Combinations

- 4 letters A, B, C, D. choose 2.

permutation → order matters.
combination → order doesnot matter.

- Permutation

order matters

AB	AC	AD
BA	BC	BC
CA	CB	CD
DA	DB	DC

permutations

- Combination

AB AC AD BC BD CD

k

- n objects, k from it. $\frac{n!}{(n-k)!}$ permutations

$\underbrace{\text{| | | | |}}_{k}$ $k \times (k-1) \times (k-2) \dots 2 \cdot 1 = k!$

$k!$ # of combination = # of permutations.

$k!$ # of comb $= \dfrac{n!}{(n-k)!}$

$\Longrightarrow$ # of combin $= \dfrac{n!}{k!\,(n-k)!} = \binom{n}{k}$ ← binomial coeff.

k out of n also n-k out of n

$$\frac{n!}{(n-k)!\,(n-(n-k))!} = \frac{n!}{(n-k)!\,k!}$$

Ex $\binom{4}{2} = \dfrac{4!}{2!\;2!} = \dfrac{4\times 3}{2} = 6$

Ex Form of committee 4 people.

4 males: R S T U

5 female: V W X Y Z.

→ R and S **both** cannot be on the committee unless at least one females.

How many possibilities are there?

$RS' \longrightarrow \binom{7}{3}$

$R'S \quad \binom{7}{3}$

$R'S' \quad \binom{7}{4}$

$RS \xrightarrow{\text{one female}} \binom{5}{1} \times \binom{2}{1}$

$RS \xrightarrow{\text{2 female}} \binom{5}{2}$

$$\binom{7}{3} + \binom{7}{3} + \binom{7}{4} + \binom{5}{1}\binom{2}{1} + \binom{5}{2} = 125$$

$$\binom{9}{4} - 1 = 125$$

RSTU

3

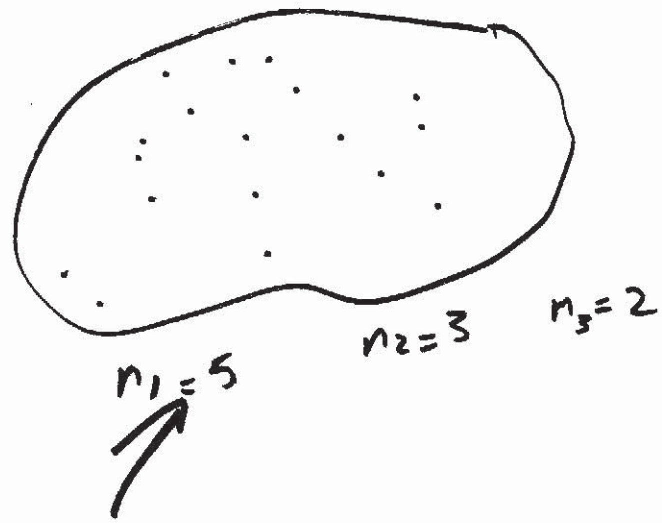
Partitions

Set of n elements.

partition into r sets.   ith set has n_i elements.

$$\sum_{i=1}^{r} n_i = n$$

How many different partitions can I have.



Form one set at a time:

$\binom{n}{n_1}$ # of ways to choose set 1

$\binom{n-n_1}{n_2}$ # of ways to choose set 2

$\vdots$

$\binom{n-n_1-n_2}{n_3}$ # of [illegible]

$$= \binom{n}{n_1}\binom{n-n_1}{n_2}\cdots\cdots\binom{n-n_1-n_2\cdots-n_{r-1}}{n_r}$$

$\longrightarrow$ # of ways of partitioning a set of n elements.

$$= \frac{n!}{n_1!\,(n-n_1)!}\;\frac{(n-n_1)!}{n_2!\,(n-n_1-n_2)!}\cdots\cdots\frac{(n-n_1-\cdots\cdots n_{r-1})!}{(n-n_1-n\cdots\cdots-n_{r-1}-n_r)!\,n_r!}$$

$$= \frac{n!}{n_1!\,n_2!\cdots\cdots n_r!} \longrightarrow \text{multinomial coeff.}$$

If $r=2 \Longrightarrow$ binomial coeff.

n

$K.$

$$\frac{n!}{K!\,(n-K)!}$$

$$\underbrace{K}_{n_1}+\underbrace{(n-K)}_{n_2}=n$$

EX TATTOO

how many differant 6 letter words can I make out of the letters.

$n=6$

$$\frac{6}{3\times3\times\cancel{3\times3}\times3\times3}$$

group A

1 letter A

group T

3 letters

group O

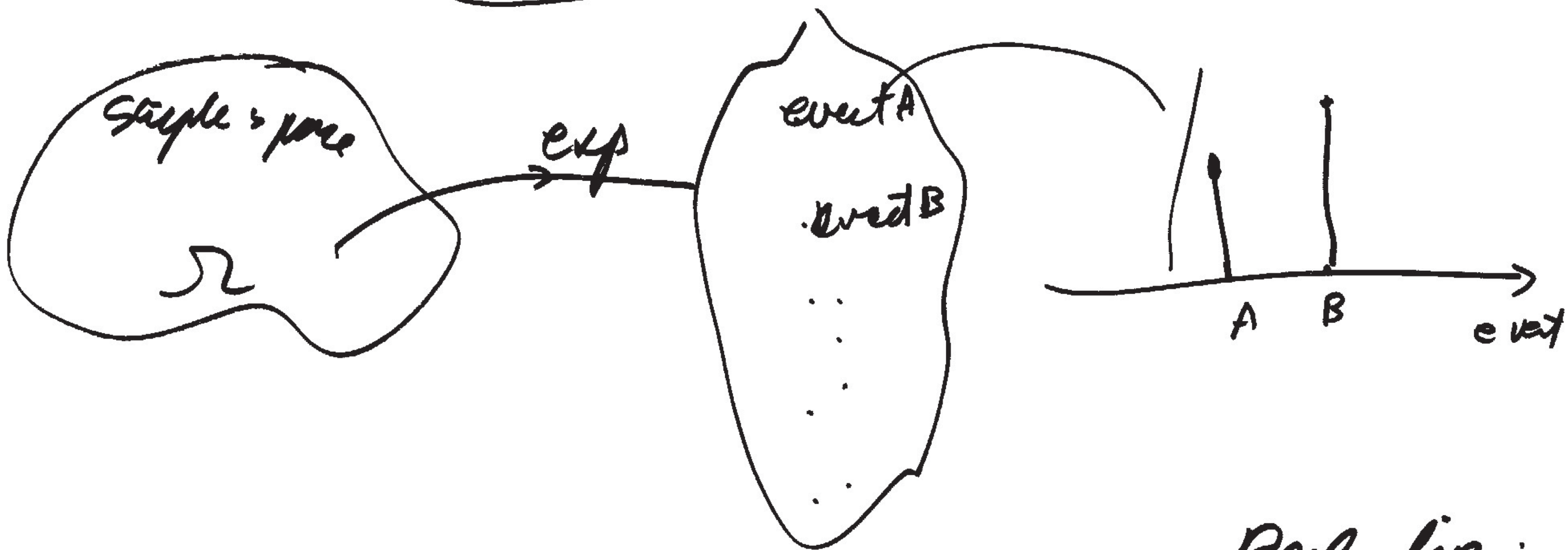
2 letter

$3+2+1=6$

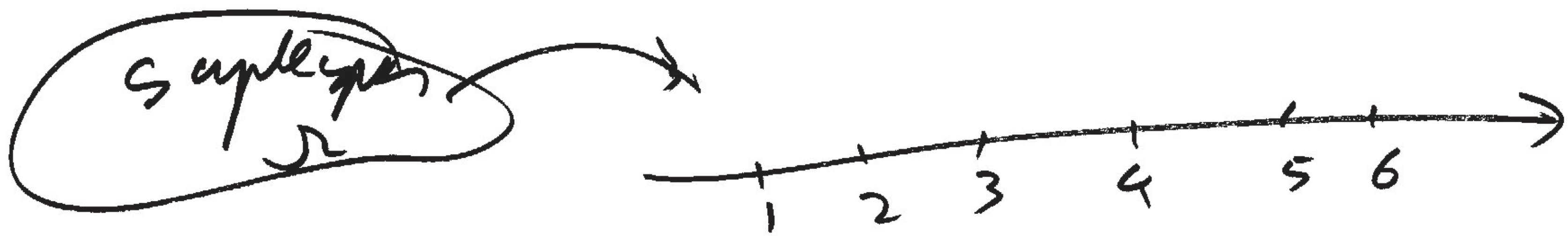
$$\frac{6!}{3!\,2!\,1!}=$$

$$\frac{6\times5\times4}{2!}=\underline{\underline{60}}$$

Random Variables



R.V. mapping from sample space onto Real line.



- Ex Roll a die. → # on Top is RV

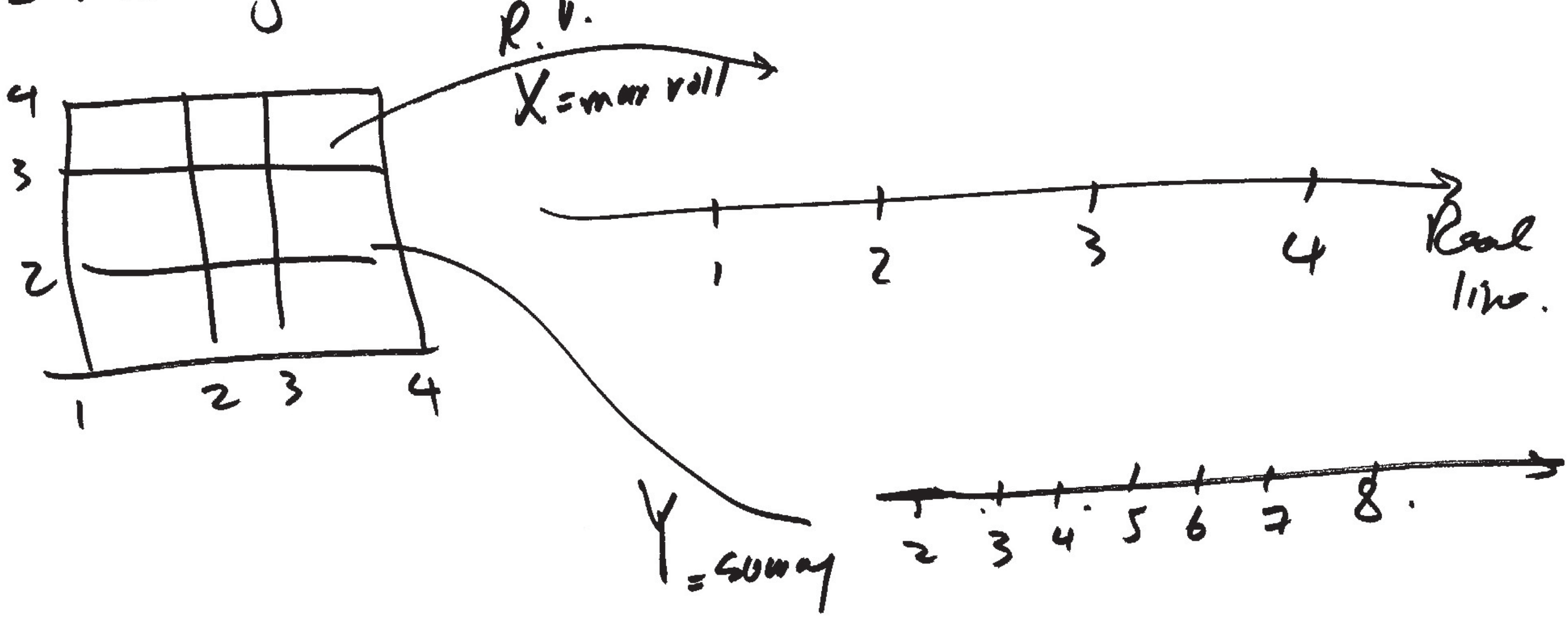
X

$$P(X=1)=\frac{1}{6}$$

$$P(X=2)=\frac{1}{6}$$

Each time we do an exp, generating a exp. value for ana. R.V.

Ex 2 rolls of a die 4 sides



Ex Flip a coin

$$X = \begin{cases} \sqrt{2} & \text{if head} \\ 55 & \text{if tail} \end{cases}$$

Ex flip a coin 3 Times.

R.V h = Total of heads

R.V. r = length of the largest run resulting from 3 flips.

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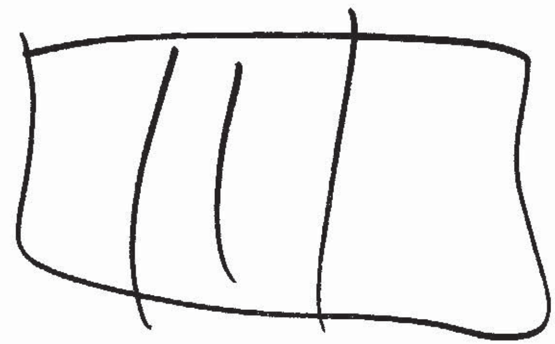
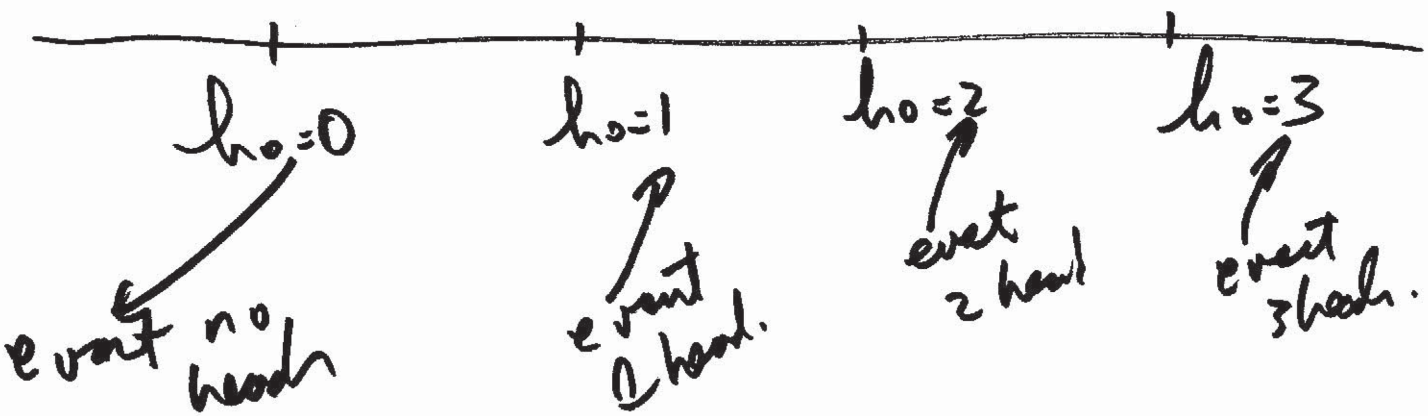
1 exp → $H_1 \underline{T_2 \; T_3}$

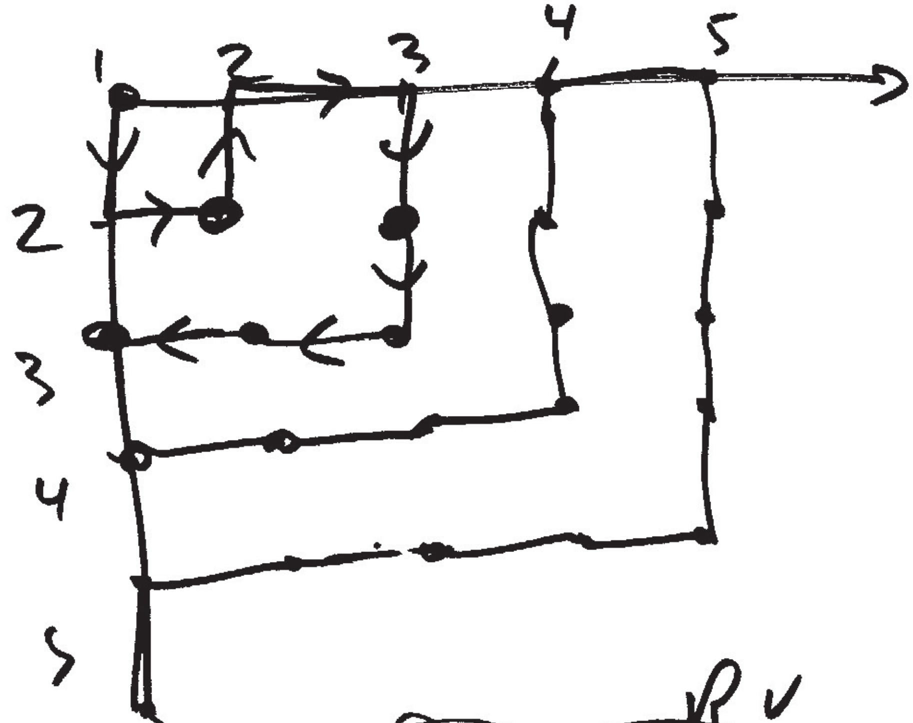
exp. value of r is 2

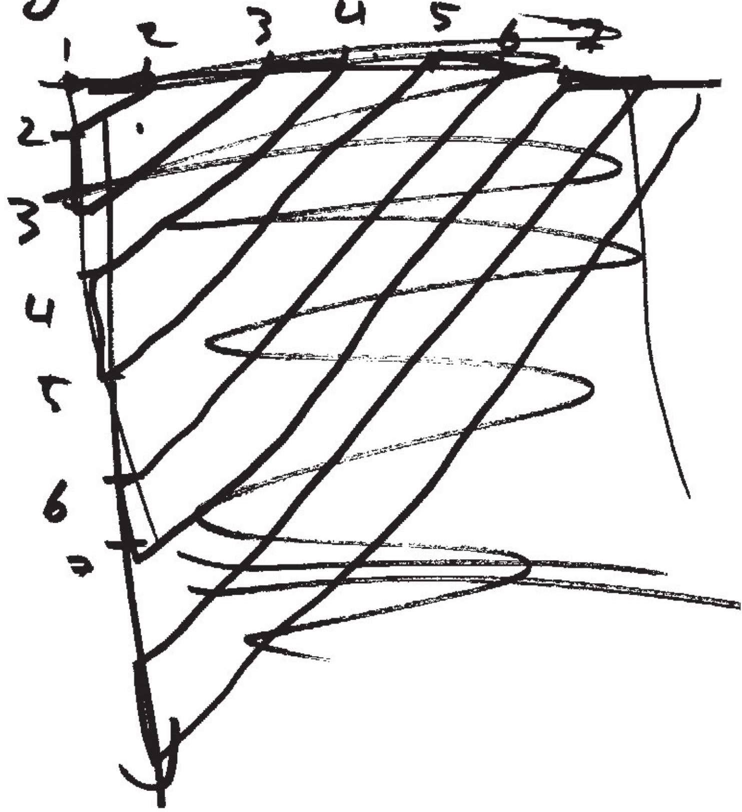
exp value of h is 1

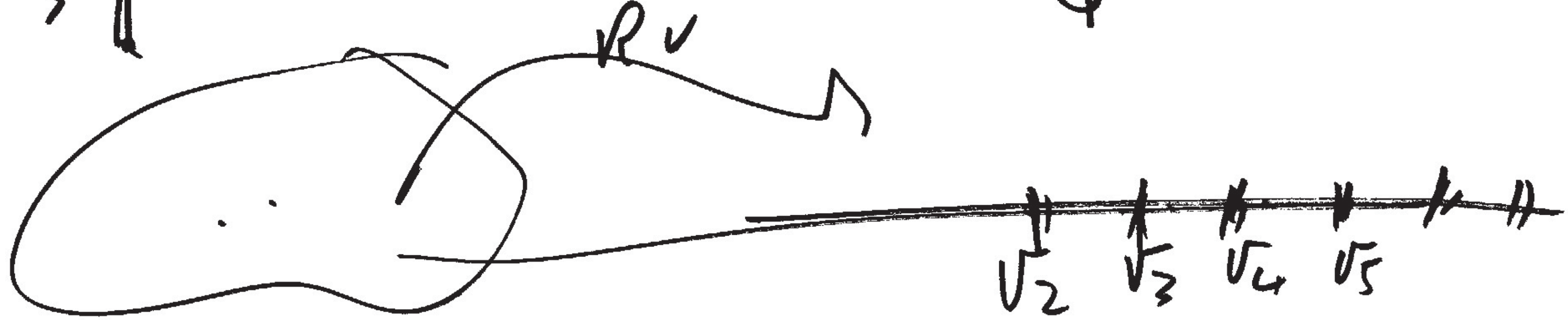




Def Discrete Random Variable if its range is finite or at most countably ∞.







Continuous Rate

choose a

between

[-1, +1]

b(k)

$$\mathrm{sgn}\{a\} = \begin{cases} 1 & a>0 \\ 0 & \text{if } a=0 \\ -1 & \text{if } a<1 \end{cases}$$

$\cos(\omega t + f(b(k)))$

Probability Mass Function PMF

PMF characterizes a R.V. through prob of the values it can take.

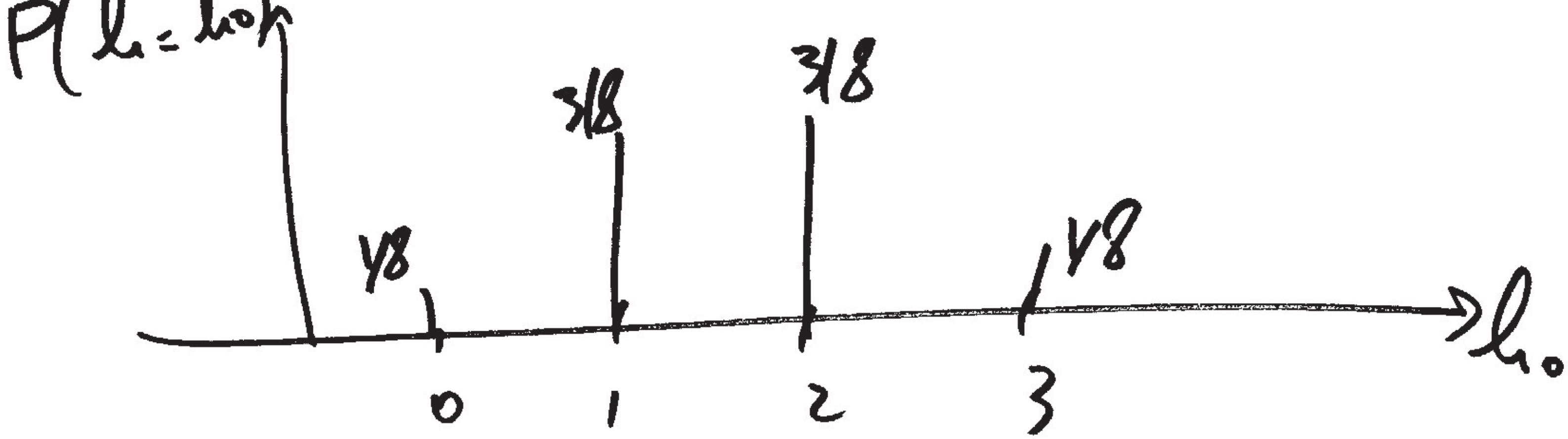
3 toss of coin
h = # of heads.

$h_0=0$ $\quad h_0=1$ $\quad h_0=2$ $\quad h_0=3$

event no head

$$P(h = h_0) = \begin{cases} \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8} & \text{if } h_0=0 \\ 3/8 & \text{if } h_0=1 \\ 3/8 & \text{if } h_0=2 \\ 1/8 & \text{if } h_0=3 \end{cases}$$

PMF of random variable h



$P_X(x)$ = Prob that R.V X takes on value x.

R.V.

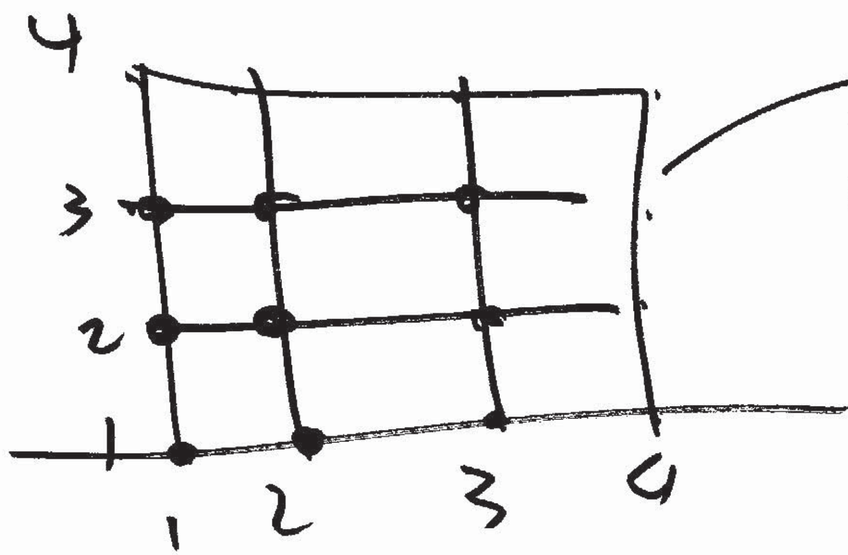
dummy variable.

$\cancel{f(x) = \frac{x}{x+1}}$

$\cancel{f(y) = \frac{y}{y+1}}$

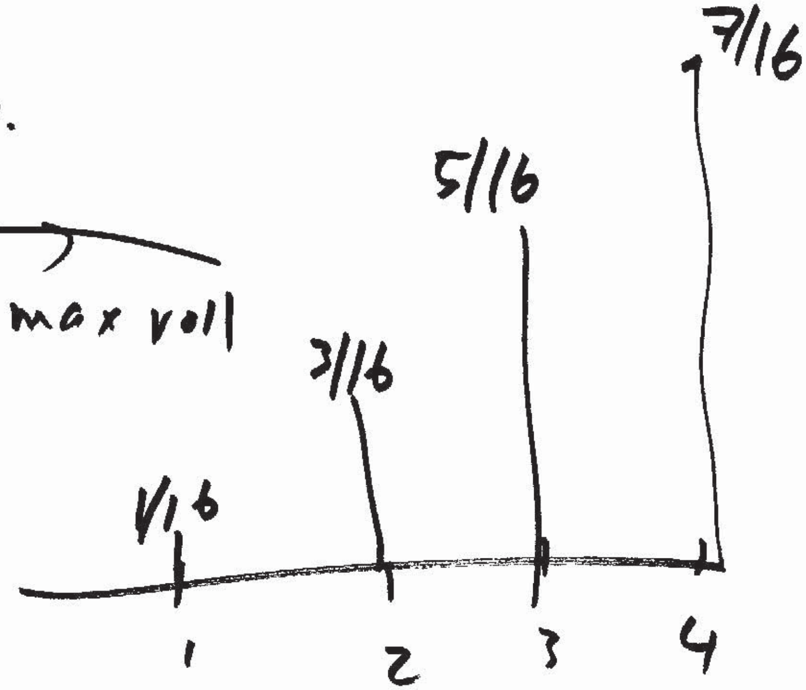
$P_x(x_0)$

2 roll die



RV.

X = max roll



$$P(X=k) = \begin{cases} 1/16 & k=1 \\ 3/16 & k=2 \\ 5/16 & k=3 \\ 7/16 & k=4. \end{cases}$$

Bernoulli Random Variable

2 outcomes $\rightarrow$ p, $1-p$. $\longrightarrow$ $P_X(x) = \begin{cases} p & \text{if } x=1 \\ 1-p & \text{if } x=0 \end{cases}$

Toss a coin

$$X = \begin{cases} 1 & \text{if head} \\ 0 & \text{if tail.} \end{cases}$$

Binomial R.V.

Toss a coin n times $\rightarrow$ heads p, tail $(1-p)$

X = # of heads in seq of n Tosses

$$P_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$p = 1/2$

symmetric

n large
p small