Example on Conditional R.V.

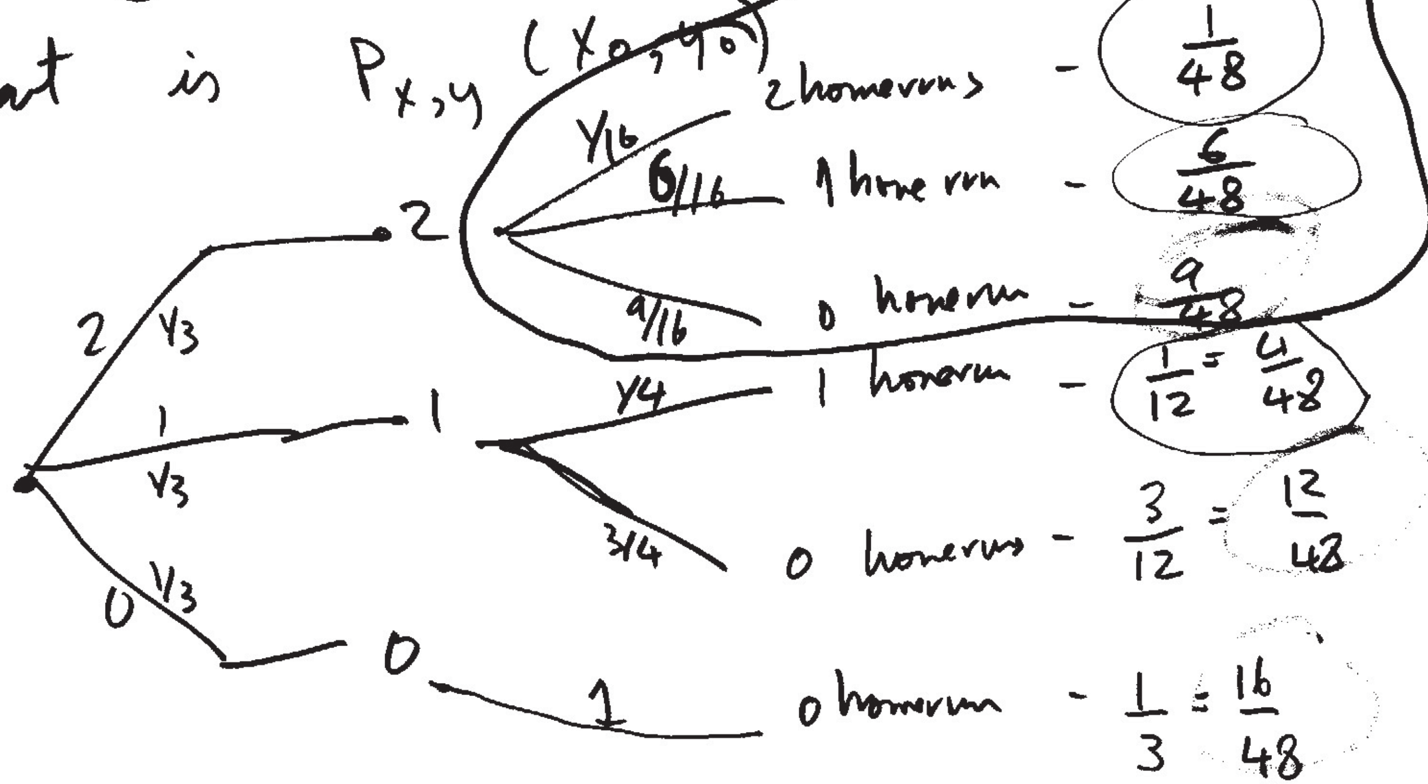
- Each game B.B is pitched to 0, 1, or 2 Times with equal prob.
- Each Time he is batting pr(home run) = $1/4$

X = # of Times he is pitched at

Y = # of home runs.

1	2	
3/4	1/4	$\frac{3}{16} \times 2 = \frac{6}{16}$
1/4	3/4	

- what is $P_{X,Y}(x_0, y_0)$



$p(\text{at least one home run})$

$= \frac{4}{48} + \frac{1}{48} + \frac{6}{48}$

$= \frac{11}{48}$

what is $p(x|y)$?

$$P(x|y) = \begin{cases} \text{if } y=2 \longrightarrow P_{x|2}(x_0/2) = \begin{cases} 1 & x=2 \\ 0 & \text{otherwise.} \end{cases} \\ \text{if } y=1 \longrightarrow P_{x|1}(x_0/\ y=1) = \begin{cases} 6/10 & x=2 \\ 4/10 & x=1 \end{cases} \\ \text{if } y=0 \longrightarrow P_{x|0}(x_0/y_0=0) = \begin{cases} \frac{9}{37} & x=2 \\ \frac{12}{37} & x=1 \\ \frac{16}{37} & x=0 \end{cases} \end{cases}$$

Conditional Exp

Event A.

- Conditional expect of a R.V. X given event A.

$$E[X|A] = \sum_{x_0} x_0 \; P_{X|A}(x_0|A)$$

- $E[g(X)|A]$ = conditional Expect of a fn of a R.V. X. given Event A

$$= \sum_{x_0} g(x_0) \; P_{X|A}(x_0|A)$$

- Expected value of r.v. X given r.v $y = y_0$

$$E[X|y=y_0] = \sum_{x_0} x_0 \; P_{X|Y}(x_0|y=y_0)$$

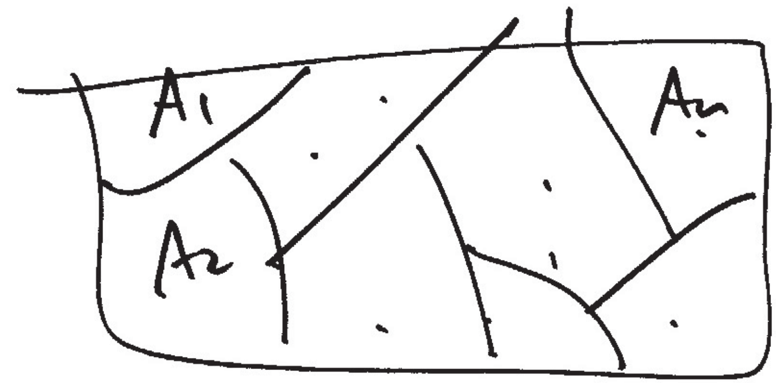
• write $E(X)$ as a fn of $E[X|y=y_0]$

$$E(X) = \sum_{y_0} P_Y(y_0) \quad E[X \mid Y = y_0]$$

Total expectation Thm

- $A_1, A_2, \dots A_n$ = disjoint events form a partition of sample space.

$$E(X) = \sum_{i=1}^{n} P(A_i) \; E[X \mid A_i]$$



Ex Barry Bond E # of home runs ?

E(homerun) = P(zero times pitched at) E(homerun / pitched 0 times)

+ P(one time " ") E(homerun / pitched once)

+ P(pitched twice) E(homerun / pitched twice)

$$= \frac{1}{3} \cdot 0$$

$$+ \frac{1}{3} \cdot \frac{1}{4}$$

$$+ \frac{1}{3} \cdot \frac{1}{4} \cdot X2$$

$$= 0 + \frac{1}{12} + \frac{2}{12} = \frac{3}{12} = \frac{1}{4}$$

independence

R.V. X is indp. from an event A

if $P_{X|A}(X = x_0 | A) = P_X(x_0)$ $\forall x_0$

$P(X = x_0 \text{ and } A) = P(X = x_0)\, p(A)$

EX roll an unbiased 4 sided R.V X

A = event. even #

$$P_X(x_0) = \begin{cases} 1/4 & x_0 = 1,2,3,4 \\ 0 & \text{otherwise.} \end{cases}$$

$\neq$

$$P_X(x_0 | A) = \begin{cases} 1/2 & x_0 = 2,4 \\ 0 & \text{otherwise} \end{cases}$$

$\Rightarrow$ X, A are not indp.

Def independence of 2 R.V.

X, y are called indp R.V. if.

$$P_{x,y}(x_0, y_0) = P_x(x_0)\, P_y(y_0)$$

or

$$P_{x|y}(x_0 / y_0) = P_x(x_0)$$

Def Conditional indp of 2. R.V. w.r.t. event A.

Given Event A, R.V. x, y are indp iff

$$P_{x,y|A}(x_0, y_0 | A) = P_{x|A}(x_0/A)\, P_{y|A}(y_0/A)$$

Def Conditional Variance of R.V.

$$\sigma^2_{x|A} = \sum_{x_0} \left[x_0 - E(X|A)\right]^2 P_{x|A}(x_0/A)$$

- If x, y indp Then

$$E(xy) = E(x) \cdot E(y).$$

$$E(xy) = \sum_{x_0} \sum_{y_0} x_0 y_0 \, P_{x,y}(x_0, y_0)$$

$$\Big\downarrow \text{indp assumption}$$

$$= \sum_{x_0} \sum_{y_0} x_0 y_0 \, P_x(x_0) \, P_y(y_0)$$

$$= \Big[\sum_{x_0} x_0 \, P_x(x_0) \Big] \Big[\sum_{y_0} y_0 \, P_y(y_0) \Big]$$

$$= E(x) \qquad E(y) \implies \text{QED.}$$

Do not need indp

$$E(x+y) = E(x) + E(y)$$

If x, y indep.

$$Var(x+y) = Var(x) + Var(y)$$

<u>EX</u> 4 sided die, roll once. biased.

n = r.v. value of face down. value.

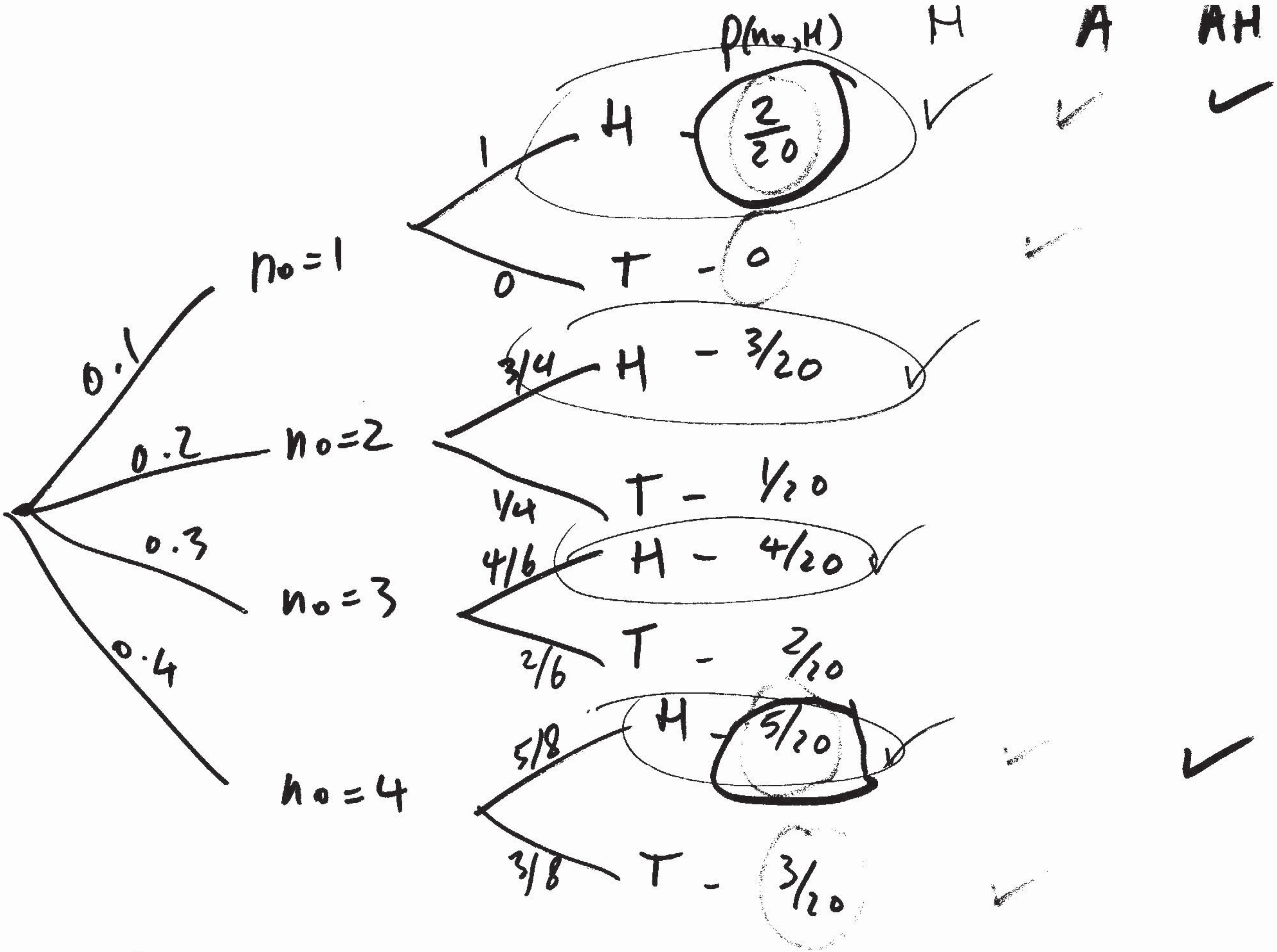
$$P_n(n_0) = \begin{cases} \frac{n_0}{10} & n_0 = 1, 2, 3, 4 \\ 0 & \text{otherwise.} \end{cases}$$

Toss a coin.

- Based on outcome,

$$p(\text{heads}) = \frac{n+1}{2n}$$

<u>Q</u> Expected value & variance of n.



Ans $E(n) = \sum_{no} no \, P_n(no) = 0.1 \times 1 + (2 \times 0.2) + (3 \times 0.3) + (4 \times 0.4) = 3$

$$Var[n] = \sum_{n_0} (n_0 - 3)^2 \; P_n(n_0)$$

$$= 0.1(1-3)^2 + 0.2(2-3)^2 + 0.3(3-3)^2 + 0.4(4-3)^2$$

$$\sigma_n^2 = 1$$

Q2 Conditional pmf, conditional exp. and conditional var of n given head.

Cond.

$$P_{n|H}(n_0|H) = \frac{P(n_0, H)}{P(H)}$$

$$P_{n|H}(n_0|H) = \begin{cases} \frac{2}{20} \times \frac{20}{14} = 2/14 & n_0 = 1 \\ \frac{3}{20} \times \frac{20}{14} = 3/14 & n_0 = 2 \\ \frac{4}{20} \times \frac{20}{14} = 4/14 & n_0 = 3 \\ \frac{5}{20} \times \frac{20}{14} = 5/14 & n_0 = 4 \end{cases}$$

$$E(n \mid H) = \sum_{n_0} n_0 \, P_{n|H}(n_0 \mid H)$$

conditional expect.

$$= 1 \cdot \frac{2}{14} + 2 \cdot \frac{3}{14} + 3 \cdot \frac{4}{14} + 4 \cdot \frac{5}{14} = \frac{20}{7}$$

$$\sigma^2_{n|H} = \sum_{n_0} \left(n_0 - \frac{20}{7} \right)^2 P_{n|H}(n_0 \mid H)$$

$$= \frac{2}{14}\left(1 - \frac{20}{7}\right)^2 + \frac{3}{14}\left(2 - \frac{20}{7}\right)^2 + \frac{4}{14}\left(3 - \frac{20}{7}\right)^2 + \frac{5}{14}\left(4 - \frac{20}{7}\right)^2 = 35/49$$

Q Define 2 events:

Event A = value of roll of die was 1 or 4

Event H = outcome of the coin is heads.

$$P(AH) \stackrel{?}{=} P(A)\,P(H)$$

$$P(H) = \frac{14}{20} \qquad P(H) = \sum_{n_0} P_n(n_0, H)$$

$$P(A) = \frac{2}{20} + 0 + \frac{5}{20} + 3/20 = \frac{10}{20}$$

$$P(AH) = \frac{7}{20}$$

$$P(AH) \stackrel{?}{=} P(A)\,P(H) =$$

$$\frac{7}{20} = \frac{14}{20} \times \frac{10}{20} = \frac{7}{20}$$

$\Longrightarrow$ A and H are indp.