\[
P[A + CD + B(A+CD)]
\]

Real *

---

\[\text{Properties of Prob laws}\]

- If \( A \subseteq B \), \( P(A) < P(B) \)
- \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \)
\[ P(A \cup B) = P(A + B) = P(A) + P(B) - P(AB) \]

\[ P(A + B) \leq P(A) + P(B) \]

Topics
- Conditional prob.
- Trees
- Total Prob. Theorem
- Bayes Rule.
Conditional Prob

provide a way to learn about outcome of an exp. based on partial information

Ex 2 successive rolls of a die.

Given Sum = 9 → event B.

what is Prob first roll was 6? event A

\[ \frac{1}{4} = P(A|B) \]

(3, 6)
(4, 5)
(5, 4)
(6, 3)
\[ P(A | B) = \frac{\text{area of blue}}{\text{area of } B} \]

\[ = \frac{\text{area of } AB}{\text{area of } B} \]

\[ = \frac{P(AB)}{P(B)} \]

\[ B = \{(3, 6), (4, 5), (5, 4), (6, 3)\} \]
\[ P(A|B) = \frac{P(AB)}{P(B)} \]

\[ P(AB) = \frac{1}{36} = \text{prob (sum is 9 and first roll is 6)} \]

\[ P(B) = \frac{4}{36} \]

\[ P(A|B) = \frac{1/36}{4/36} = \frac{1}{4} \]

Reminder:

\[ AB = A \cap B \]

\[ A+B = A \cup B \]
Ex

A fair die is rolled. Given that the outcome is even, what is \(\Pr(2)\)?

\[
\begin{align*}
\frac{\Pr(2)}{\Pr(\text{even})} &= \frac{n(2, \text{even})}{n(\text{even})} \\
\Pr(2) &= \frac{1}{2} \times \frac{1}{6} \\
\Pr(\text{even}) &= \frac{1}{2} \times \frac{1}{6}
\end{align*}
\]
Ex Fair coin twice.

Given; observed: at least one toss was a head. \( \rightarrow B \)

What is the prob both tosses were heads

\[ P(A \mid B) \]

Two ways: 1) intuitively:

- \( H_1, H_2 \)  \( \frac{1}{4} \)
- \( H_1, T_2 \)  \( \frac{1}{4} \)
- \( T_1, H_2 \)  \( \frac{1}{4} \)
- \( T_1, T_2 \)  \( \frac{1}{4} \)

2) Equation:

\[ P = \frac{P(A \cap B)}{P(B)} \]

\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \]
Prob Trees for Sequential Exps

- for Exp. seq character, can use conditional prob to determine unconditional prob.

\[ P(A_1 B_1) = P(A_1) P(B_1 | A_1) \]
Ex. If aircraft present, prob radar registers is 0.99.
- If aircraft is NOT present, prob radar registers something is 0.1
- Aircraft is present 5 x Time.

Compute prob false alarm and prob miss

\[ A = \{ \text{aircraft present} \} \]
\[ A^c = \{ \text{aircraft absent} \} \]
\[ B = \{ \text{radar says, "yes"} \} \]
\[ B^c = \{ \text{radar says \text{ no}} \} \]

\[ P(\text{false alarm}) = P(A^c B) \]
\[ P(\text{miss}) = P(A B^c) \]
\[ P(\text{miss}) = 0.05 \times 0.01 = 0.0005 \]

\[ P(\text{false}) = 0.095 \]
Multiplication Rule

\[ P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(A)P(B|A)}{P(A)} \]

or any set of events

\[ P(A_1, A_2, \ldots, A_n) = P(A_1)P(A_2|A_1)\cdots P(A_n|A_1, A_2, \ldots, A_{n-1}) \]
Ex: 52 card deck

Draw 3 cards without replacement.

\[ P( \text{none of cards in heart}) \]

\[ A_i = i^{th} \text{ card is not a heart} \]

\[ P( A_1 \text{ and } A_2 \text{ and } A_3) = \]

\[ P(A_i) = \frac{39}{52} \]

\[ P(A_2 | A_1) = \frac{38}{51} \]

\[ P(A_3 | A_1 A_2) = \frac{37}{50} \]

\[ P(A_1 A_2 A_3) = P(A_1) P(A_2 | A_1) P(A_3 | A_1 A_2) \]
\[
\frac{39}{52} \times \frac{36}{51} \times \frac{37}{50}
\]

**Total Probab Thm.**

- **A, ...,** An disjoint events that form partition of sample space.

For any event \( B \),

\[
P(B) = P(A_1B) + P(A_2B) + P(A_3B)
\]

\[
= P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + P(A_3)P(B|A_3)
\]

\( p(A_i) \neq 0 \)
\[
P(B) = P\left(\frac{A_1B}{A_1B}\right) + P\left(\frac{A_2B}{A_2B}\right) + P\left(\frac{A_3B}{A_3B}\right)\]

\[
= P\left(\frac{A_1B}{A_1B}\right) + P\left(\frac{B}{A_2}\right) + P\left(\frac{B}{A_3}\right) + P\left(\frac{B}{A_4}\right)
\]

Diagram:

- Node A_1B
- Node A_2B
- Node A_3B
- Node B_c
- Node B_e
- Node B_f
- Node B_g
- Node B_h
- Node B_i
- Node B_j
- Node B_k
- Node B_l

 probabilities:

- A_1B
- A_2B
- A_3B
- B_c
- B_e
- B_f
- B_g
- B_h
- B_i
- B_j
- B_k
- B_l

connections:

- A_1B to A_2
- A_1B to A_3
- A_2B to B_c
- A_2B to B_e
- A_2B to B_f
- A_2B to B_g
- A_2B to B_h
- A_2B to B_i
- A_2B to B_j
- A_2B to B_k
- A_2B to B_l
- A_3B to B_c
- A_3B to B_e
- A_3B to B_f
- A_3B to B_g
- A_3B to B_h
- A_3B to B_i
- A_3B to B_j
- A_3B to B_k
- A_3B to B_l
Ex. chess: Type 1 = 50%  
Type 2 = 1/4  
Type 3 = 1/4  
\[ P(\text{winning against type 1}) = 30\% \]  
\[ P(\text{winning against type 2}) = 40\% \]  
\[ P(\text{winning against type 3}) = 50\% \]  
\[ B = \text{event of winning} \]  
\[ A_i = \text{event of playing against type } i \]  
\[ P(\text{winning}) = P(B) = P(A_1) = 30\% \]  
\[ P(A_2) = 25\% \]  
\[ P(A_3) = 25\% \]  
\[ P(B) = P(\text{type 1}) P(\text{win/1}) + P(2) P(\text{win/2}) + P(3) P(\text{win/3}) \]  
\[ = 50\% \times 30\% + 25\% \times 40\% + 25\% \times 50\% \]  
\[ = 0.375 \]