# <u>Indp of Multiple Events</u>

Events $A_1 A_2 \ldots A_n$ are indp. if .

$$P\left( \bigcap_{i \in S} A_i \right) = \prod_{i \in S} P(A_i)$$

for <u>EVERY</u> subset $S$ of $\{1,2,3, \ldots n\}$.

<u>EX</u> 3 events $A_1 \; A_2 \; A_3$

what are <u>ALL</u> possible subsets of $\{1,2,3\}$.

$\{1,2\}$ $\{1,3\}$ $\{2,3\}$, $\{1,2,3\}$

$$\left.\begin{aligned} P(A_1 A_2) &= P(A_1)\,P(A_2) \\ P(A_1 A_3) &= P(A_1)\,P(A_3) \\ P(A_2 A_3) &= P(A_2)\,P(A_3) \end{aligned}\right\} \rightarrow \text{pairwise independent.}$$

$P(A_1A_2A_3) = P(A_1)P(A_2)P(A_3)$

for us to call $A_1, A_2, A_3$ indp.

<u>EX</u> 2 indp. fair coin tosses.

HH
TH.
HT.
TT

$H_1 = \{$1st toss is heads$\}$

$H_2 = \{$2nd toss is heads$\}$

$D = \{$2 tosses have different results$\}$.

$\Rightarrow P(D) = 2/4 = \frac{1}{2}$

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<u>Q</u>: ~~Are~~ $H_1$, $H_2$ and $D$ indp?

$H_1$ & $H_2$ are indp. $P(H_1H_2) = P(H_1)P(H_2)$.

Are $H_1$ and $D$ indp?

$$P(D \mid H_1) = \frac{P(H_1D)}{P(H_1)} = \frac{1/4}{\frac{1}{2}} = 1/2 = P(D)$$

$\Rightarrow$ $D$ & $H_1$ are also indp.

How about $P(H_1 H_2 D) \stackrel{?}{=} P(H_1)\ P(H_2)\, P(D)$

$\phi$ not True $\neq 0$

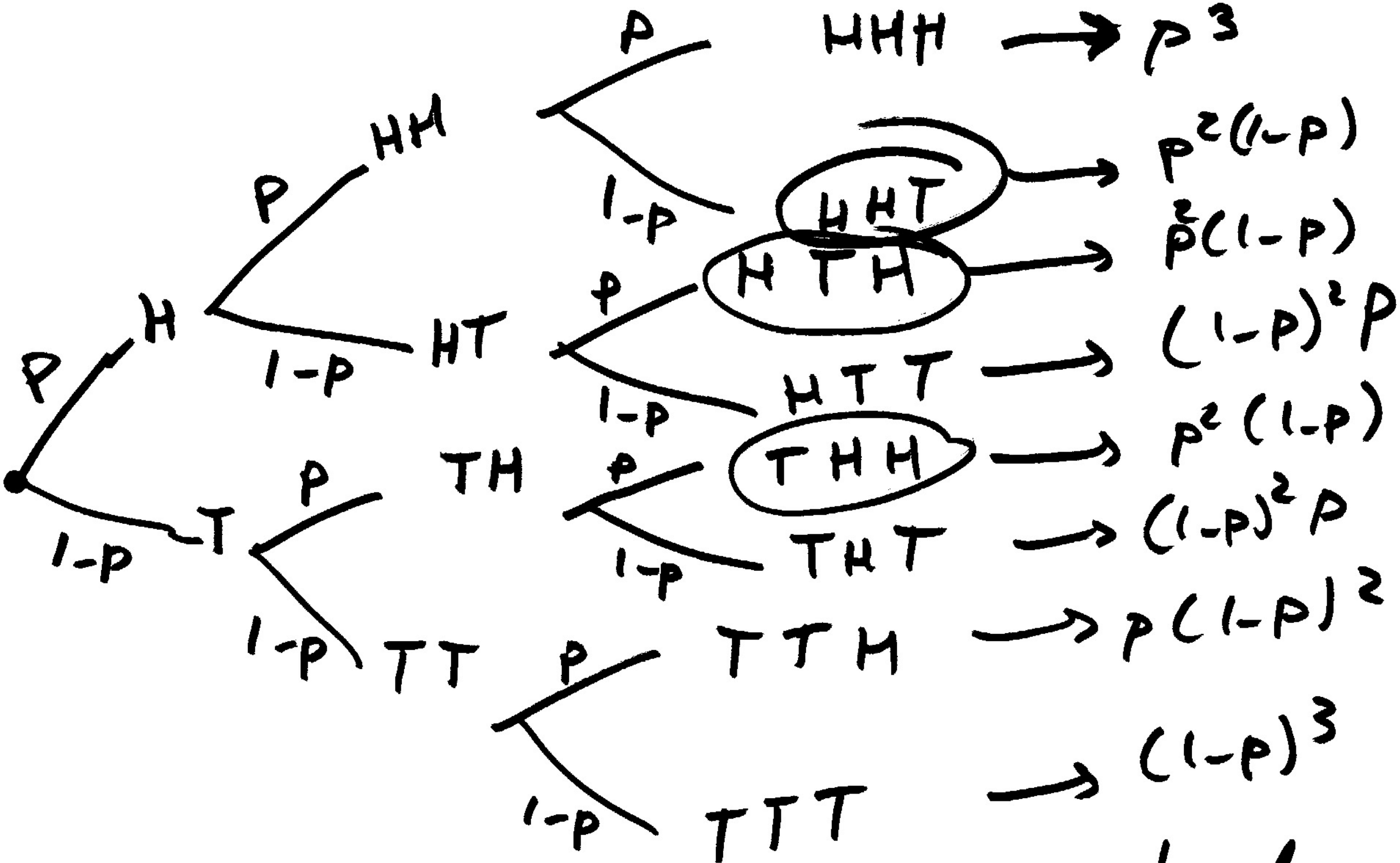
$\Rightarrow$ $H_1, H_2,$ and $D$ are not indp.

# Independent Trials

- Seq of $\overbrace{\text{independent, but identical}}^{\text{stages}}$ in an experiment.

- If 2 outcomes $\longrightarrow$ Bernoulli Trials.

Ex $n$ indp tosses of a coin.




$\underline{Q}$ what is prob that I get $k$ heads in a seq. of $n$ trials.

for one seq of $k$ heads and $n-k$ Tails.

$$P(\cdot) = p^k (1-p)^{n-k}$$

How many such seq are There?

$$\binom{n}{k} \triangleq \frac{n!}{k!\,(n-k)!} = \binom{n}{n-k}$$

$n$ choose $k$ $\quad\quad$ $n$ choose $n-k$

Ex $n=3 \quad k=2 \quad \frac{3!}{2!\,1!} = 3$

$$P(k \text{ heads}) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \frac{n!}{k!\,(n-k)!} p^k (1-p)^{(n-k)}$$

binomial prob.

binomial coeff.

$$\sum_{k=0}^{n} P(k \text{ heads}) = 1 \Rightarrow$$

$$\boxed{\sum_{k=0}^{n} \frac{n!}{k!\,(n-k)!} p^k (1-p)^{n-k} = 1}$$

identity


n = 100

Binomial distribution.

Ex

$$\frac{100{,}000{,}000}{4{,}000} = 25{,}000$$

$$\frac{100{,}000}{4{,}000} = 25$$

0.001

~~0.01~~

~~0.1~~

~~0.0001~~

Q : what is prob

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Ex

c modems $c \ll n$

n subscribers

each customer dials up with prob (p) indp. of the others.

Q: what is prob That customers get rejected?

Define $p(K) = p($ $K$ subs are connected$)$

$$p(\cdot) = \sum_{K=C+1}^{n} p(K) = p(C+1) + p(C+2) \cdots$$

$$p(K) = \binom{n}{K} p^{K} (1-p)^{n-K}$$

$$p(\cdot) = \sum_{K=C+1}^{n} \binom{n}{K} p^{K} (1-p)^{n-K}$$

<u>Ex</u> $n = 100$ $p = 10\%$ $C = 15$

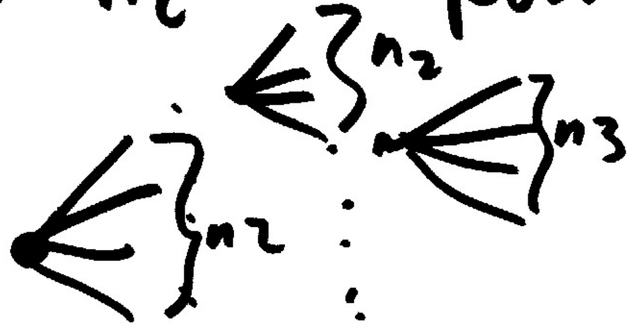
$$\Rightarrow p(\cdot) = 0.0399$$

# Counting

- helps us do prob. calc.

Process $r$ stages.

1. $n_1$ possible results in stage 1
2. For **ALL** possible results of 1st stage, There are $n_2$ possible results at stage 2
3. $n_i$ possible results at stage $i$




total # of possibilities $= n_1 n_2 \cdots n_r$

<u>EX</u> 7 digit phone #:

$$\underbrace{10}_{1} \times \underbrace{10}_{2} \times \underbrace{10}_{3} \times \underbrace{10}_{4} \times \underbrace{10}_{5} \times \underbrace{10}_{6} \times \underbrace{10}_{7} = 10^7$$

if no 0 or 1 in 1st digit

$$8 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 8 \times 10^6$$

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Selecting k objects out of n

Order matters → permutation

order doesn't matter → combination

Ex  4 balls

#1, #2, #3, #4.

Choose 2 out of 4 balls, in draw #1 & #2. how many diff. sequences can I get?

order matter.

$$\left.\begin{array}{l} 1\ 2 \\ 1\ 3 \\ 1\ 4 \\ 2\ 3 \\ 2\ 4 \\ 2\ 1 \\ 3\ 4 \\ 3\ 1 \\ 3\ 2 \\ 4\ 1 \\ 4\ 2 \\ 4\ 3 \end{array}\right\} \underline{\underline{12}}$$

$4 \times 3 = 12$

n objects choose k

① first Time 1 out of n
left with n-1 objects.

② 2nd Time 1 out of n-1

③ 3rd 1 out of n-2

kth

$n$
$n-1$
$n-2$
$\vdots$
$n-k+1$

$= n \times (n-1) \times (n-2) \ldots (n-k)$ # of permutations.

$$= \frac{n(n-1)(n-2)\ldots(n-k+1)(n-k)(n-k-1)\ldots\ldots 2 \cdot 1}{(n-k)(n-k-1)(n-k-2)\ldots\ldots 2 \cdot 1}$$

$$= \frac{n!}{(n-k)!}$$

n objects, choose k.

$$\frac{n!}{(n-k)!}.$$

Ex How many distict 4 letter words are there.

$$\frac{26!}{22!} = 26 \times 25 \times 24 \times 23$$

$$\approx 358000$$