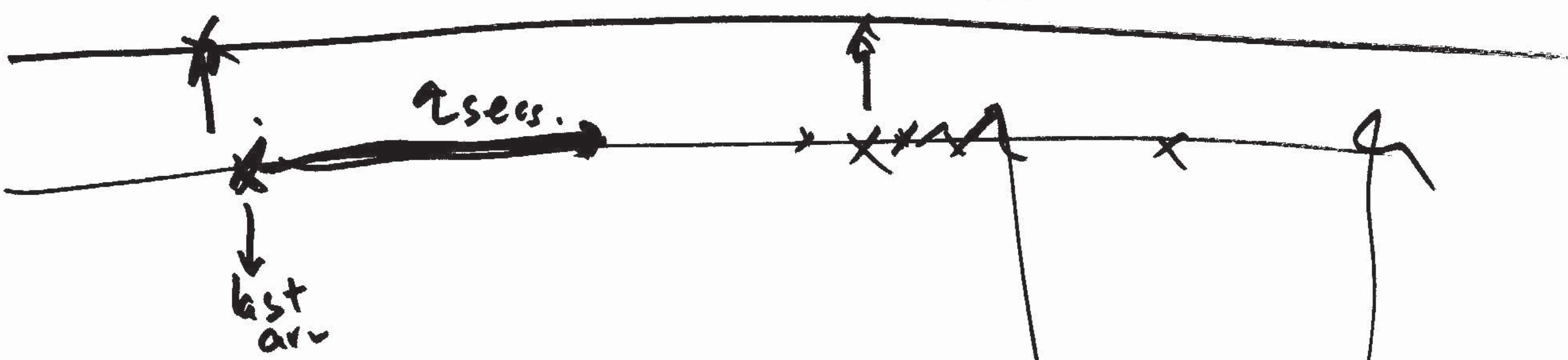
Poisson Process (cont'd).

Memoryless Property

Start a Poisson process;
τ seconds has elapsed since the last arrival.
Conditional pdf of $l_1 - \tau$ is still poisson indp of τ



Fresh Start: $\forall t$, the history of the process after time t is also a poisson. and indp of history of the process until time t.

merging + splitting of ~~a~~ Poisson Process

merging

X, Y ~~expect~~ 1st order interarrival Time for 2 indp. poisson process

Posson $E(X)$

①

Poisson $E(Y)$

②

merged poisson $E(X) + E(Y)$

$M_X(s) = e^{E(X)(e^s - 1)}$ $M_Y(s) = e^{E(Y)(e^s - 1)}$

$W = X + Y$.

$$M_W(s) = M_X(s)\, M_Y(s)$$

$$= e^{[E(X)+E(Y)](e^s - 1)}$$

$$P_W(w_0) = \frac{(E(X)+E(Y))^{w_0}}{w_0!}\, e^{-(E(X)+E(Y))}$$

Sum of 2 Poisson Processe with arrival rate of λ_1 & λ_2 is anothe poisson.

Splitting of Poisson

original poisson. λ = arrival.

P keep

1-P Throwaway

New process = poisson.

λp.

Ex pdf independent interarrival times between successive cars in highway is

$$f_t(t_0) = \begin{cases} \frac{1}{12} e^{-t_0/12} & t_0 \geq 0 \\ 0 & t_0 < 0 \end{cases} \longrightarrow \lambda = \frac{1}{12}$$

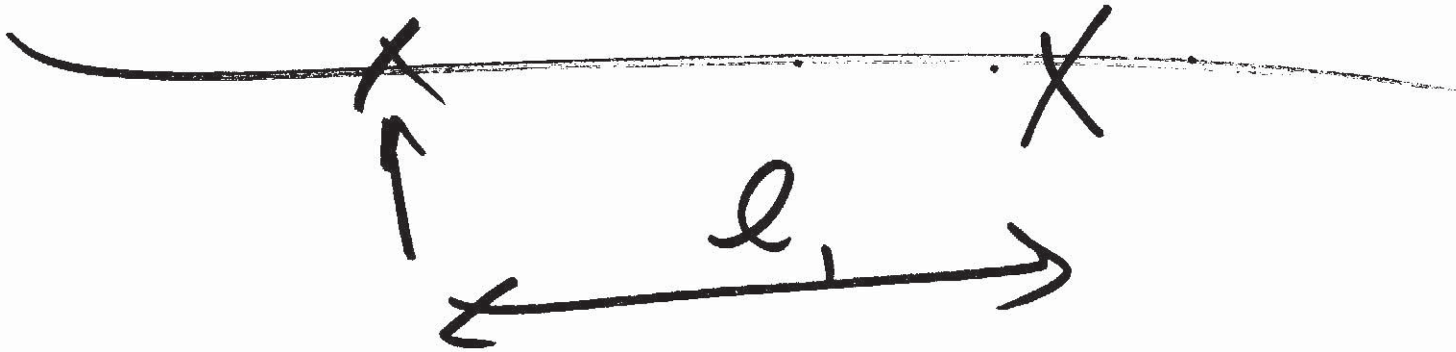
$$\int_{t_0=0}^{\infty} f_t(t_0)\, dt_0 = 1$$

Q Wombat requires 12 secs to cross. Starts immediately after a car.

Prob of survives?

$$P(K;t) = \frac{\left(\frac{t}{12}\right)^K e^{-t/12}}{K!} \qquad K = 0, 1, 2 \ldots$$

$$P(0;12) = \frac{e^{-1}}{0!} = 0.368.$$



survial $= Pr(l_1 > 12)$

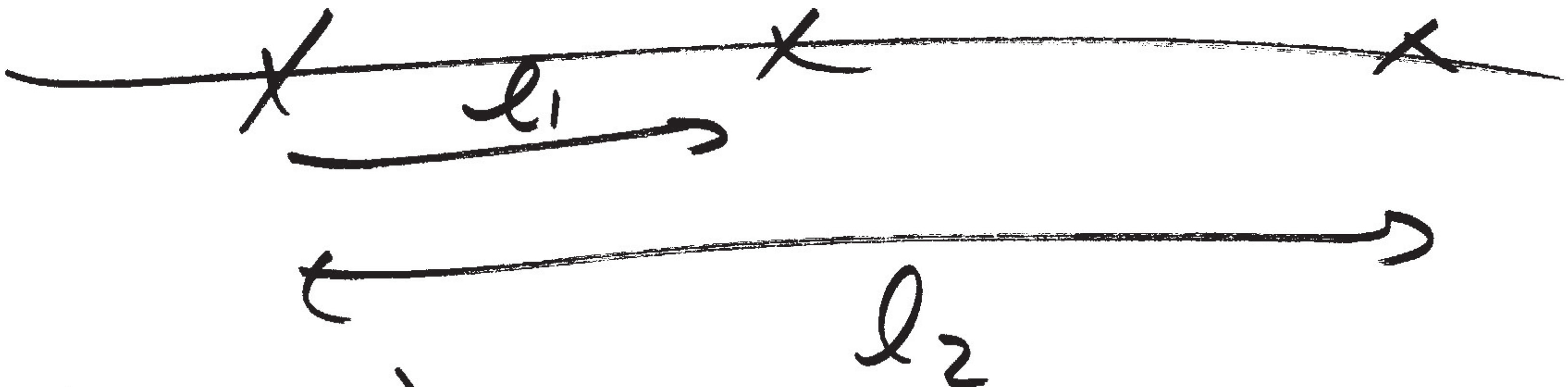
$$= \int_{t_0=12}^{\infty} f_t(t_0)\, dt_0 =$$

$$= \int_{t=12}^{\infty} \frac{1}{12} e^{-t/12}\, dt = 0.368$$

Q slow wombat: 24 secs To crosss

2 cars To die.

what is prob of survival.



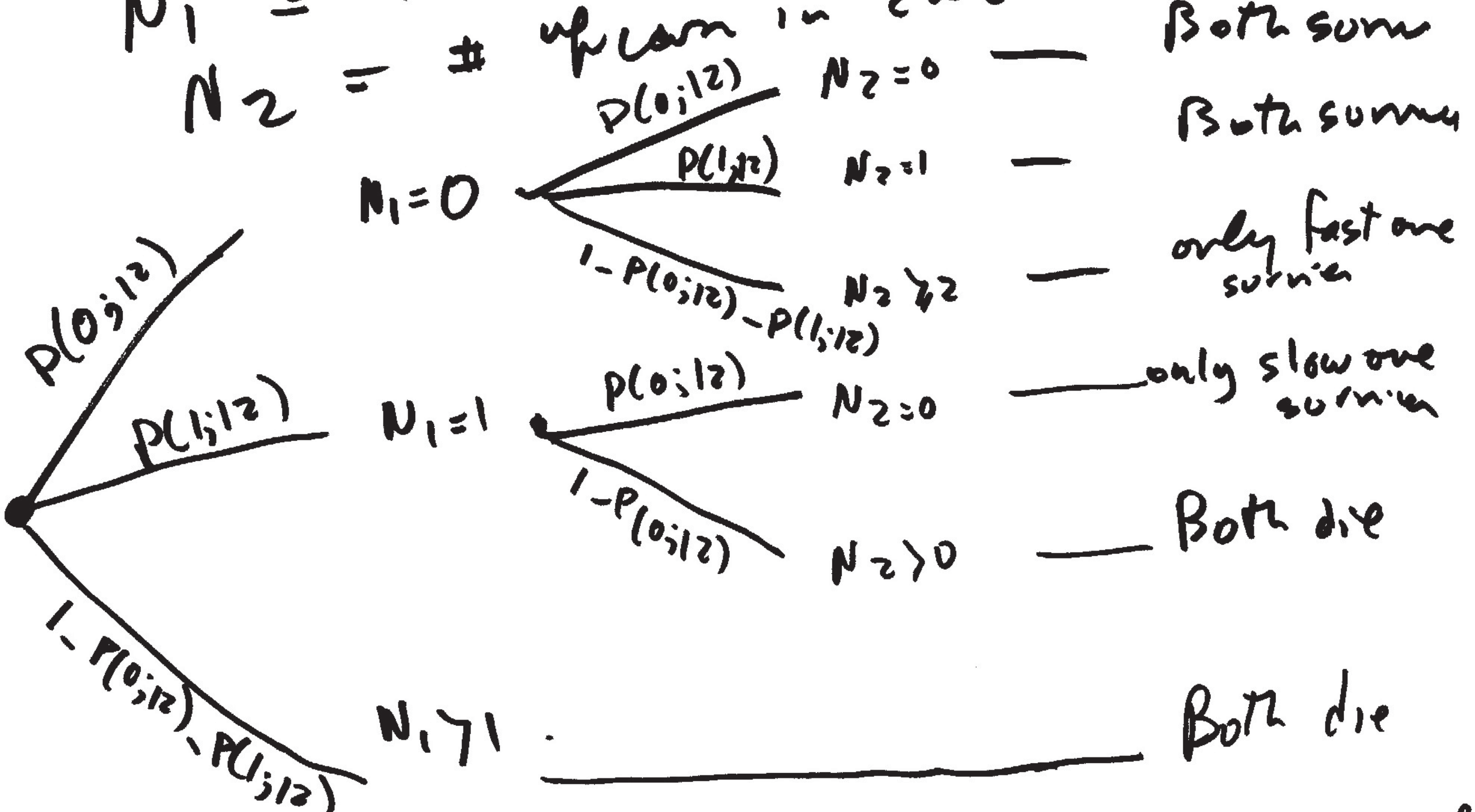
① $P(l_2 > 24)$

② $P(0;24) + P(1;24) = 3e^{-2} = 0.406$

Q B & R leave at the same time. immediately after a car.
What is Prob. that exactly one survive.

N_1 = # cars in first 12 second

N_2 = # of cars in 2nd 12 second.



P(exactly one wombat sorting) =

$$P(N_1=0,\ N_2 \geq 2) + P(N_1=1,\ N_2=0)$$

indep

$$= p(0;12)\left[1 - p(0;12) - p(1;12)\right]$$

$$+ \ p(1;12)\ p(0;12)$$

$$= e^{-1} - e^{-2} = 0.233.$$

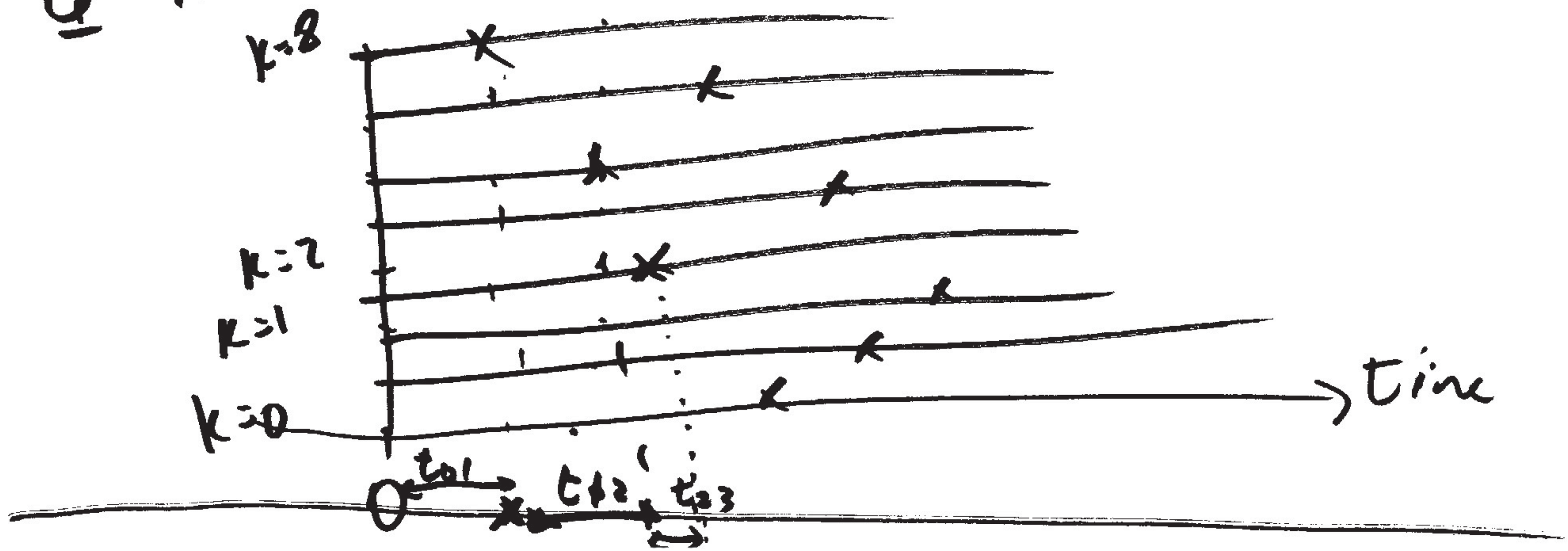
Ex $\quad f_t(t_0) = \begin{cases} \lambda e^{-\lambda t_0} & t_0 \geq 0 \\ 0 & \text{otherwise.} \end{cases}$

lifetime of a bulb.

$t=0$. we turn 8 bulbs on.

y = R.V. time until 3rd failure.

Q mean + variance and Transform of y.



Define t_{01} = Time from start until 1ST failure.

t_{12} = time from 1st failure to 2nd failure

t_{23} = time from 2nd failure to 3rd failure

$$y = t_{01} + t_{12} + t_{23}$$

merger a sum of 8 indp poisson R.P. with 8λ.

Poisson process with arrival rate of 7λ.

Poisson process with arrival rate of 6λ.

t_{ij} are indp

$$E(y) = E(t_{01}) + E(t_{12}) + E(t_{23})$$

$$= \frac{1}{8\lambda} + \frac{1}{7\lambda} + \frac{1}{6\lambda}$$

$$\sigma_y^2 = \sigma_{t_{01}}^2 + \sigma_{t_{12}}^2 + \sigma_{t_{23}}^2$$

$$= \left(\frac{1}{8\lambda}\right)^2 + \left(\frac{1}{7\lambda}\right)^2 + \left(\frac{1}{6\lambda}\right)^2$$

$$M_y(s) = M_{t_{01}}(s) \times M_{t_{12}}(s) \times M_{t_{23}}(s)$$

$$M_y(s) = \frac{8\lambda}{s+8\lambda} \cdot \frac{7\lambda}{s+7\lambda} \cdot \frac{6\lambda}{s+6\lambda}$$

<u>Ex</u> Joe waiting for a bus. for:

Newport To Fresno

$N \longrightarrow F.$ Right

$F \longrightarrow N$ wrong.

Arrival $\left\{ \begin{matrix} NF \\ FN \end{matrix} \right\}$ is poisson with rates

$$\left\{ \begin{matrix} \lambda_{NF} \\ \lambda_{FN} \end{matrix} \right\} = \left\{ \begin{matrix} \lambda_{right} \\ \lambda_{wrong} \end{matrix} \right\}$$

K = # of wrong buses until right bus arrives.

<u>Q</u>: PMF of K? mean.

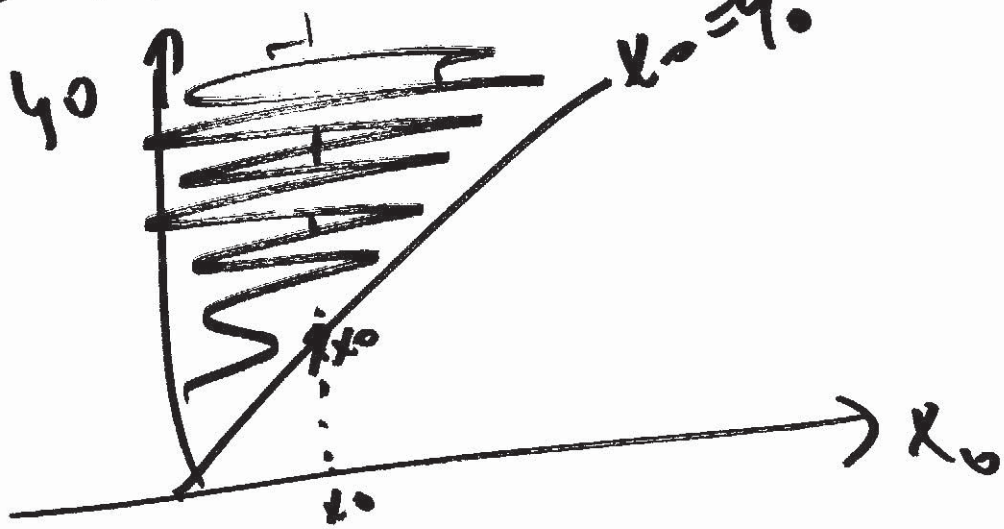
X = R.V. wait for wrong bus.

Y = R.V. Time to wait for the right bus

P(next bus is wrong) = Pr (X < Y)

$f_X(x_0) = \lambda_w e^{-\lambda_w x_0}$ $\quad x_0 > 0$

$f_Y(y_0) = \lambda_r e^{-\lambda_r y_0}$ $\quad y_0 > 0$

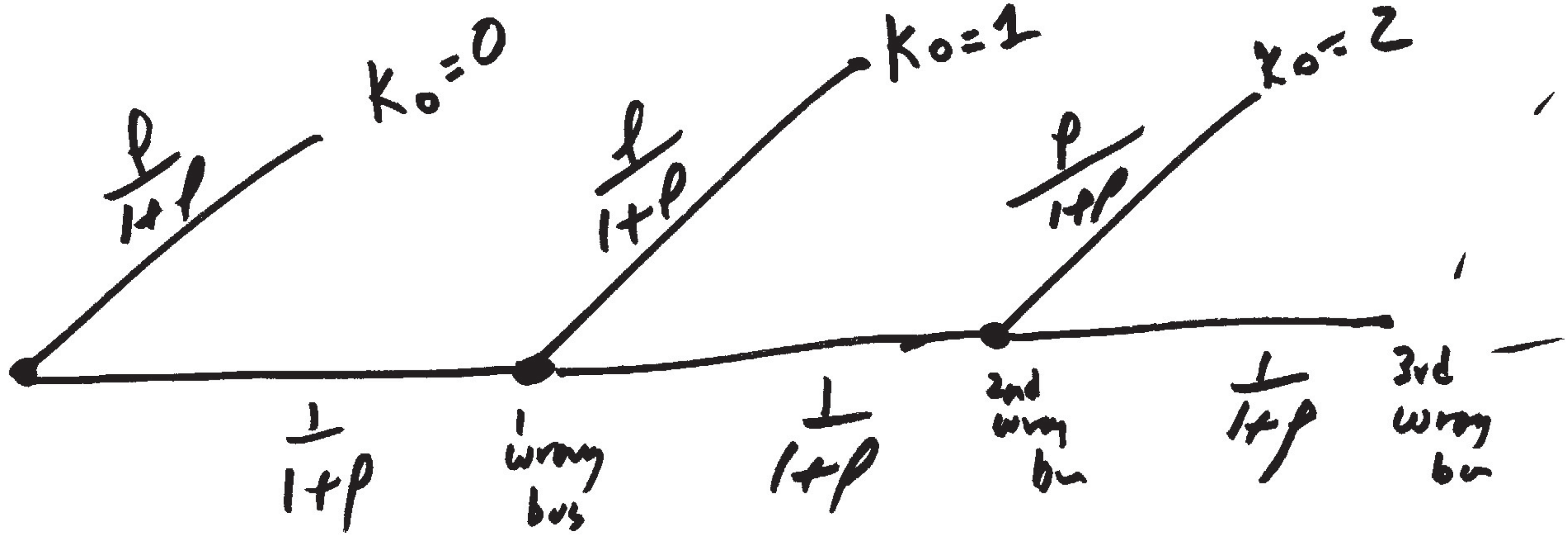


P(next bus is wrong) = P(x<y) =

$$= \int_{x_0=0}^{\infty} dx_0 \int_{y_0=x_0}^{\infty} dy_0 \; \lambda_w \lambda_r \, e^{-\lambda_w x_0} \, e^{-\lambda_r y_0}$$

$$\rho = \frac{\lambda_r}{\lambda_w}$$

$$P(\text{next bus is wrong}) = \frac{1}{1+\rho}$$



$K_0 = 0, 1, 2 \ldots$

$$P_K(K_0) = \frac{\rho}{(1+\rho)^{K_0+1}}$$

To compute mean Take transform, Then $s=0$

$\frac{1}{\rho} = E(K)$

$\rho \gg 1 \Rightarrow \lambda_r \gg \lambda_w$

$\Rightarrow E(K) = 0$

$\rho \ll 1 \Rightarrow \lambda_r \ll \lambda_w \Rightarrow E(K) \rightarrow \infty$