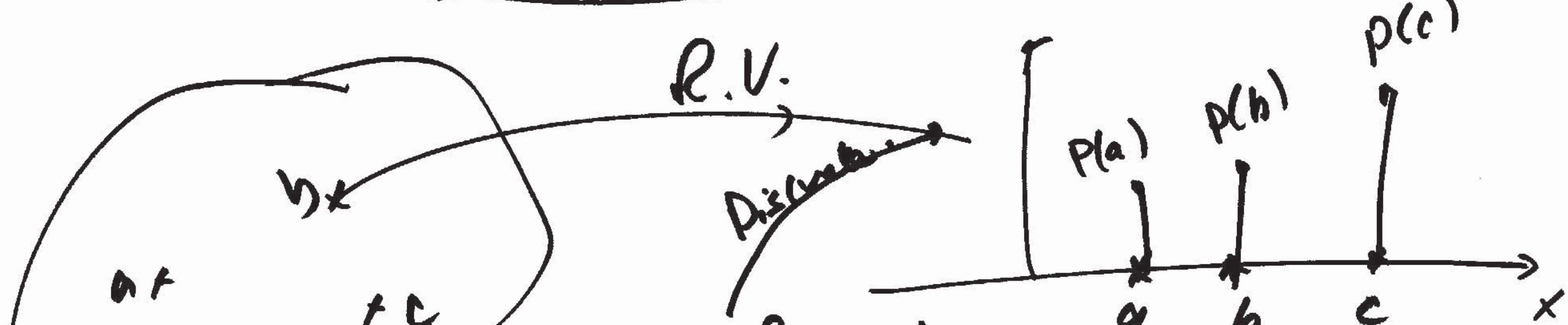
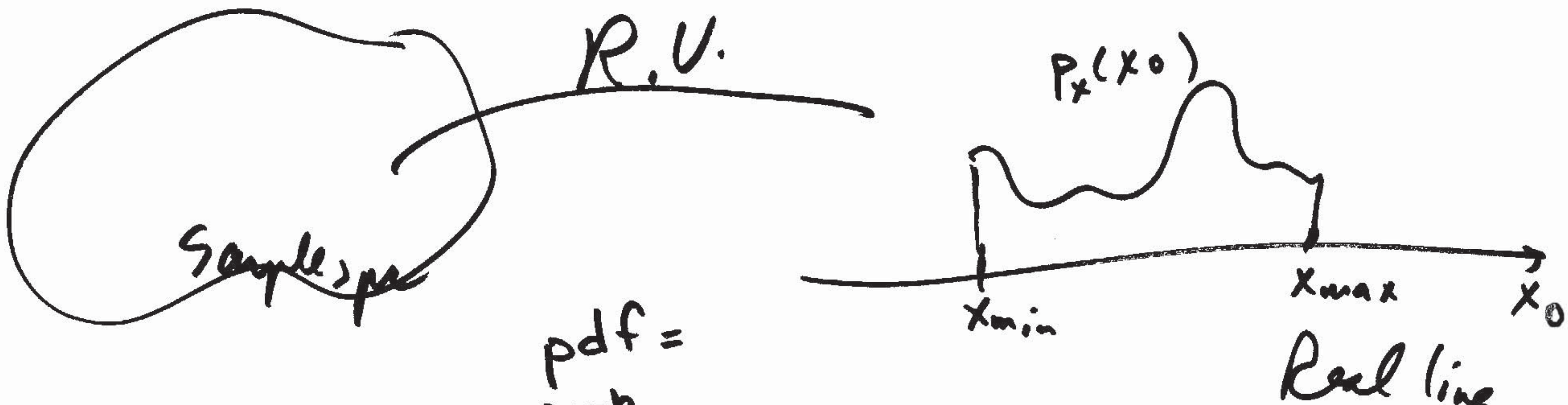
Continuous R.V.



pmf = prob mass function



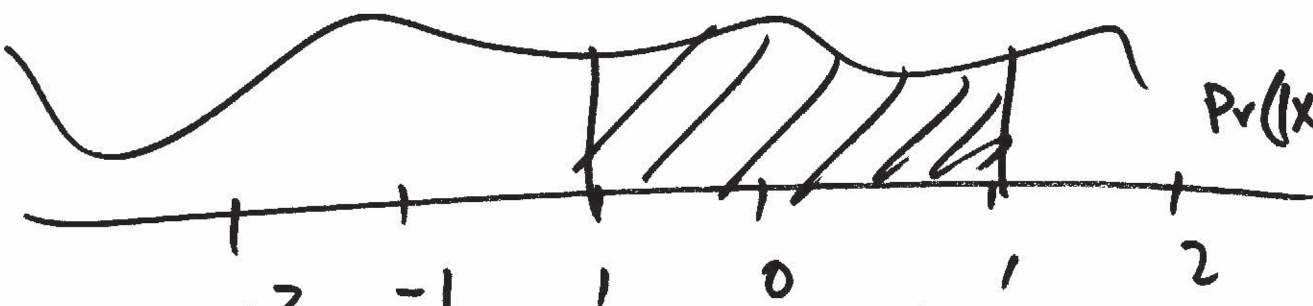
pdf = prob. dist. fn.

Def. pdf of R.V. x whose event space.
real line $x_0 \to -\infty$ to $x_0 \to +\infty$.

$$Pr(a \le x \le b) = Pr(a < x < b) = Pr(a < x \le b)$$
$$= Pr(a \le x < b)$$
$$= \int_a^b f_x(x_0)\, dx_0$$

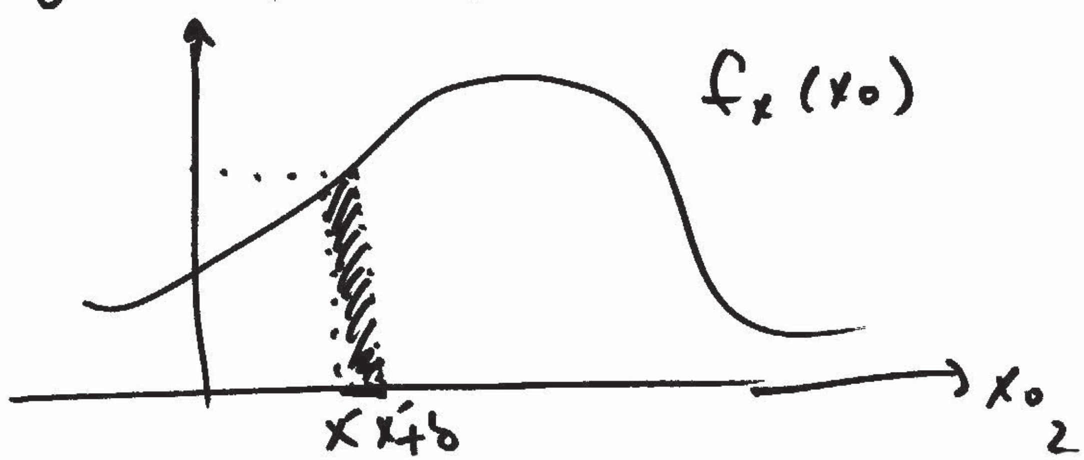
$f_x(x_0)$

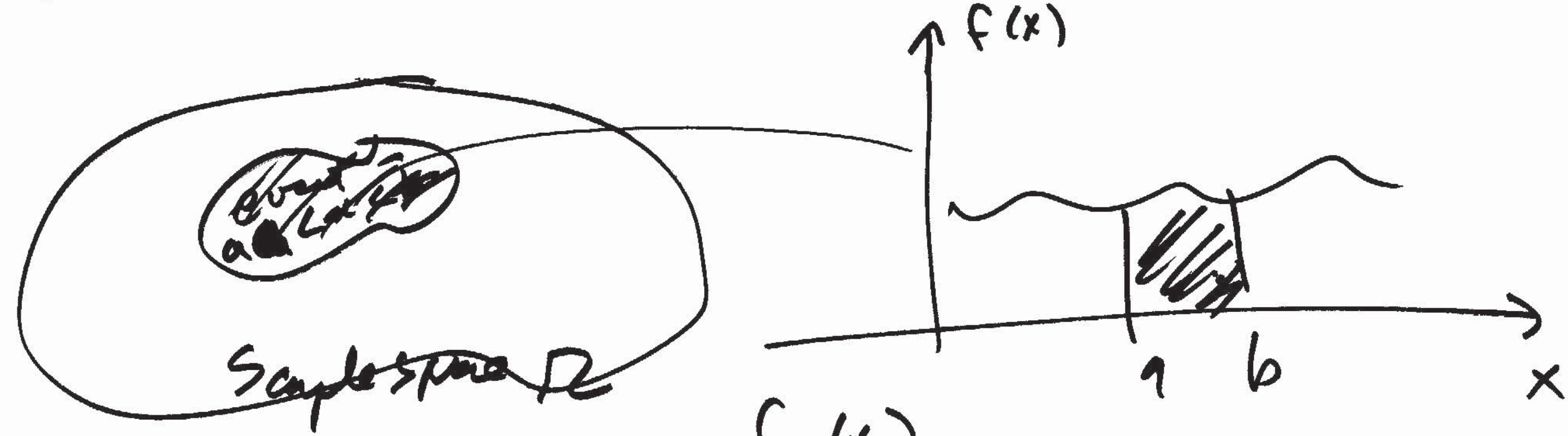
Event $|X| < 1$

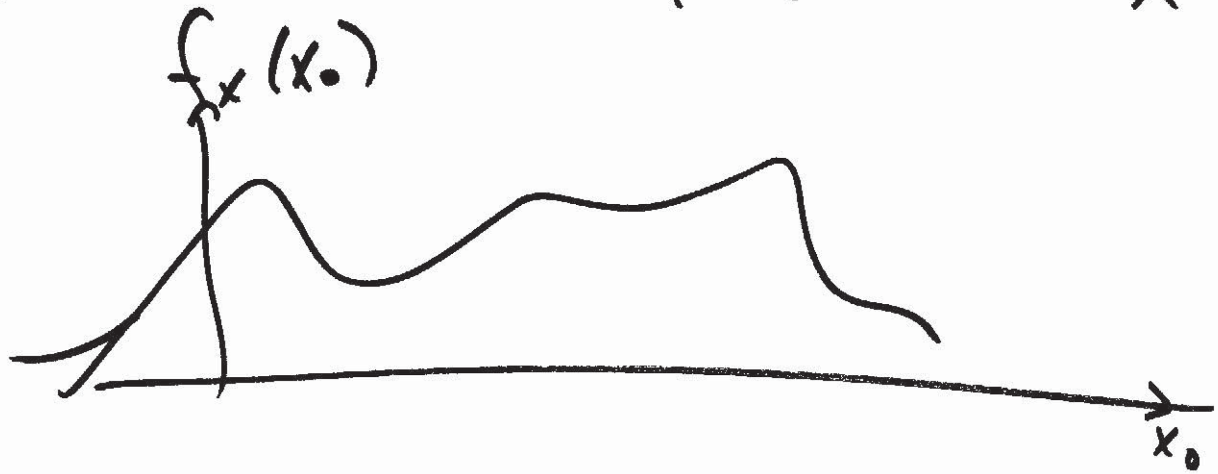


$$Pr(|X|<1) = \int_{-1}^{+1} f_x(x_0)\, dx_0$$

$$Pr([x', x'+\delta])$$
$$= \int_{x'}^{x'+\delta} f_x(x_0)\, dx_0 \simeq \delta f_x(x_0')$$





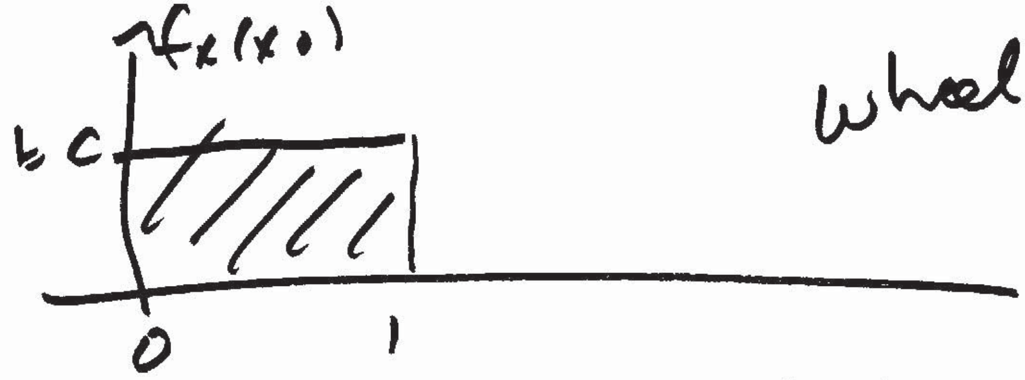


<u>Constraints on pdf</u> :

① $\displaystyle\int_{-\infty}^{+\infty} f_X(x_0) = 1$ ← normalization property.

② $f_X(x_0) \geq 0$ ← nonnegative.

Ex

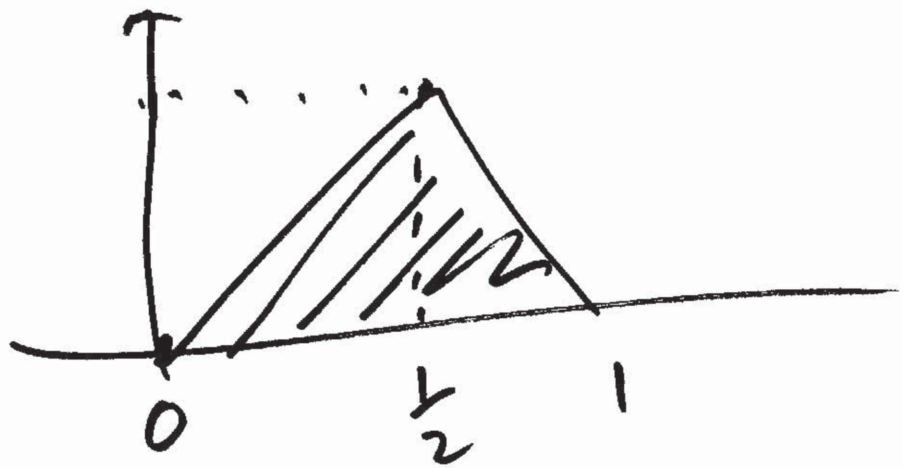


wheel of fortune.
a number between 0 & 1.

$$f_x(x_0) = \begin{cases} c & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$\int_0^1 f_x(x_0)\,dx_0 = 1 \Rightarrow \boxed{c = 1}$$

Ex



Expectation

$$E(X) = \int_{-\infty}^{+\infty} x_0 \, f_X(x_0) \, dx_0$$

Expected value of a fn of a R.V.

$$g(X) = Y$$

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x_0) \, f_X(x_0) \, dx_0$$

$$Var[X] = E[(X - \bar{X})^2]$$

$$= \int_{-\infty}^{+\infty} (x_0 - \bar{X})^2 f_X(x_0) \, dx_0$$

$$Var(X) \geq 0$$

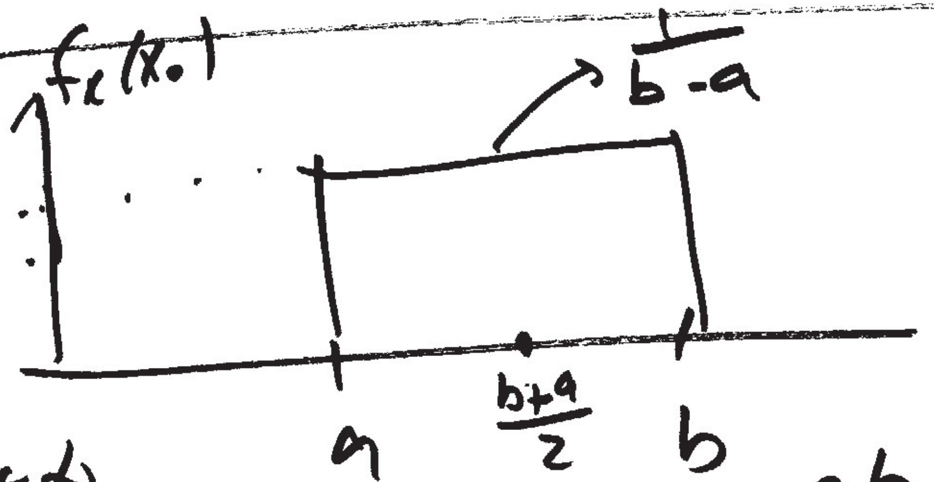
$$Var[X] = E(X^2) - E^2(X)$$

$$y = ax + b \Longrightarrow$$

$$E(y) = a\,E(x) + b$$

$$\mathrm{Var}(y) = a^2\,\mathrm{Var}(x)$$

Ex



$$E(x) = \int_{-\infty}^{+\infty} x_0\, f_x(x_0)\, dx_0 = \int_a^b x_0 \frac{1}{b-a}\, dx_0$$

$$= \frac{1}{b-a}\left[\frac{1}{2}x^2\right]_a^b$$

$$= \frac{a+b}{2}$$

$$E(X^2) = \int_a^b \frac{x^2}{b-a}\, dx = \frac{1}{b-a} \int_a^b x^2\, dx$$

$$= \frac{1}{b-a} \left(\frac{x^3}{3} \right]_a^b$$

$$= \frac{a^2+ab+b^2}{3}$$

$$\boxed{Var[X] = E[X^2] - E^2(X) = \frac{(b-a)^2}{12}}$$

<u>Ex</u> Exp. R.V.

$$f_X(x_0) = \begin{cases} \lambda e^{-\lambda x_0} & x_0 \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

<u>Note</u>: $\int_{-\infty}^{+\infty} \lambda e^{-\lambda x_0} dx_0 = 1$ ✓

$E(X) = \frac{1}{\lambda}$ $\quad Var(X) = \frac{1}{\lambda^2}$

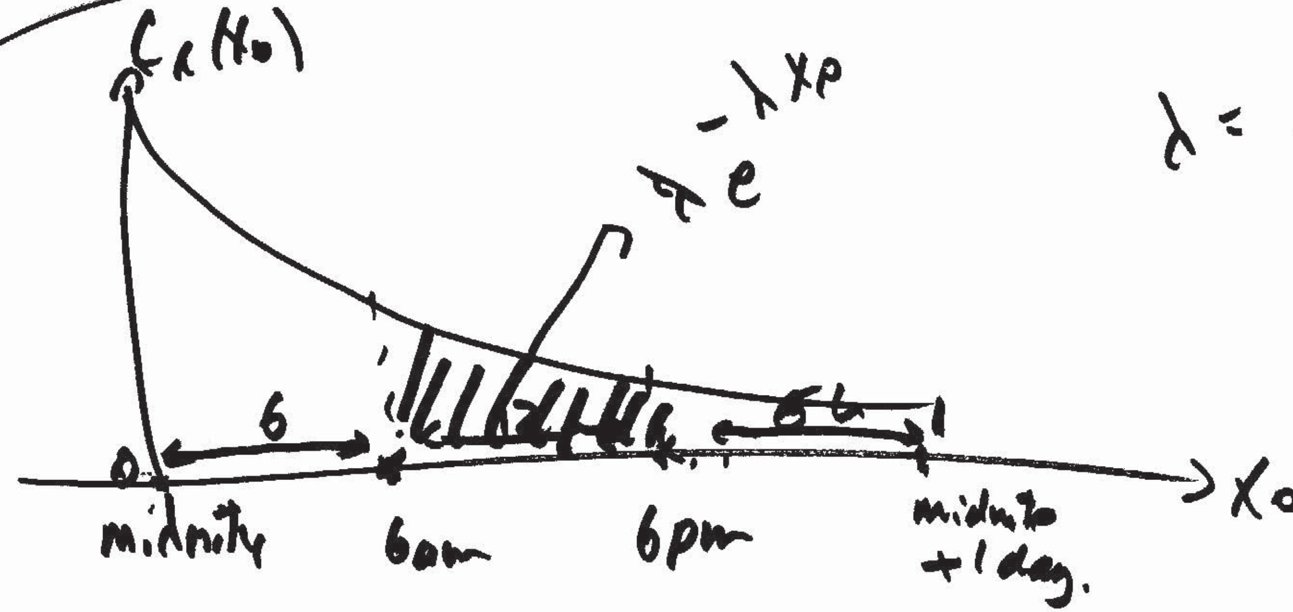
s.d. $= \frac{1}{\lambda}$

Exp R.V., mean = s.d.

$$Pr(X \geq a) = \int_a^\infty \lambda e^{-\lambda x}\, dx = e^{-\lambda a}$$

<u>EX</u> time to bulb failure is exp.
mean 10 day.

Right now = midnite. prob (failure 6:00am, 6:00pm of same day)



$\lambda = \frac{1}{10}$ 1/day.

$$p\left(\tfrac{1}{4} < X < \tfrac{3}{4}\right) = p\left(X > \tfrac{1}{4}\right) - p\left(X > \tfrac{3}{4}\right)$$

$$= e^{-1/40} - e^{-3/40}$$

$$= 0.0476$$

CDF = Cumulative Distribution Fn

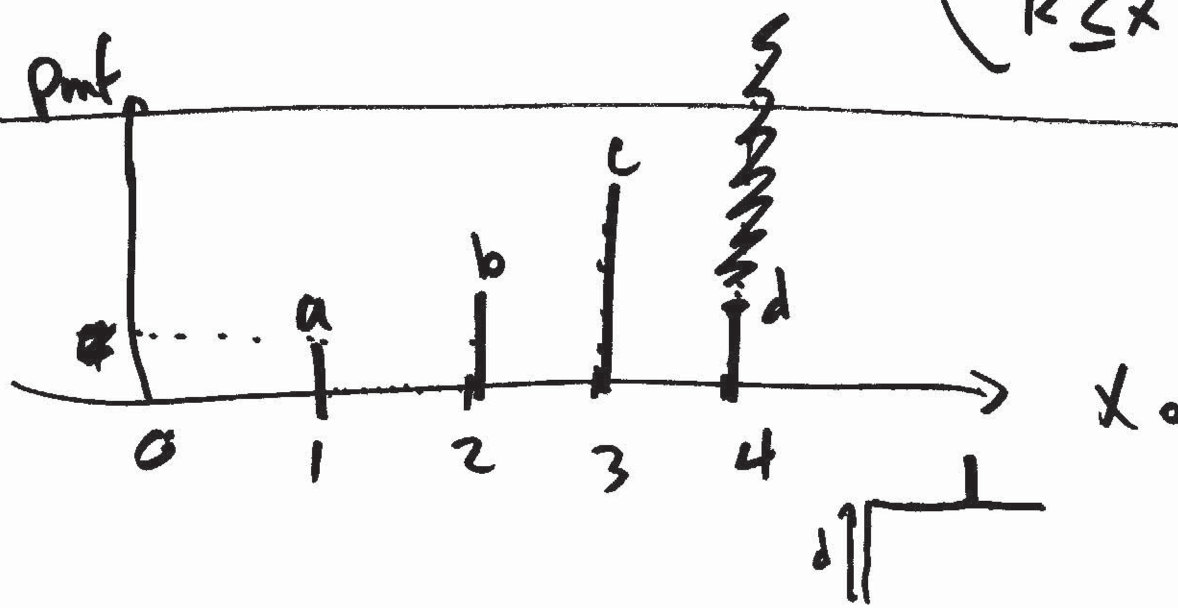
CDF is integral of pdf.

pdf $f_X(x_0)$ or pmf $P_X(x_0)$

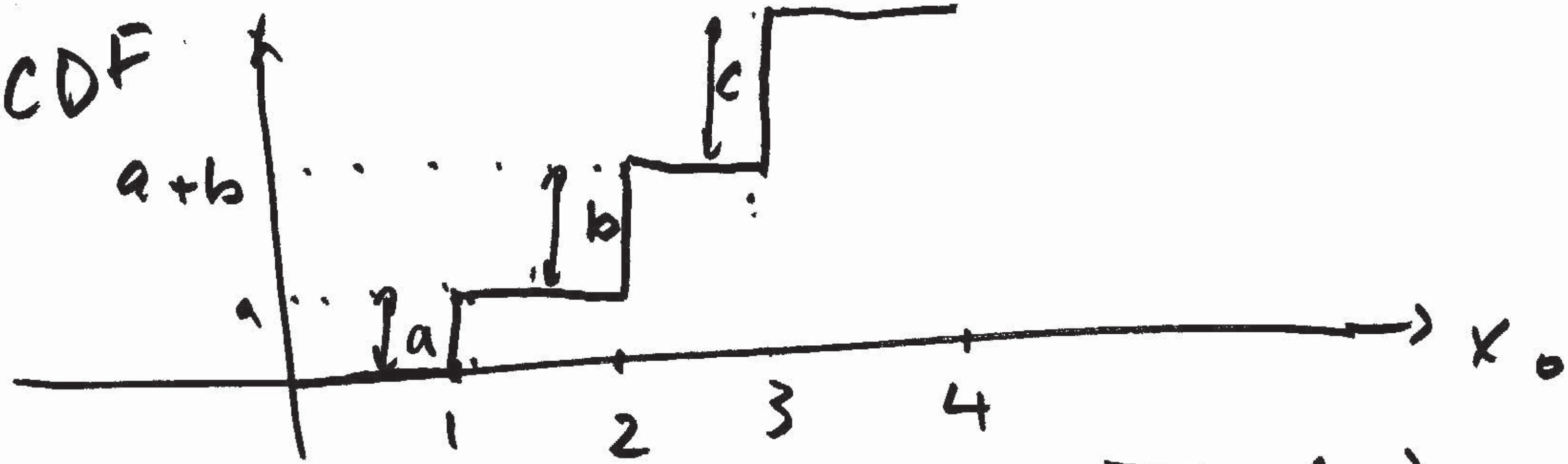
$$F_X(x) \triangleq pr(X \le x) = \begin{cases} \int_{-\infty}^{x} f_X(x_0)\,dx_0 & X \text{ continuous} \\ \sum_{k \le x} P_X(k) \end{cases}$$

CDF

Ex

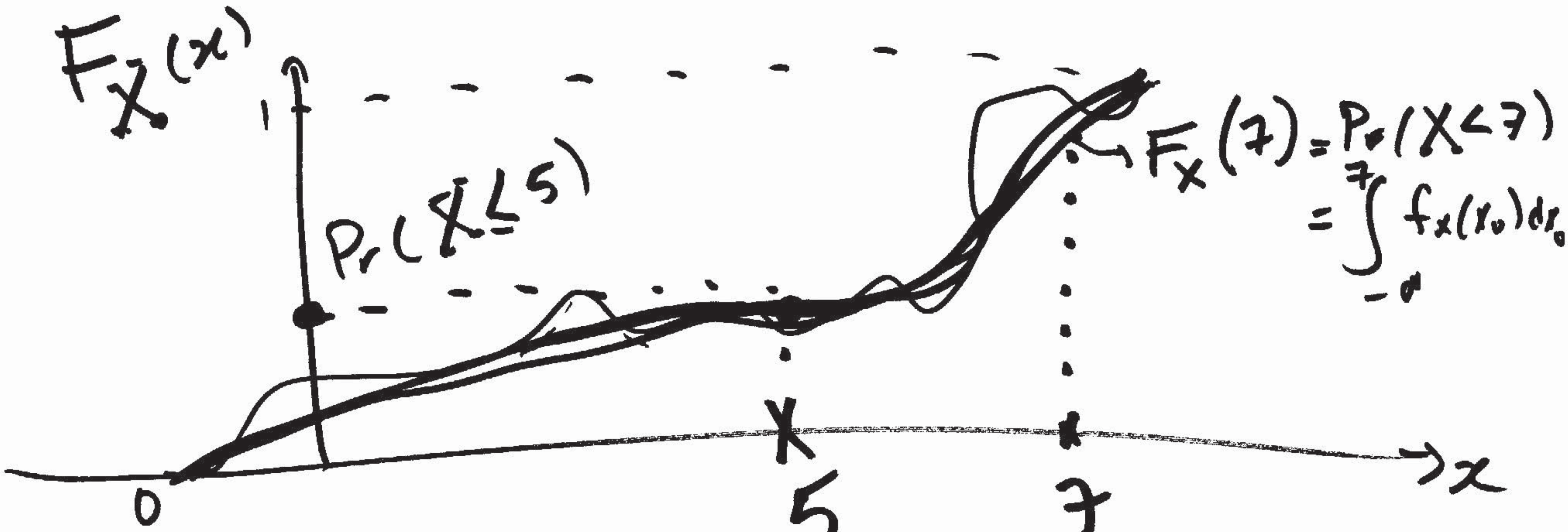


F CDF



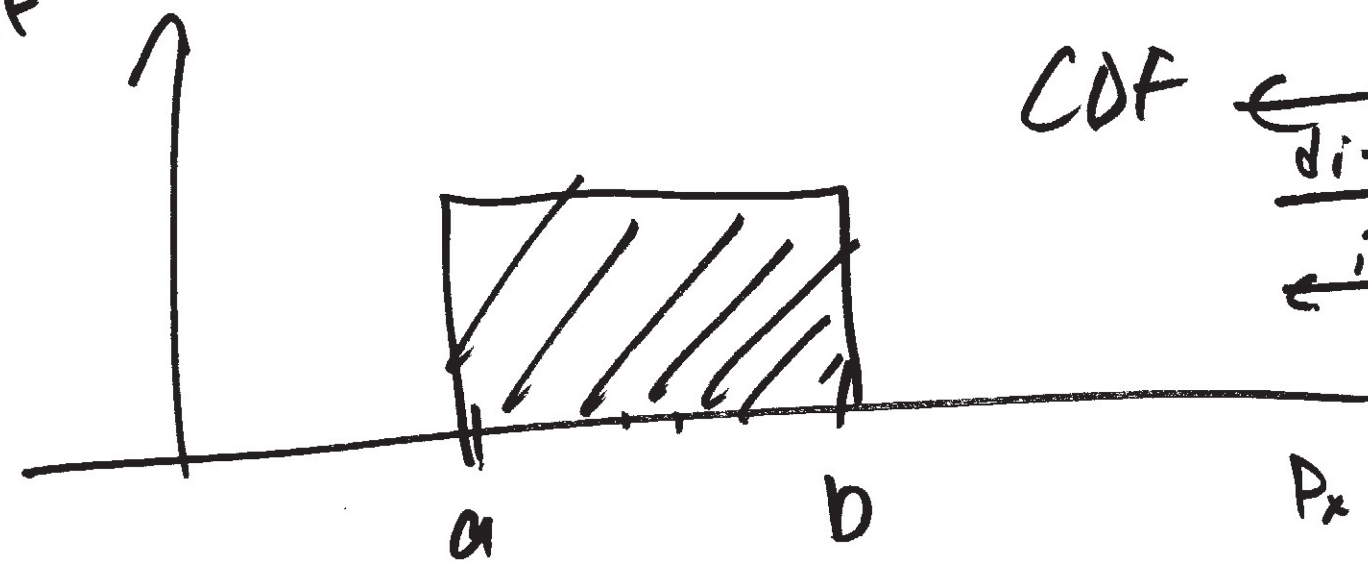
$$Pr(X < 2) = a = F_X(2) = a$$

$$Pr(X \le 4) = F_X(4^+) = a+b+c+d = 1$$



$$F_X(7) = Pr(X < 7) = \int_{-\infty}^{7} f_x(x_0)\,dx_0$$

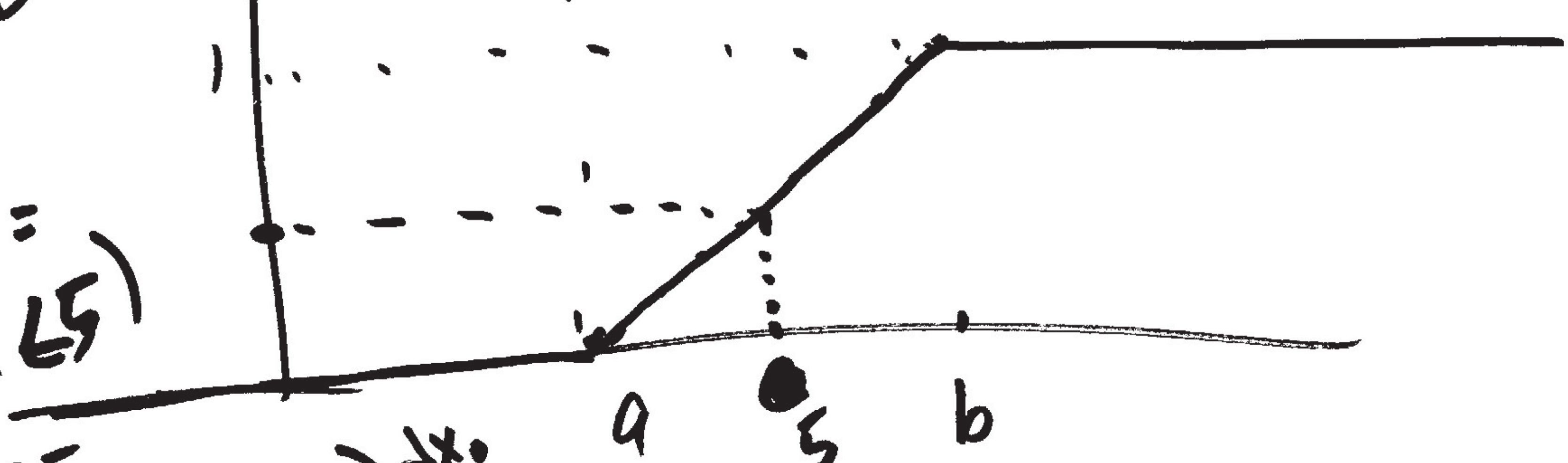
pdf



$$CDF \underset{\xleftarrow{\text{int}}}{\xrightarrow{\text{diff}}} pdf$$

$$p_x(x_0) = \frac{d F_x(x_0)}{dx_0}$$

CDF



$F_X(5) =$

$Pr(x \le 5)$

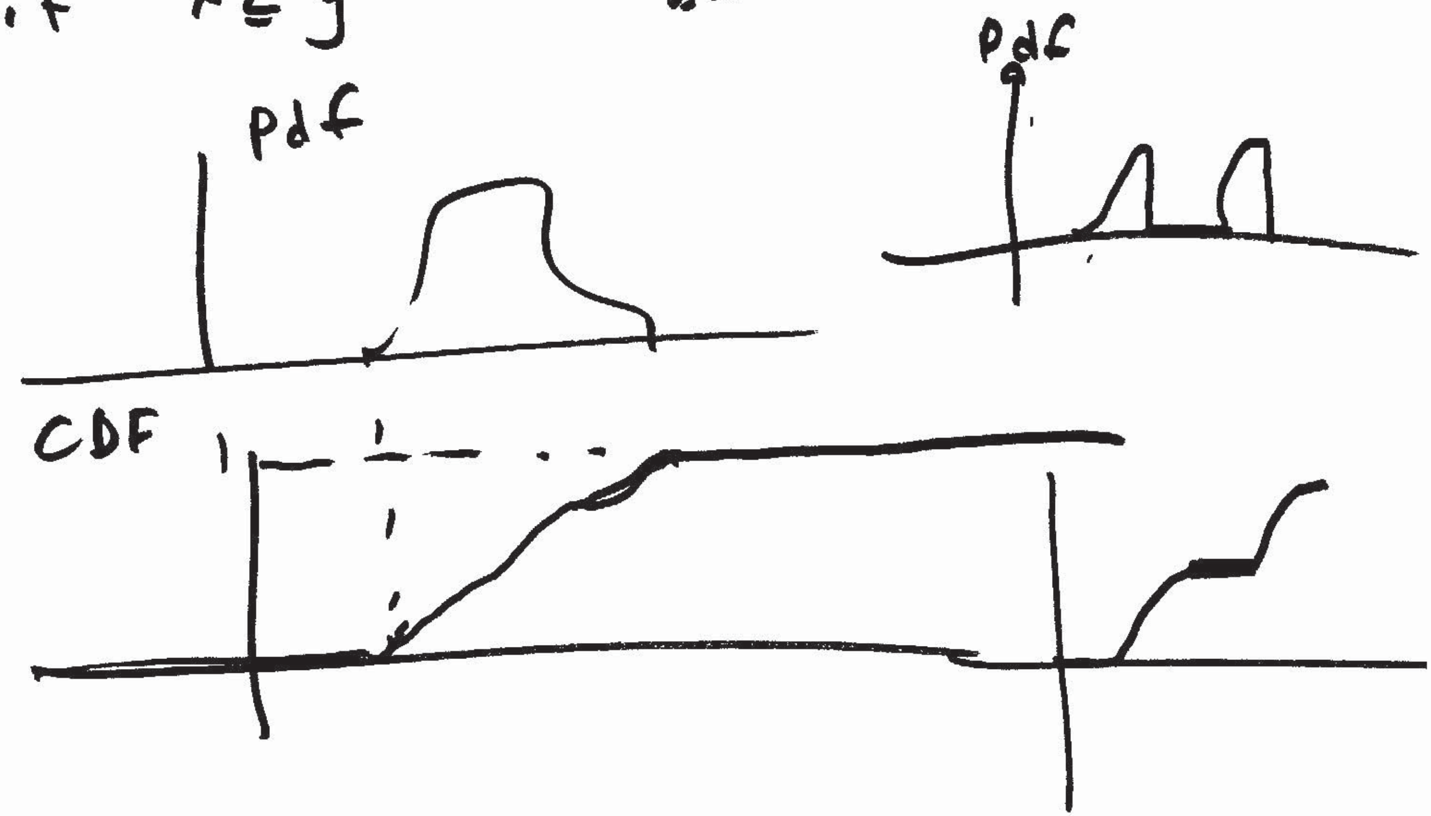
$= \int_{-\infty}^{5} f_x(x_0)\,dx_0$

Prop of CDF

① $F_X(x) \triangleq P_r(X < x) \quad \forall x.$

② F_X is monotically non-decreasing.

if $x \le y \Rightarrow F_X(x) \le F_X(y)$



3 $F_X(x) \to 0$ as $x \to -\infty$

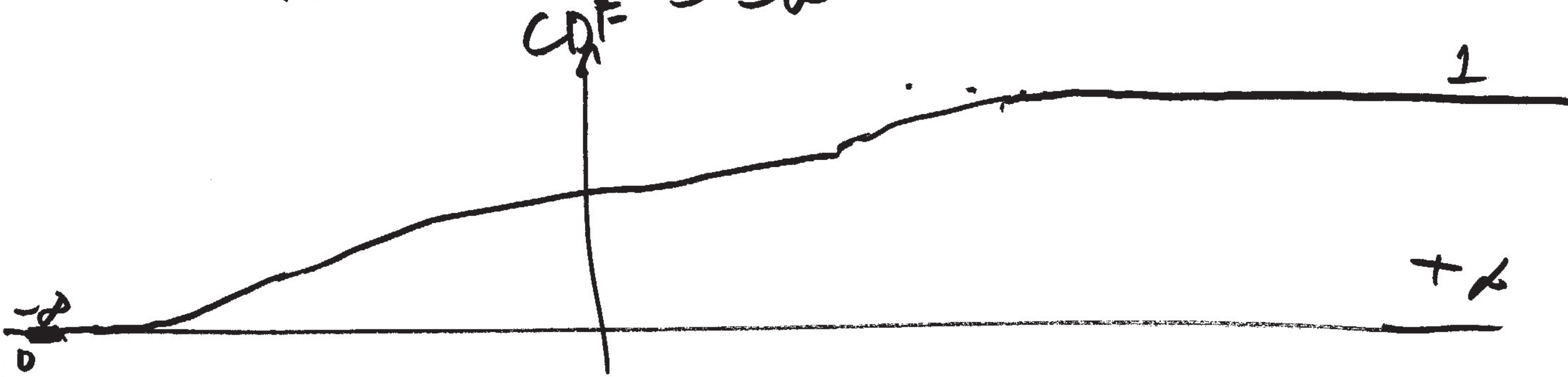
$\to 1$ as $x \to +\infty$

as $x \to -\infty$

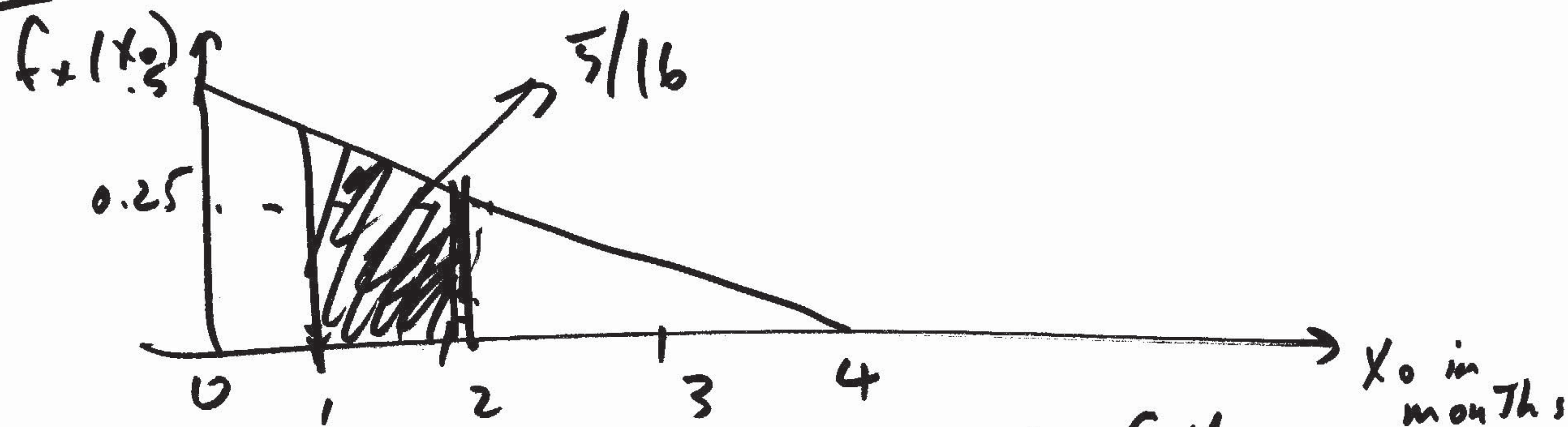
$$F_X(x) = \int_{-\infty}^{-\infty} f_x(x_0)\,dx_0 = 0$$

as $x \to +\infty$

$$F_X(x \to +\infty) = \int_{-\infty}^{+\infty} f_x(x_0)\,dx_0 = 1$$



Ex life time of a component R.V.



Q what is prob that component fails in the second month?

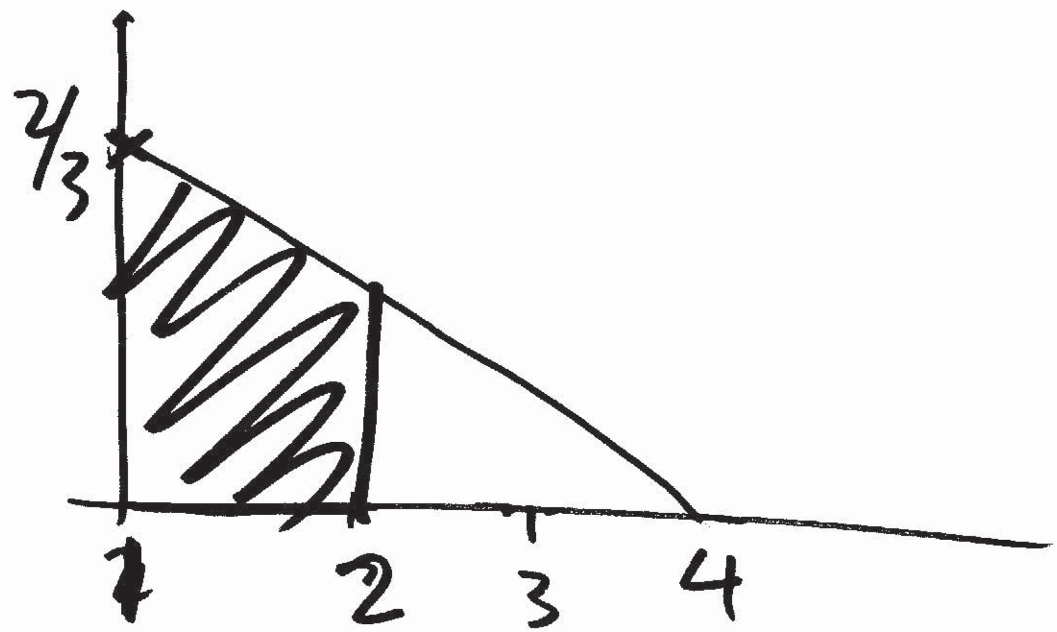
$$\Pr(1 \le x \le 2) = \int_1^2 f_X(x_0)\,dx$$

$$= 5/16$$

Q Given it did<u>not</u> fail 1st month, what is prob it fails in 2nd month?

Event : A = componet failed in 1st month

$f_X(x_0 | A')$



$$\int_1^2 f_X(x_0 | A')\, dx_0 = 5/9$$

Event B = component fail in 2nd month.

Event A = Component failed first month
Event B = " " 2nd month

$$P(B \mid A') = \frac{P(A'B)}{P(A')} = \frac{P(B)}{P(A')} = \frac{5/16}{9/16} = 5/9$$

$P(A'B) = P(B)$

Event B is included in Event A'

$$P(A') = \int_1^4 f_X(x_0)\,dx_0 = 9/16$$