Geometric & Exponential CDF

- x geometric R.V.
  \# of bernoulli trials until first success.
  in a series of exp.

pr(success in each trial is p)

$$P(X=K) = p\,(1-p)^{K-1} \leftarrow \text{pmf.}$$

$\leftarrow$ CDF

CDF : $$\boxed{F_{geo}(n)} = \sum_{K=1}^{n} p(1-p)^{K-1}$$

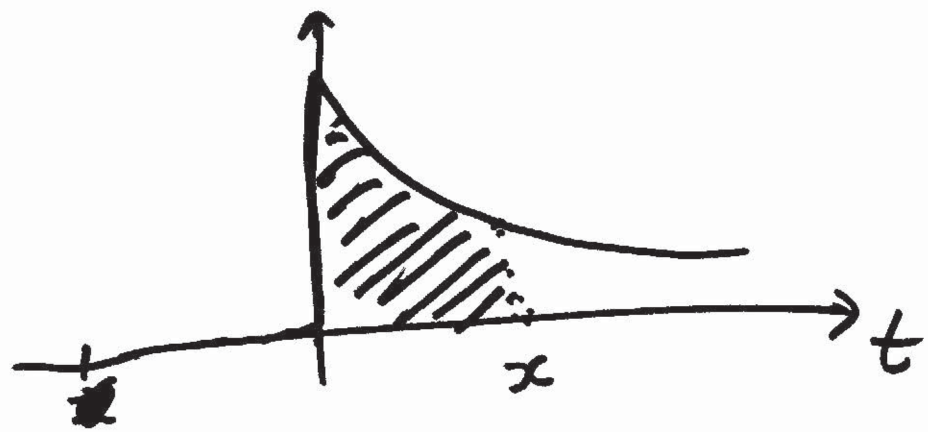
$$= p\,\frac{1-(1-p)^n}{1-(1-p)} = \boxed{1-(1-p)^n}$$

CDF for Exp. R.V.

$$\text{pdf} \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$F_{exp}(x) = P(X < x) = \begin{cases} 0 & \text{if } x < 0 \\ \displaystyle\int_0^x \lambda e^{-\lambda t}\, dt & x > 0 \end{cases}$$

$$\boxed{F_{exp}(x) = 1 - e^{-\lambda x}} \quad x > 0$$



~~Let~~ $\cancel{e^{-\lambda\delta} = 1-p}$ $\qquad e^{-\lambda\delta} = 1-p$

$\cancel{F_{geo}(n) = 1 - e^{-\lambda\delta}}$ $\qquad$ $\left.\begin{aligned} F_{geo}(n) &= 1 - e^{-\lambda\delta n} \\ F_{exp}(x) &= 1 - e^{-\lambda x} \end{aligned}\right\} \Rightarrow$

$\cancel{F_{exp}(x) = 1 - e^{-\lambda x}}$

$$F_{geo}(n) = F_{exp}(x) \quad \text{if} \quad x = \delta n$$

$$n = 1, 2, \ldots\ldots$$

$$F_{exp}(n\delta) = F_{geo}(n) \qquad n = 1, 2, \ldots\ldots$$

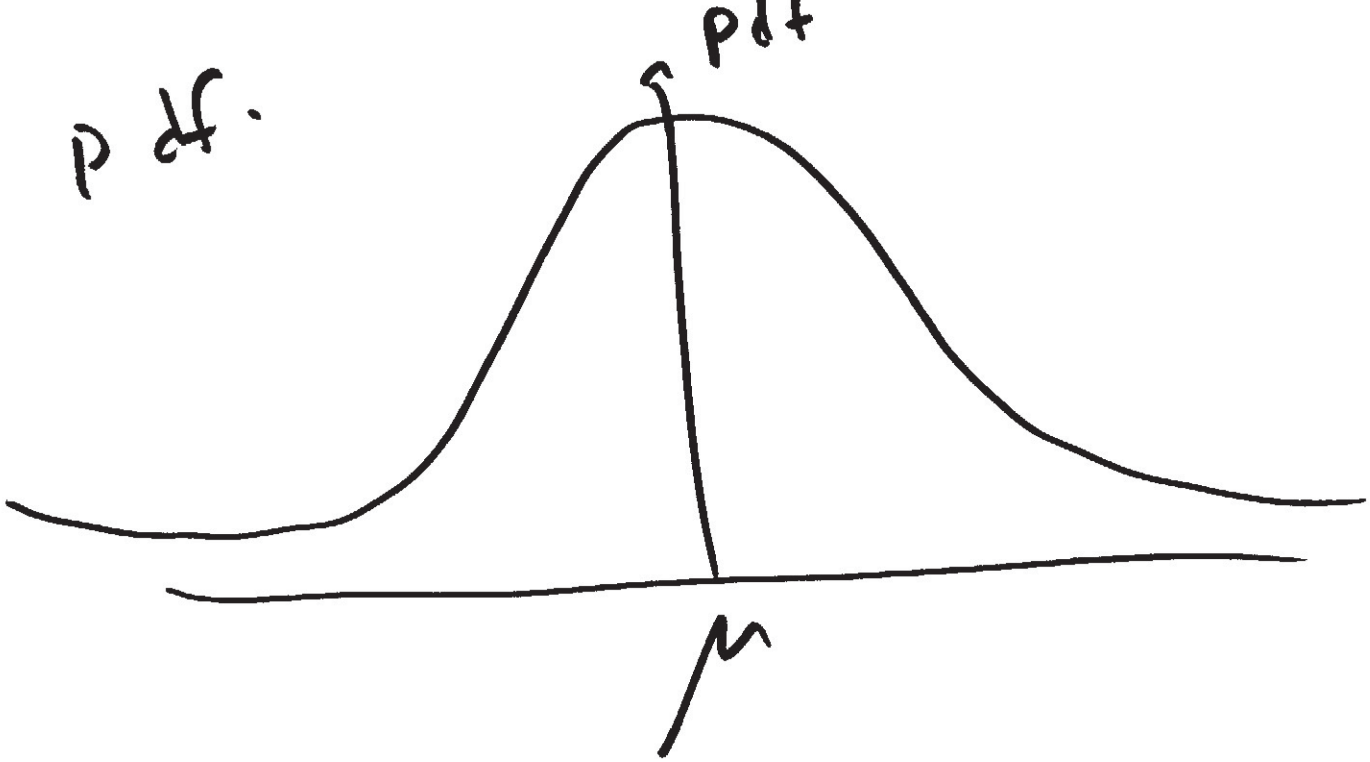
$\Longrightarrow$ Conclusion $x = \delta n$ geo/exp are same.

Gaussian $\longleftarrow$ Normal R.V.

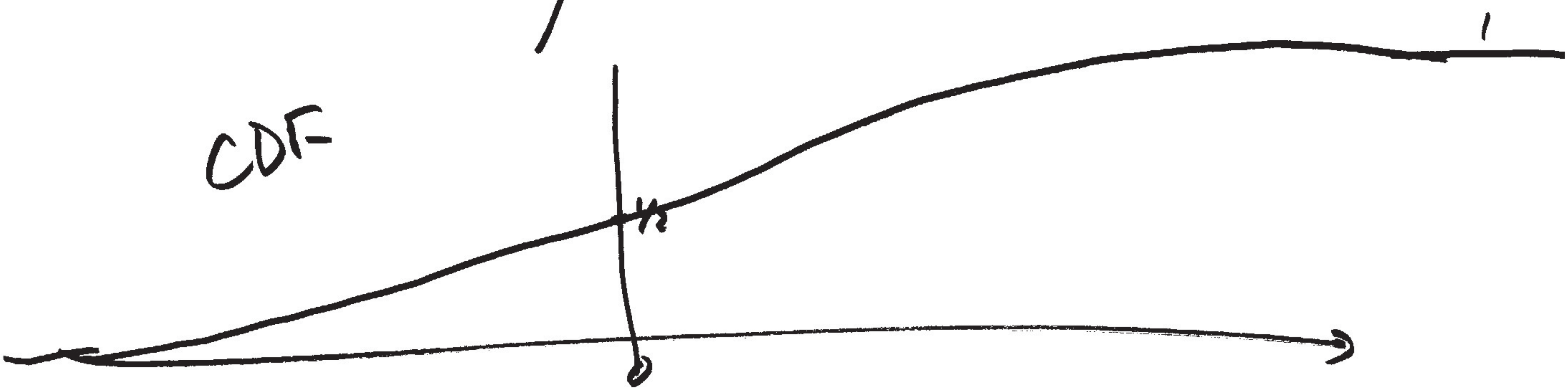
$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

σ = standard variation μ = mean.

p df.



CDF



$$\mathrm{Var}(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{+\infty} (x-\mu)^2 \, e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx$$

change of var $\quad y = \dfrac{x-\mu}{\sigma}$

$$\mathrm{Var}(x) = \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} y^2 \, e^{-y^2/2} \, dy$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \left[-y\, e^{-y^2/2} \right]_{-\infty}^{+\infty}$$

$$+ \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-y^2/2} \, dy$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{-y^2/2} \, dy$$

$$= \sigma^2$$

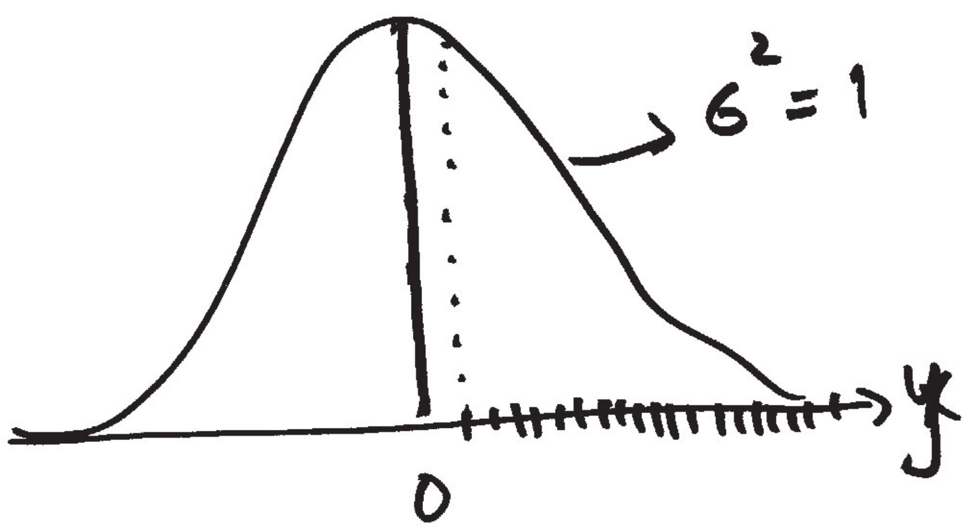
Will show: x normal μ, σ

$$\boxed{y = ax + b \quad \text{is also normal}}$$

$$\text{mean}(y) = a\mu + b$$

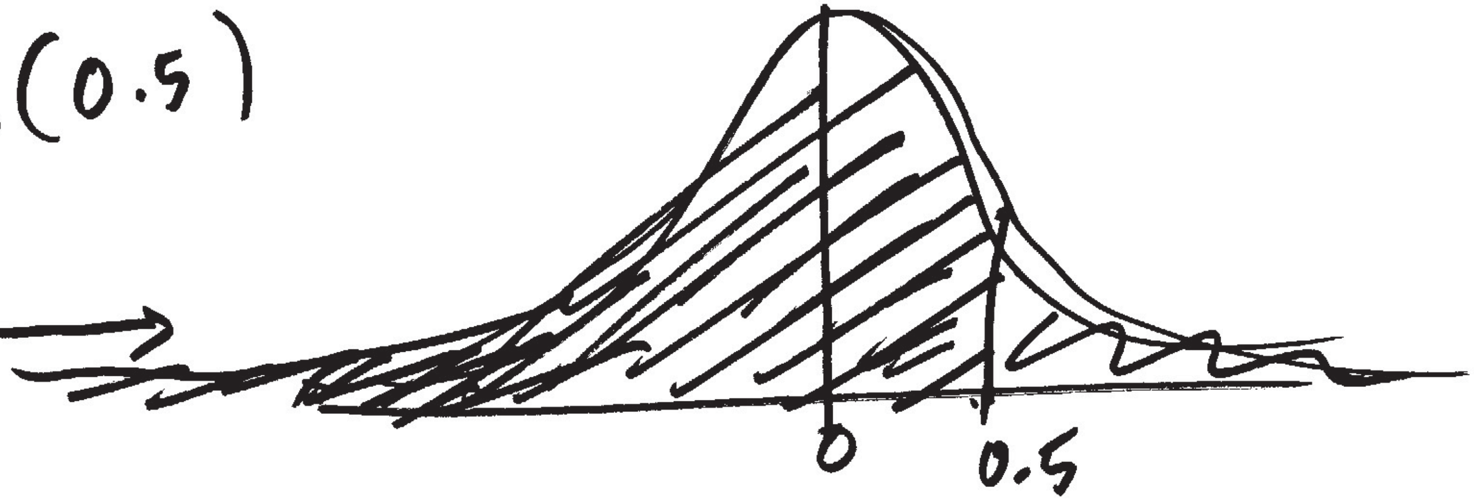
$$\text{var}(y) = a^2\sigma^2$$

Assume $\mu = 0$, $\sigma = 1 \Rightarrow Y$ normal RV. standard normal

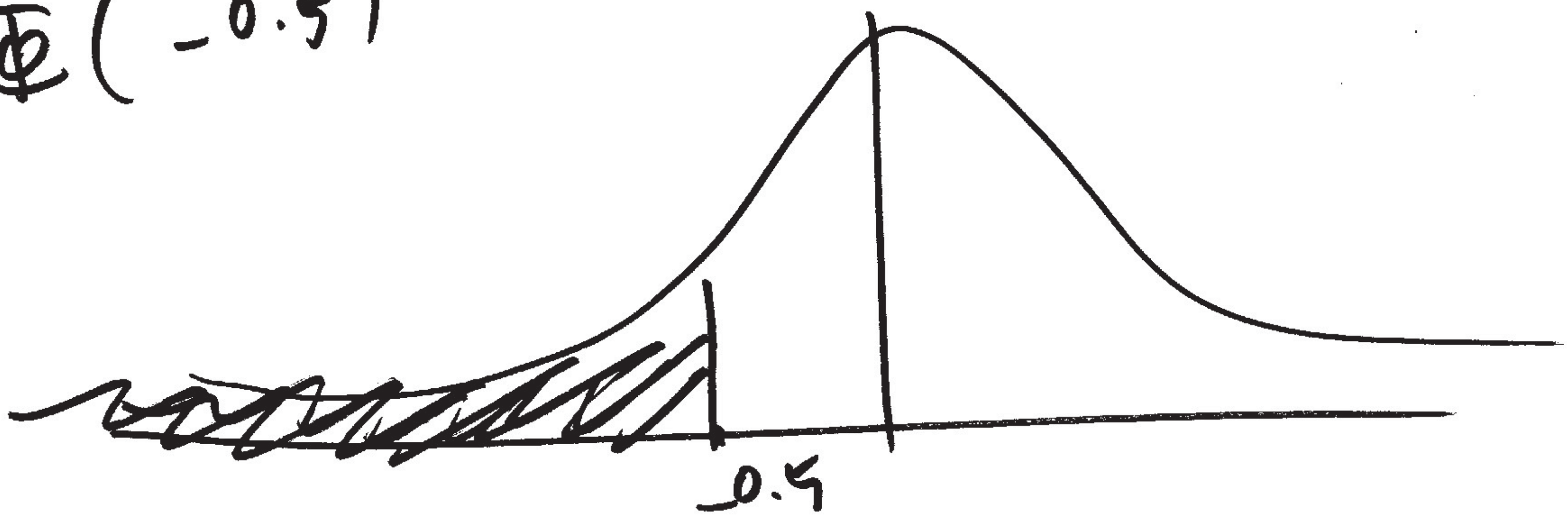


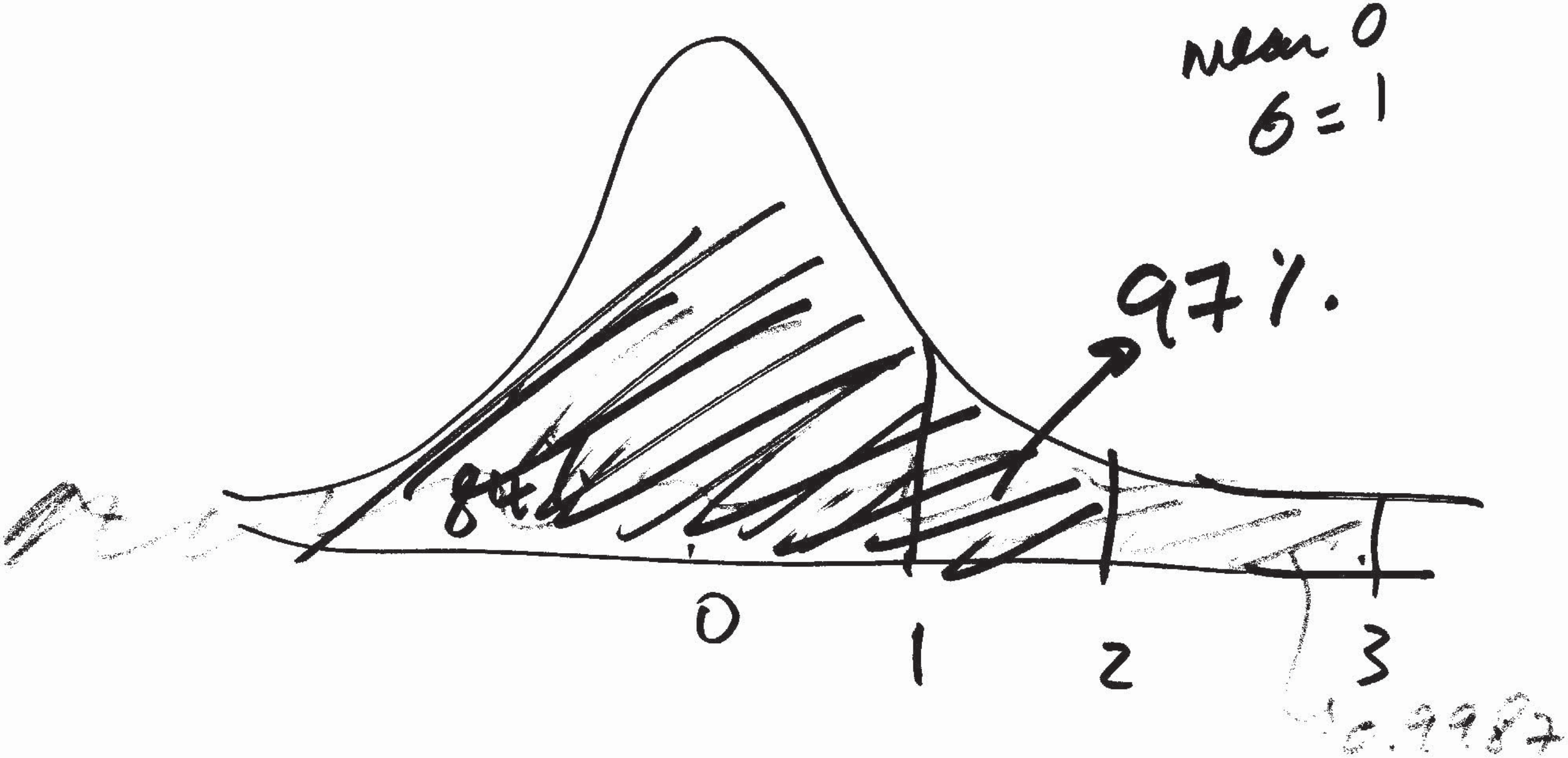
$$\Phi(y) = \Pr[Y < y] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-t^2/2} \, dt$$

$\Phi(0.5)$



$\Phi(-0.5)$




mean 0
σ = 1
97%
84
0
1
2
3

$\Phi(-0.5) = \Pr(Y \le -0.5) = \Pr(Y \geq 0.5)$

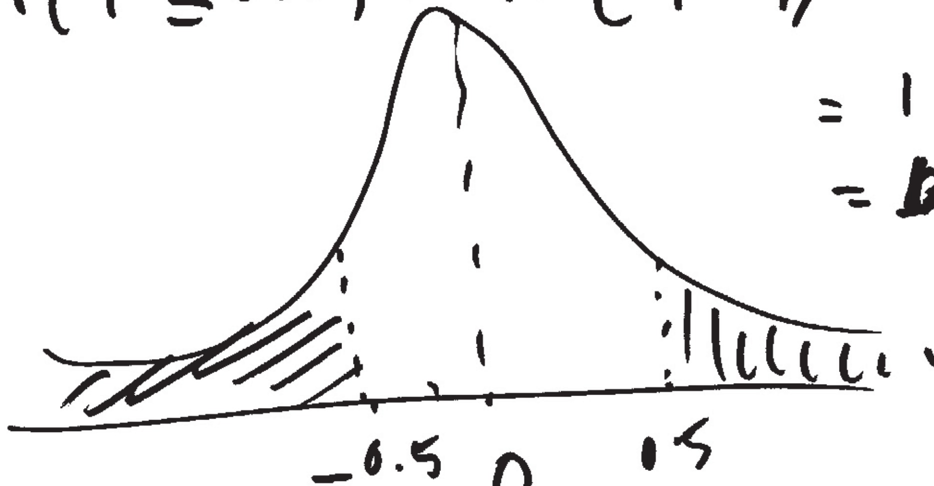
$\Phi(0.5)$

$= 1 - \Pr(y < 0.5)$

$= 1 - \Phi(0.5)$

$= 1 - 0.6915$

$= 0.3085$



Generally $\Phi(-y) = 1 - \Phi(y)$.

Suppose x normal μ, σ^2

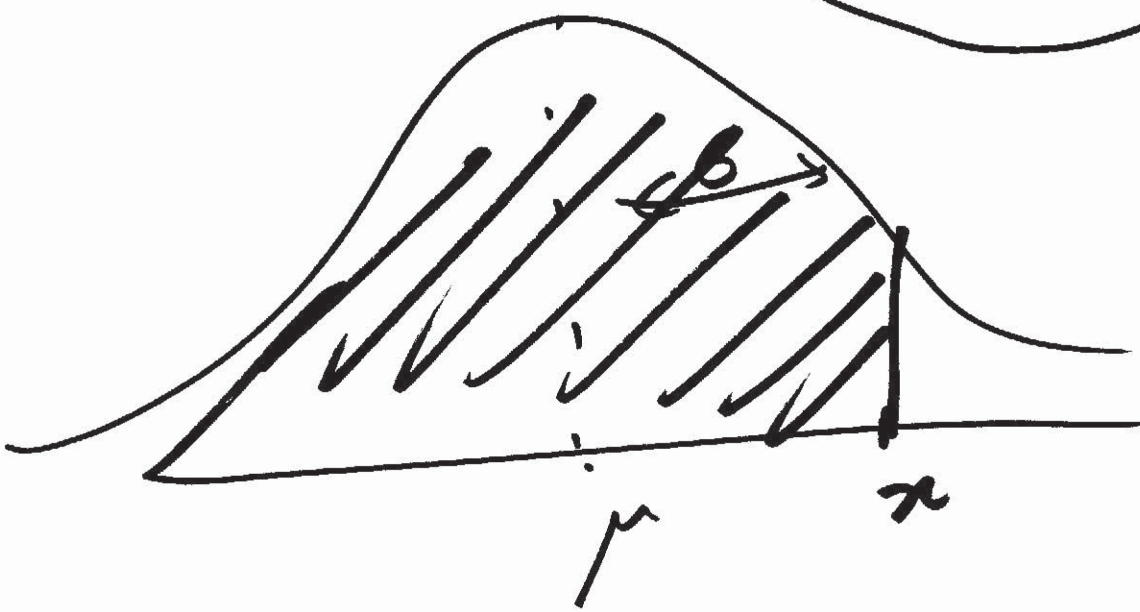
Define $y = \dfrac{x-\mu}{\sigma}$ is normal.

mean $y = 0$ $\quad$ $\mathrm{var}(y) = 1$

$$\Pr(X < x) = P\left(\frac{X-\mu}{\sigma} < \frac{x-\mu}{\sigma}\right)$$

$$= \Pr\left(Y < \frac{x-\mu}{\sigma}\right) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

Standard normal.

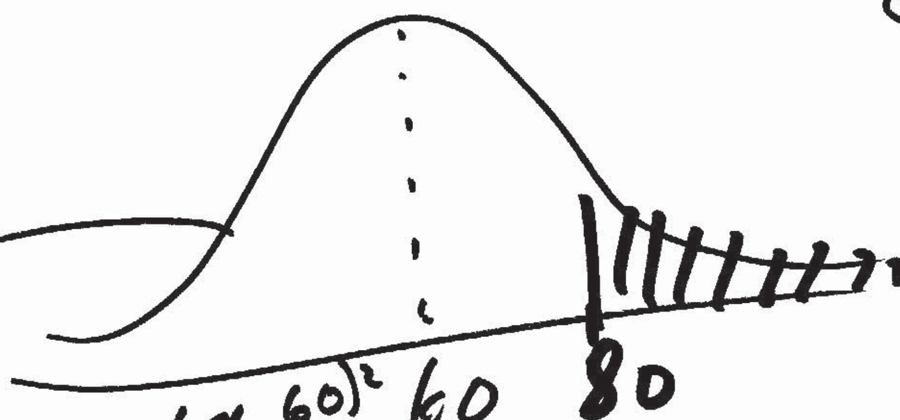


EX normal R.V. mean = 60 inch

s.d. = 20 inch. = σ

what is prob that at least 80 inches

$\sigma = 20$



$$y = \frac{x-60}{20}$$

$\to$ mean 0, $\sigma = 1$

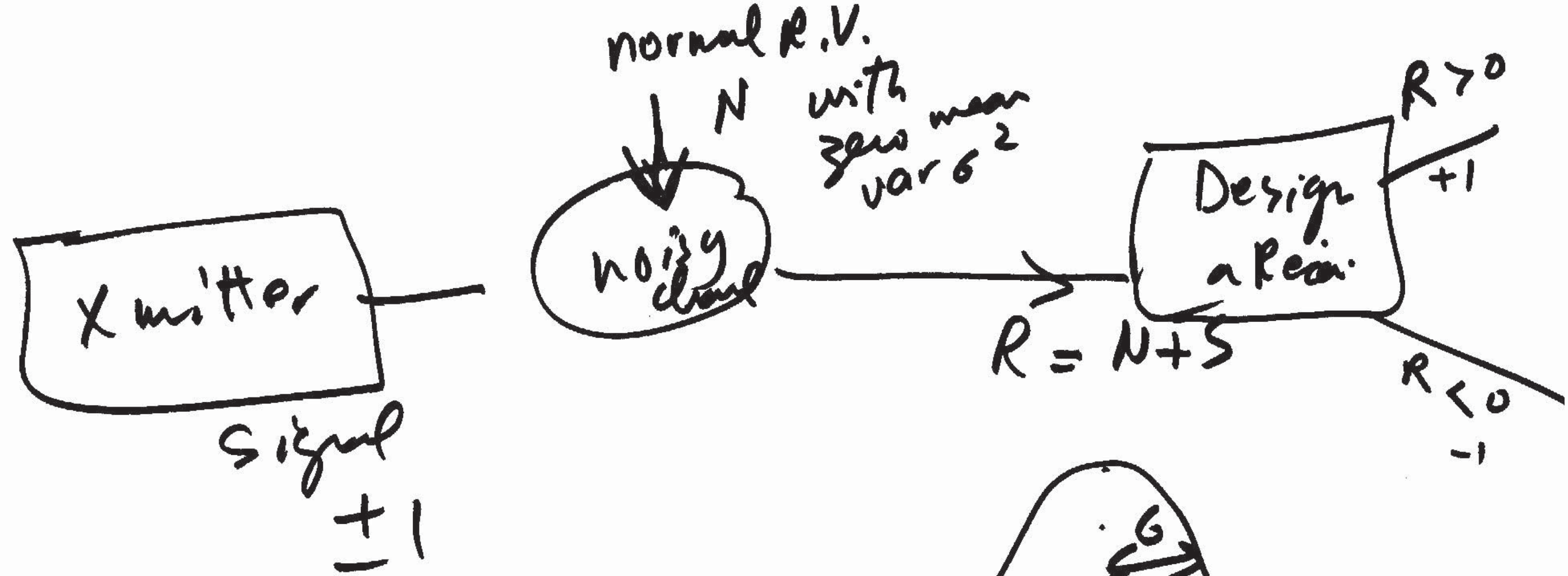
$$\int_{80}^{\infty} \frac{1}{\sqrt{2\pi}\,20} e^{-\frac{(x-60)^2}{2\times 20^2}} \, dx .$$

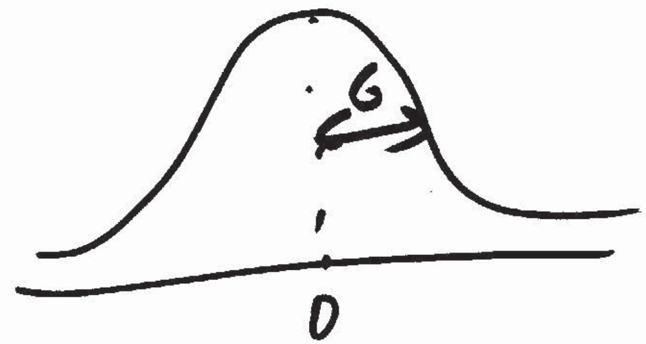
$$\Pr(X > 80) = \Pr\left(\frac{X-60}{20} > \frac{80-60}{20}\right)$$

$$= \Pr(y > 1)$$

$$= 1 - \Phi(1) = 0.15$$

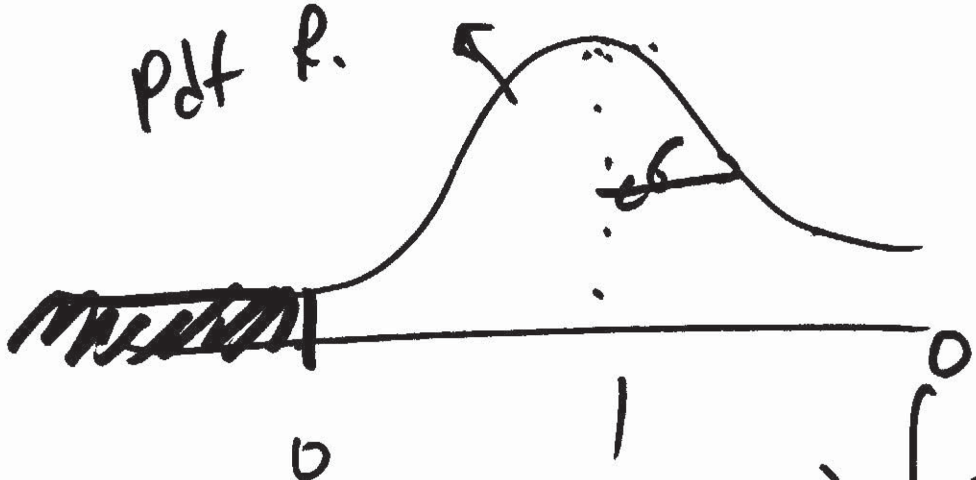
Ex





Xmit a +1

R normal
mean 1
var σ^2

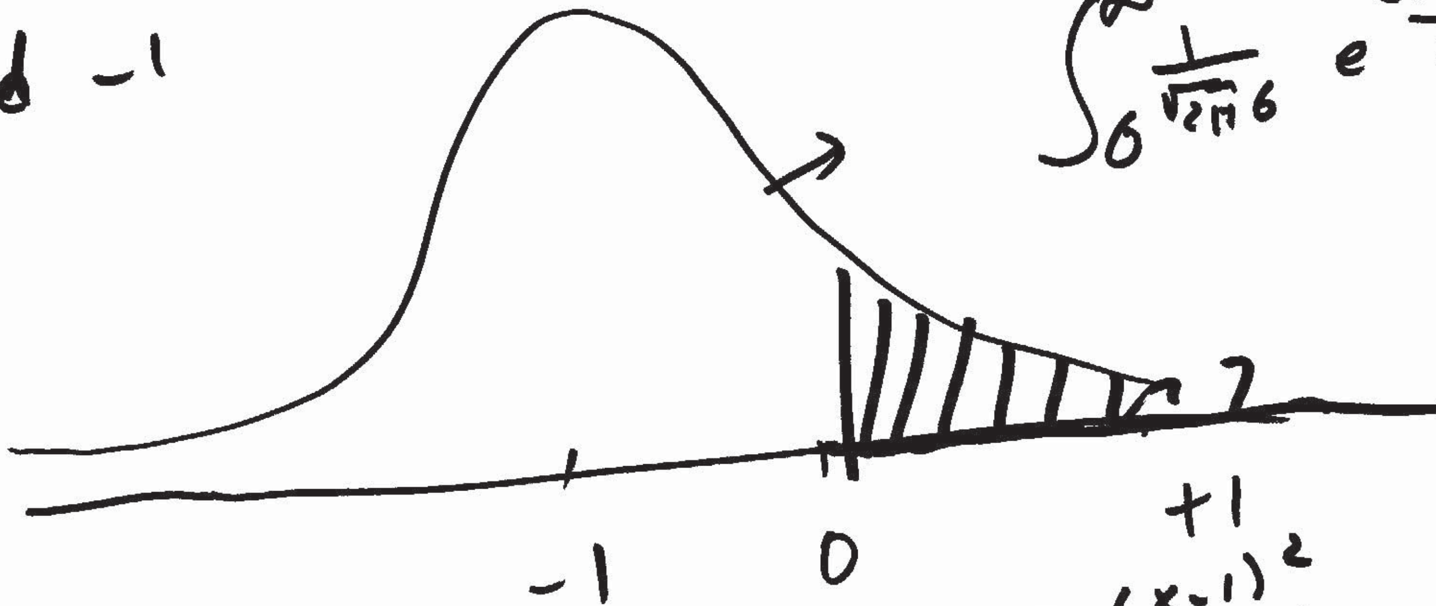


given +1 transmitted

$$\Pr(\text{error}) = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-1)^2}{2\sigma^2}}\,dx$$

If send -1

$$\int_0^\infty \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x+1)^2}{2\sigma^2}}\, dx$$



$$P(\text{error}) = \Pr(\text{send } 1) \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-1)^2}{2\sigma^2}}\, dx$$

$$+ \Pr(\text{send } -1) \int_0^\infty \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x+1)^2}{2\sigma^2}}\, dx$$

$$< \int_0^\infty \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x+1)^2}{2\sigma^2}}\, dx$$

$$P(\text{error}) = 1 - \Phi\left(\frac{1}{\sigma}\right)$$

$\sigma = 1 \Rightarrow Pr(error) = 0.1587$

$\sigma = 2 \Rightarrow pr(error) = 1 - \Phi\left(\frac{1}{2}\right)$

$= 1 - \cancel{\Phi}\, 0.6915$

$= 0.31$