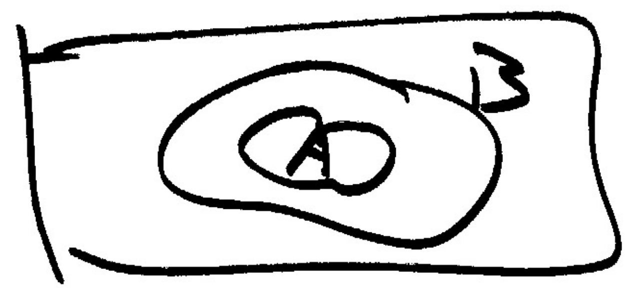
$$P\Big[\underbrace{A + CD + B(A + C'D)}_{\text{event}}\Big]$$

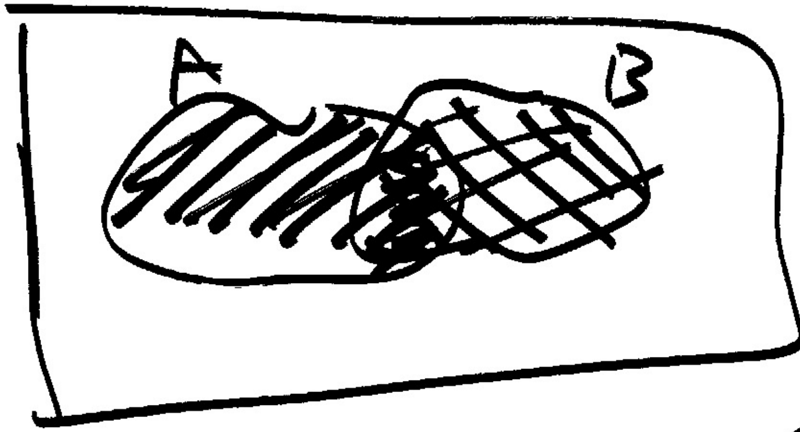
Real #

Properties of prob laws

- If $A \subset B$ (subset) $\quad P(A) < P(B)$



- $P(A \cup B) = P(A) + P(B) - P(AB$



$P(A \cup B) =$
$P(A + B) =$
$P(A) + P(B)$
$- P(AB)$

$P(A+B) \leqslant P(A) + P(B)$

4 Topics

- Conditional prob.
- Trees
- Total Prob Thm
- Bayes Rule.

Conditional Prob

provides a way to reason about outcome of an exp. based on Partial Information

Ex 2 successive rolls of a die.

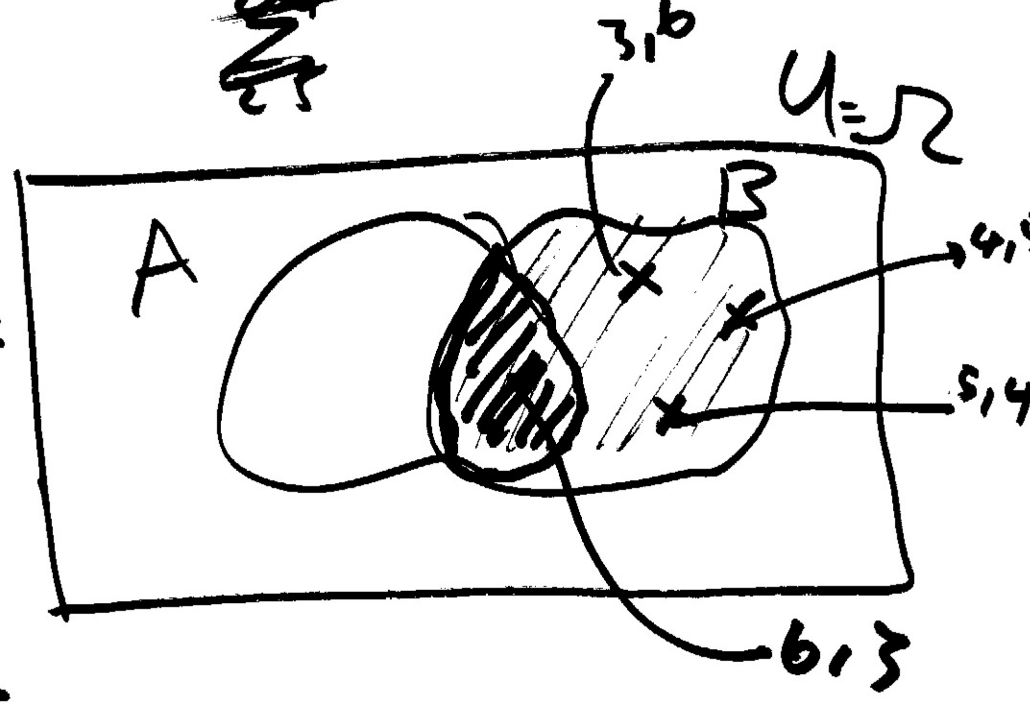
Given Sum = 9 $\longrightarrow$ event B.

what is Prob first roll was 6? $\longrightarrow$ event A

(3, 6)
(4, 5)
(5, 4)
(6, 3)

$$\frac{1}{4} = P(A|B)$$

$$P(A|B) = \frac{\text{area Blue}}{\text{area of B}} = \frac{\text{area}(AB)}{\text{area of}(B)} = \frac{P(AB)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$



$B = \{(3,6), (4,5), (5,4), (6,3)\}$

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(AB) = \frac{1}{36} = \text{prob(sum is 9 and first roll is 6)}$$

$$P(B) = \frac{4}{36}$$

$$P(A|B) = \frac{1/36}{4/36} = 1/4$$

Reminder:

$$AB = A \cap B$$

$$A + B = A \cup B$$

Ex roll a fair die.
given outcome is even what is $Pr(2)$?

Even $\underbrace{\underline{2}, 4, 6}$ $\Rightarrow Pr(2) = \frac{1}{3}$

$$P(2 / \text{even}) = \frac{P(2 \text{ and even})}{P(\text{even})} = \frac{1/6}{1/2}$$

$$= \frac{1}{3}$$

Ex Fair coin twice.

Given; observed: at least one toss was a head. $\longrightarrow B$

what is the prob $\underbrace{\text{both tosses were heads}}_{A}$

$P(A|B)$

Two ways: ① intuitively: $\underbrace{HT,\ TH,\ HH}_{B}$ $\quad \frac{1}{3}$

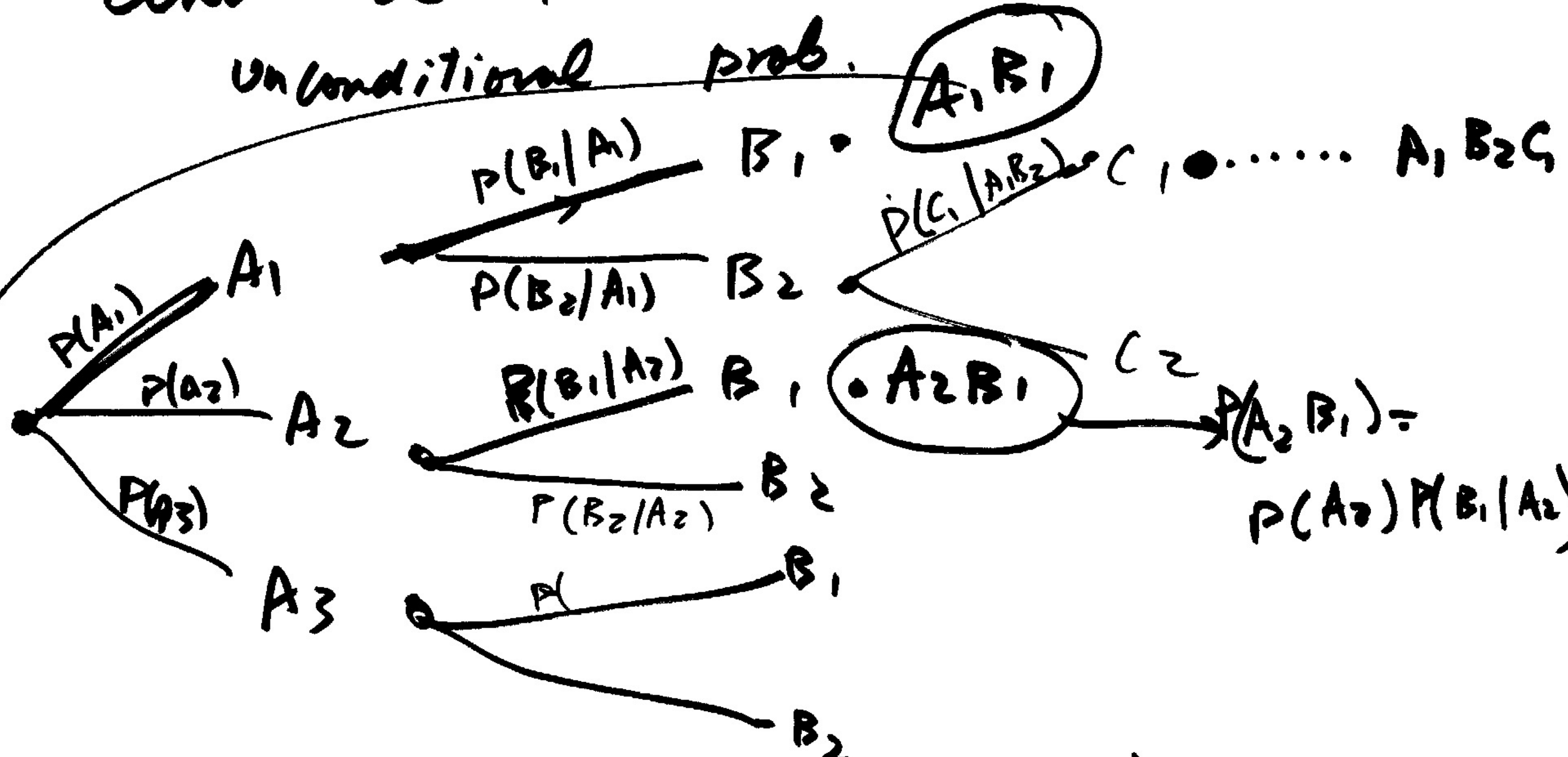
② Equation:

	P	A	B	AB
• H_1H_2	$1/4$	✓	✓	✓
• H_1T_2	$1/4$		✓	
• T_1H_2	$1/4$		✓	
• T_1T_2	$1/4$			

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{1/4}{3/4} = 1/3$$

Prob Trees for Sequential Exps

- for Exp. seq character, can use conditional prob to determine unconditional prob.



$$P(A_1 B_1) = P(A_1)\,P(B_1 | A_1)$$

Ex

- If aircraft present, prob radar registers is 0.99.
- If aircraft is NOT present, prob radar registers something is 0.1
- aircraft is present 5 % Time.

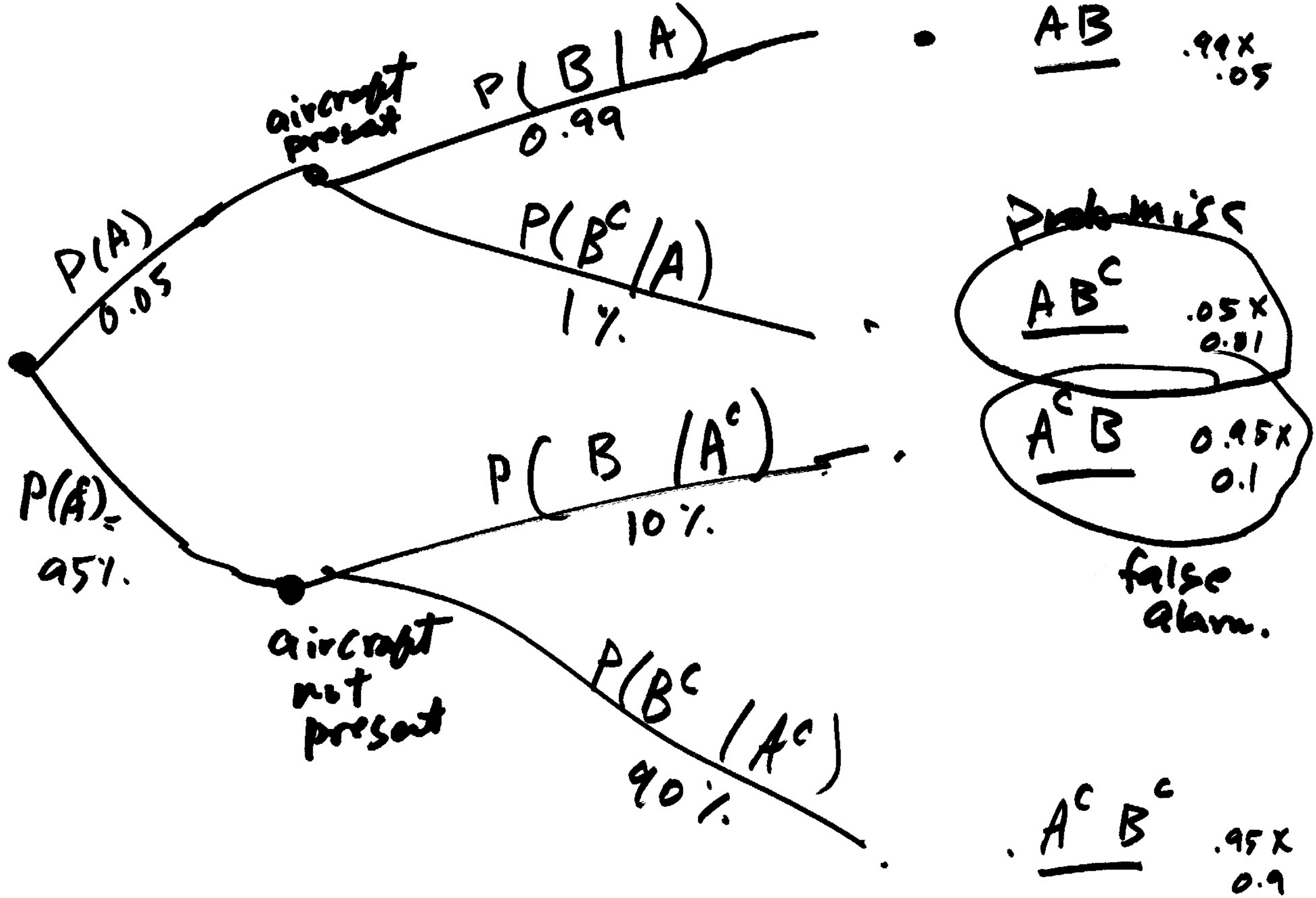
Compute prob false alarm and prob miss

$A = \{$ aircraft present $\}$ $\qquad A^c = \{$ aircraft absence $\}$

$B = \{$ radar says "yes" detects airplane. $\}$ $\qquad B^c = \{$ radar says no $\}$

$P(\text{false Alarm}) = P(A^c B)$

$P(\text{miss}) = P(A B^c)$



P(miss) = $0.05 \times .01 = .0005$

P(false) = $.095$

Multiplication Rule

$$P(B|A) = \frac{P(AB)}{P(A)} \longrightarrow$$

$$P(AB) = P(A) \cdot P(B|A)$$

or many sets/events.

$$P(A_1 A_2 \ldots A_n) = P(A_1)\,P(A_2|A_1)\,P(A_3|A_1 A_2) \ldots P(A_n|A_1 \ldots A_{n-1})$$

Ex 52 card dec.

Draw 3 cards without replacement.

p(none of cards in heart).

$A_i = \{$ ith card is not a heart $\}$

$p(A_1$ and A_2 and $A_3) =$

$P(A_1) = \frac{39}{52}$

$P(A_2 \mid A_1) = \frac{38}{51}$

$P(A_3 \mid A_1A_2) = \frac{37}{50}$

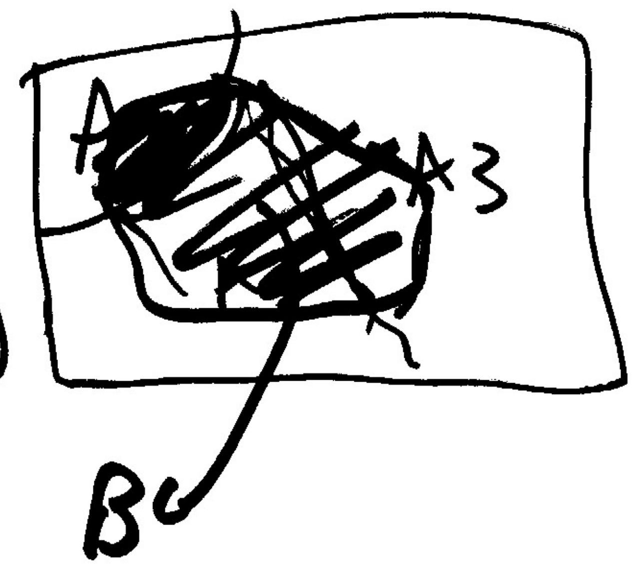
$P(A_1A_2A_3) = P(A_1)P(A_2 \mid A_1)\, P(A_3 \mid A_1A_2)$

$$= \frac{39}{52} \times \frac{38}{51} \times \frac{37}{50}$$

Total Probab Thm.

$A_1 \ldots A_n$ disjoint events that form partition of sample space.

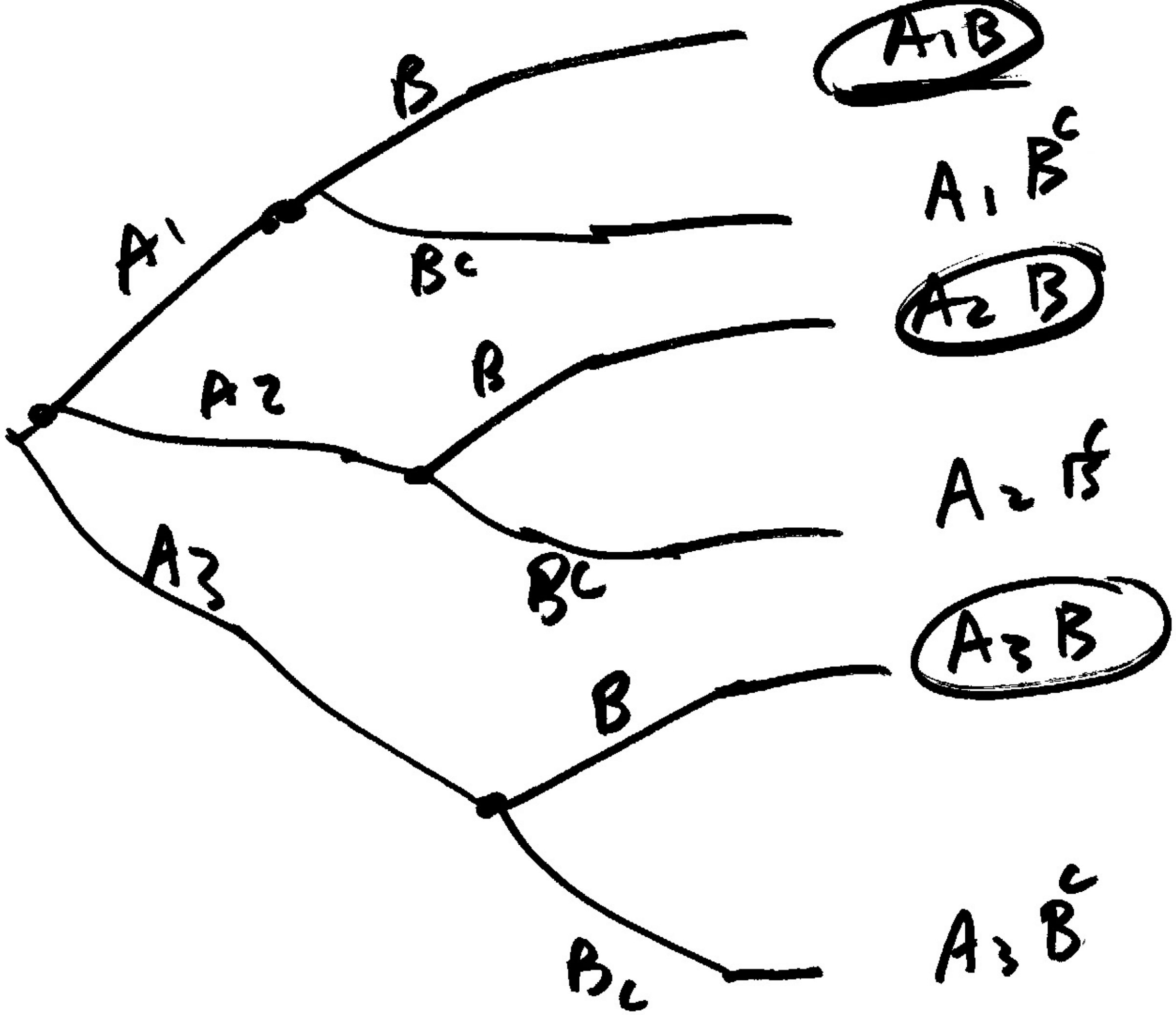
For any event B.



$$P(B) = P(A_1B) + P(A_2B) + P(A_3B)$$

$$= P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$$

$P(A_i) \neq 0$



$$P(B) = P(A_1B) + P(A_2B) + P(A_3B)$$
$$= P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)$$

Ex chess.

Type 1 = 50%

Type 2 = 1/4

Type 3 = 1/4

P(winning against type 1) = 30%

P(" " " 2) = 40%

P(" " " 3) = 50%

B = event of wining.

A_i = event of playing against Type i.

P(wining) = P(B)?

$P(A_1) = 30\%$. $P(A_2) = 25\%$. $P(A_3) = 25\%$.

$P(B) = P(\text{Type }1)P(\text{win}|1) + P(2)P(\text{win}|2) + P(3)P(\text{win}|3)$

$= 50\% \times 30\% + 25\% \times 40\% + 25\% \cdot 50\%$

$= 0.375$