty} e^{-sy_0} f_{Y|A_i}(y_0|A_i)\,dy_0$$

$$= E(e^{-sy}|A_i)$$

$$\Rightarrow \boxed{M_y(s) = \sum_i P(A_i)\, M_{Y|A_i}(s)}$$

valid s Transform

$$\text{Inv}\left[\underbrace{\tfrac{1}{9}}_{P(A_1)}\,1 + \underbrace{\tfrac{8}{9}}_{P(A_2)}\,\frac{\lambda/9}{s+\lambda/9}\right]$$

valid s Trasform

$$\cancel{\frac{1}{9}\mathrm{Inv}[\;]} = \frac{1}{9}\,\mathrm{Inv}[1] + \frac{8}{9}\,\mathrm{Inv}\left[\frac{\lambda/9}{s+\lambda/9}\right]$$

$$= \frac{1}{9}\,\delta(\tau_0) + \frac{8}{9}\,e^{-\lambda\tau_0/9}\,u(\tau_0)$$

step fn

Covariance + Correlation

R.V. X, Y.

- $\text{Cov}(X,Y) \triangleq E[(X-\bar{X})(Y-\bar{Y})]$

  $Y = X \Longrightarrow \text{Cov}(X,X) = \text{Var}(X)$

- Def If $\text{Cov}(X,Y) = 0$

  $\Longrightarrow X, Y$ uncorrelated)

$$\text{Cov}(X,Y) = E[XY] - E[X\bar{Y}] - E[\bar{Y}\bar{X}] + E[\bar{X}\bar{Y}]$$

$$= E[XY] - \bar{Y}\bar{X} - \bar{X}\bar{Y} + \bar{X}\bar{Y}$$

$$= E[XY] - \bar{X}\bar{Y} =$$

$$\boxed{Cov(X,Y) = E[XY] - E[X]E[Y]}$$

If X, Y independent

$$Cov(X,Y) = E(X)E(Y) - E(X)E(Y)$$
$$= 0$$

$\Longrightarrow$ If X, Y indp $\Longrightarrow$ Also uncorrelated

$\cancel{\Longleftarrow}$

if uncorrelated does <u>not</u> mean indp.

indp.

$P(x,y) = P(x)P(y)$

uncorrl. $E(xy) = E(x)\,E(y)$

Difference between indp. and uncorrelated.

Indp $\longrightarrow$ deals with pdfs.

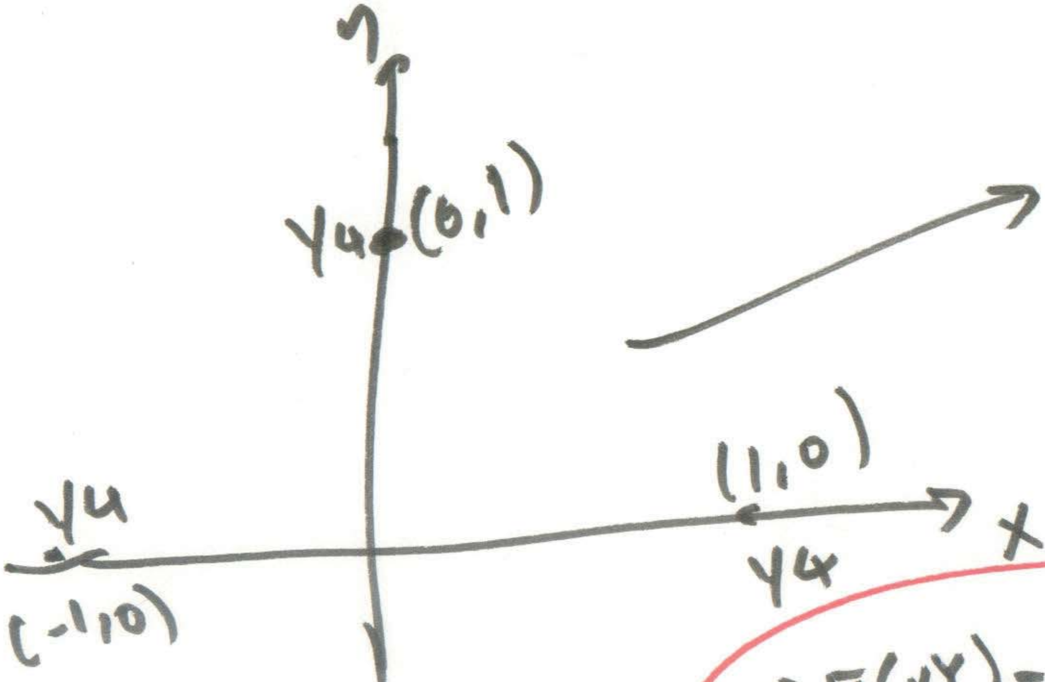
Uncorrelation only deals with Expect. 5.5

$\Rightarrow$ Intuitively

positive covariance $\Longrightarrow$ $x - \bar{x}$ and $y - \bar{y}$ have same sign.

negative covariance $\Longrightarrow$ $X - \bar{X}$, $y - \bar{y}$ have different sign.

Ex. 2 R.V. uncorrelated, But not indp.



Clear: X, Y are not independent

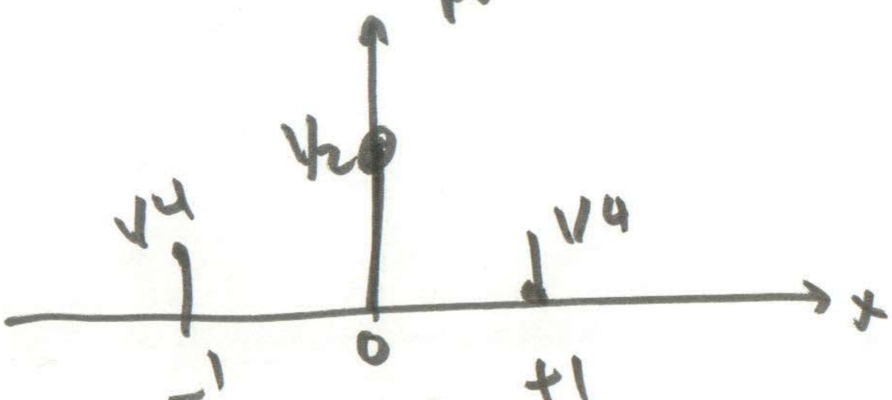
$E(XY) = 0 = \frac{1}{4} \cdot 0 + \frac{1}{4} 0 + \frac{1}{4} 0 + \frac{1}{4} 0 = 0$

$\Rightarrow E(XY) = E(X)E(Y)$

↓

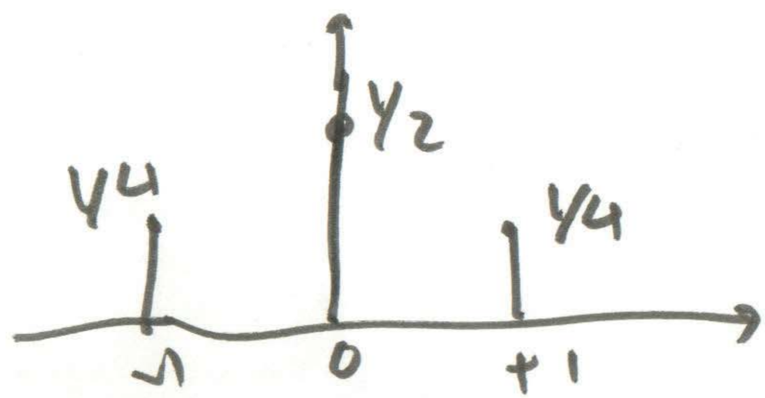
uncorrelated

p(x)



$E(X) = 0$

p(y)



$E(Y) = 0$

Def <u>Correlation coefficient</u>:

$$\rho(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}$$

Can show $|\rho| < 1$

- $X - \bar{X}$ and $Y - \bar{Y}$ have same sign $\xrightarrow{\text{on average}} \rho > 0$
- $X - \bar{X}$ and $Y - \bar{Y}$ have opposite sign $\Rightarrow \rho < 0$

$|\rho|$ Tells us how much these are True

Ex n indp. tosses of a coin.

$p(\text{head}) = p$.

$X = \#$ of heads $Y = \#$ of Tails.

$X + Y = n \implies E(X) + E(Y) = n$

$X + Y = E(X) + E(Y)$

$\implies \boxed{X - E(X) = -(Y - E(Y))}$

$\operatorname{cov}(X, Y) = E[(X - \bar{X})(Y - \bar{Y})]$

$= E(-(X - \bar{X})^2)$

$= -E((X - \bar{X})^2)$

$= -\operatorname{Var} X$

$$\left.\begin{array}{l} \rho(X,Y) = \dfrac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}} \\[2ex] \text{Show} \quad \mathrm{Var}(X) = \mathrm{Var}(Y) \end{array}\right\} \Longrightarrow$$

$$\rho(X,Y) = \frac{-\mathrm{Var}(X)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(X)}} = -1$$

$$\boxed{\rho(X,Y) = -1}$$

More generally can show:

$$|\rho| = 1 \overset{\text{iff}}{\Longleftrightarrow} \exists\, c \text{ s.t. } Y - \bar{Y} = c(X - \bar{X})$$

$|\rho| = 1 \overset{}{\underset{\text{iff}}{\Longleftrightarrow}} \exists c$ s.th. $y - \bar{y} = c(x - \bar{x})$

$$E\left[(y-\bar{y})^2\right] = c^2 E\left[(x-\bar{x})^2\right]$$

$$\text{var}\, y = c^2 \text{var}(x)$$

Can show that:

X_1, X_2.

$$\text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2) + 2\,\text{Cov}(X_1, X_2)$$

$X_1, X_2, \ldots, X_n$. (not necessarily indep).

$$\mathrm{Var}(X_1 + X_2 + \cdots + X_n) = \sum_{i=1}^{n} \mathrm{Var}(X_i) + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \mathrm{Cov}(X_i, X_j)$$