Prob. rses. Set operation

Def Set = collection of objects which are elements of the set.

if x is an element of set S

$x \in S$

if not $x \notin S$

Set no elemith $\emptyset$ = empty set.

Ways to specify set.

① finite # of elemith.

$$S = \{x_1, x_2, \ldots, x_n\}$$

$S = \{1, 2, 3, 4, 5, 6\}$ 6 elements.

$S = \{$ Tail, Head $\}$

(2) ∞ly many elements can be enumerated in list. → countably ∞.

set of even integer $= S = \{0, 2, -2, 4, -4, 6, -6, \ldots\}$

(3) specify by certain property.

$\{x \mid x$ satisfies $P\}$ ← property.

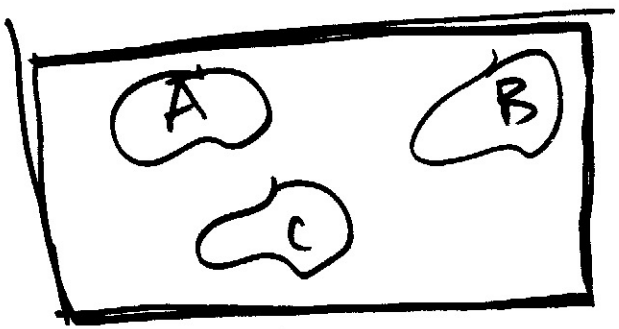
→ $\{k \mid \frac{k}{2}$ is an integer$\}$ → countably ∞ # of elements.

$\{x \mid 0 \leq x \leq 1\}$ → uncountable. elements cannot be written in a list.

1. Algebra of Events 2. Sample Space 3. probability measure.

Algebra of Events

Event or set collection of points or areas in a space.

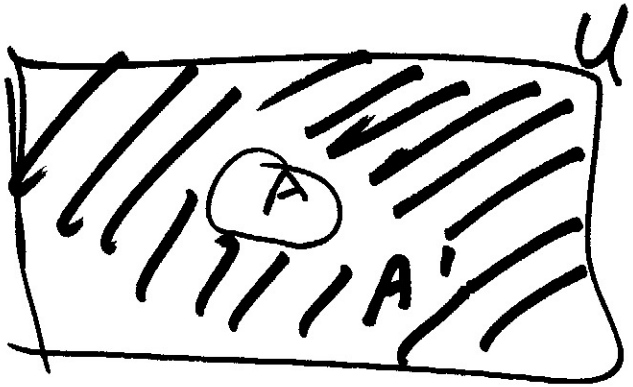


Universal Set or Universal Events: Collection of all points in the entire space. → U Ω

Complement of event or set

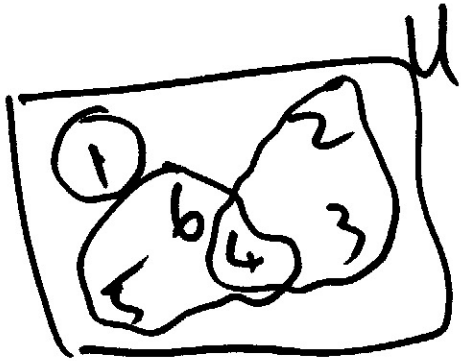
Compliment of A, A' or A^c

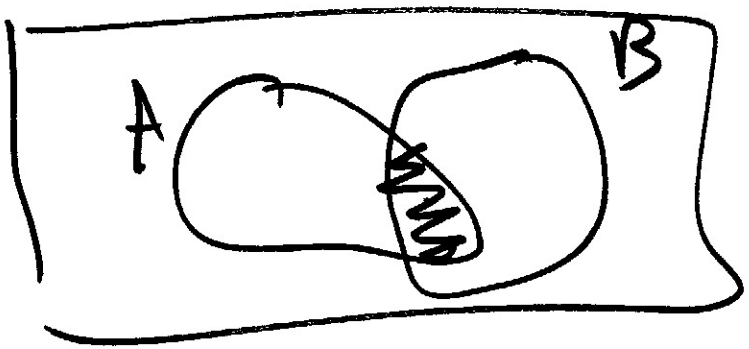
Collection of all points in The universal set, not in A.



Intersection of sets or events A, B.

collection of all points in Event A and in Event B AB or $A \cap B$





Event A = die ≤ 3

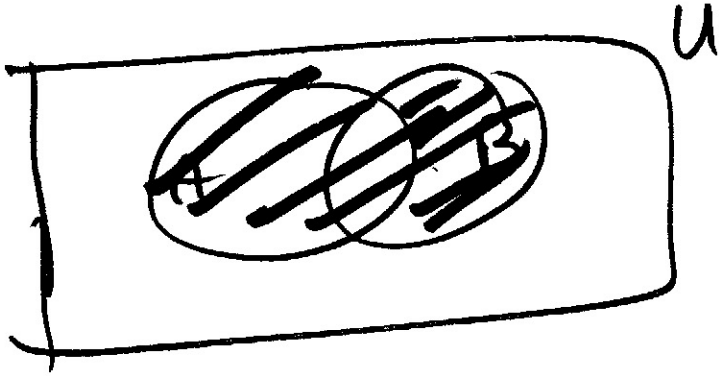
Event B = even.

$A \cap B$ = (2)

Union of 2 event A, B — that

collection of all point of U are in in B or A.

A ∪ B
A + B

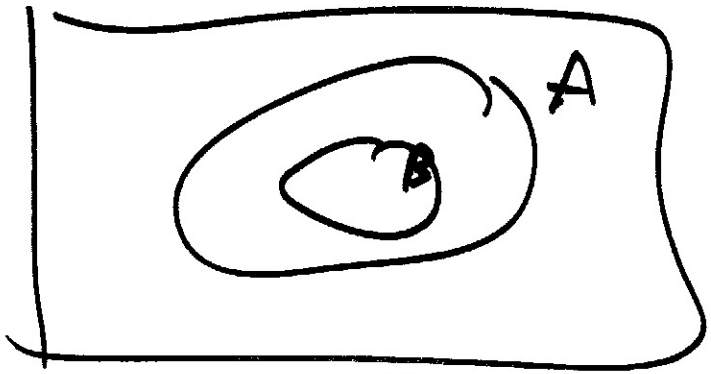


Event A: die < 3
Event B die even

A ∪ B = {1, 2, 3, 4, 6}

Inclusion

if all point of U in B are also in A, then event B is included in event A



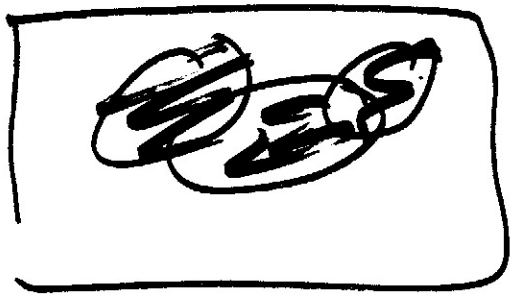
$B \subset A$

Axioms defining Algebra of Events

1. Commutative Law: $A + B = B + A$; $A \cup B = B \cup A$

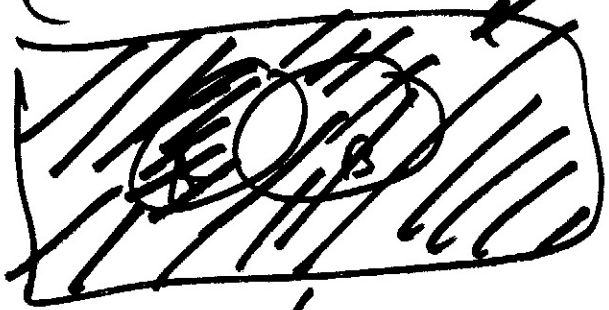
2. Associative. $A + (B + C) = (A + B) + C$

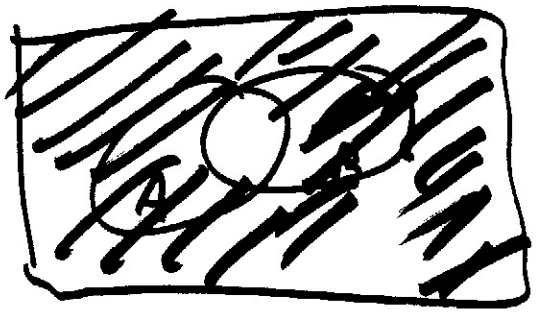
3. Distributive $\begin{cases} A \bullet (B + C) = AB + AC. \\ A \wedge (B \cup C) = (A \wedge B) \cup (A \wedge C) \end{cases}$



4. $(A')' = A$

5. $(AB)' = A' + B'$





6. $AA' = \emptyset$

7. $A \cup = A$

From axioms I can prove anything.

Ex $CD = DC$

Start with axioms let $A = C'$ $B = D'$

$A + B = B + A$

$C' + D' = D' + C'$

Take compliment of both side

$(C' + D')' = (D' + C')'$ ← axiom 5

$((CD)')' = ((DC)')'$

$CD = DC$

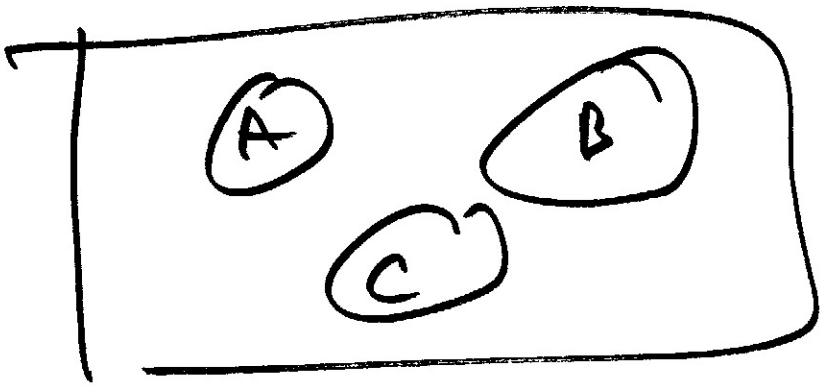
Mutually exclusive Events

$A_1, \ldots A_2 \ldots \quad A_N$.

are mutually exclusive events iff

$$A_i A_j = \begin{cases} A_i & i = j \\ \emptyset & \text{if } i \neq j \end{cases} \qquad i = 1, 2, \ldots N.$$

<u>English</u> no point in Universal set is in more than <u>one</u> event in the list.

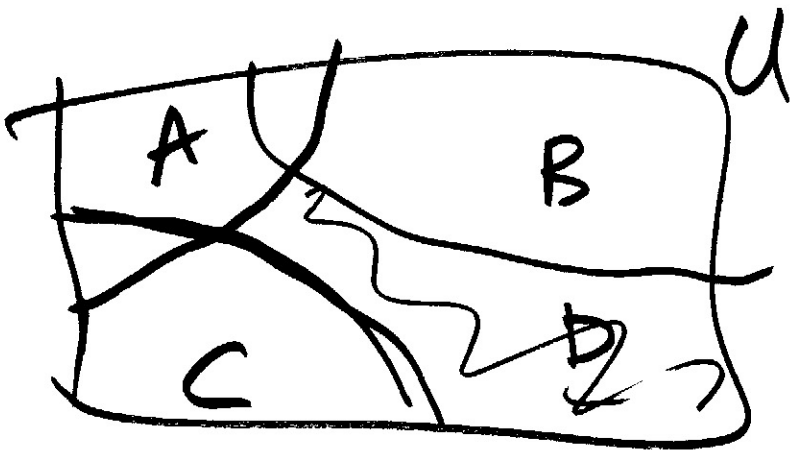


Collectively Exhaustive Event

$A_1, A_2, \ldots A_N$ iff

$A_1 + A_2 + \ldots + A_N = U$

English Each point in U is in at least one event



Space for Models of Experiments

Sample

Experiment Any process which is to some particular observer is **non-deterministic.**

Uncertainty : due to nature of process.
- state of the knowledge of observer
both.

Sample Space All possible outcomes of an experiment.

Ex **Exp**: Flip a coin once.

Sample Space = $\{H, T\}$

Types of Sample Space

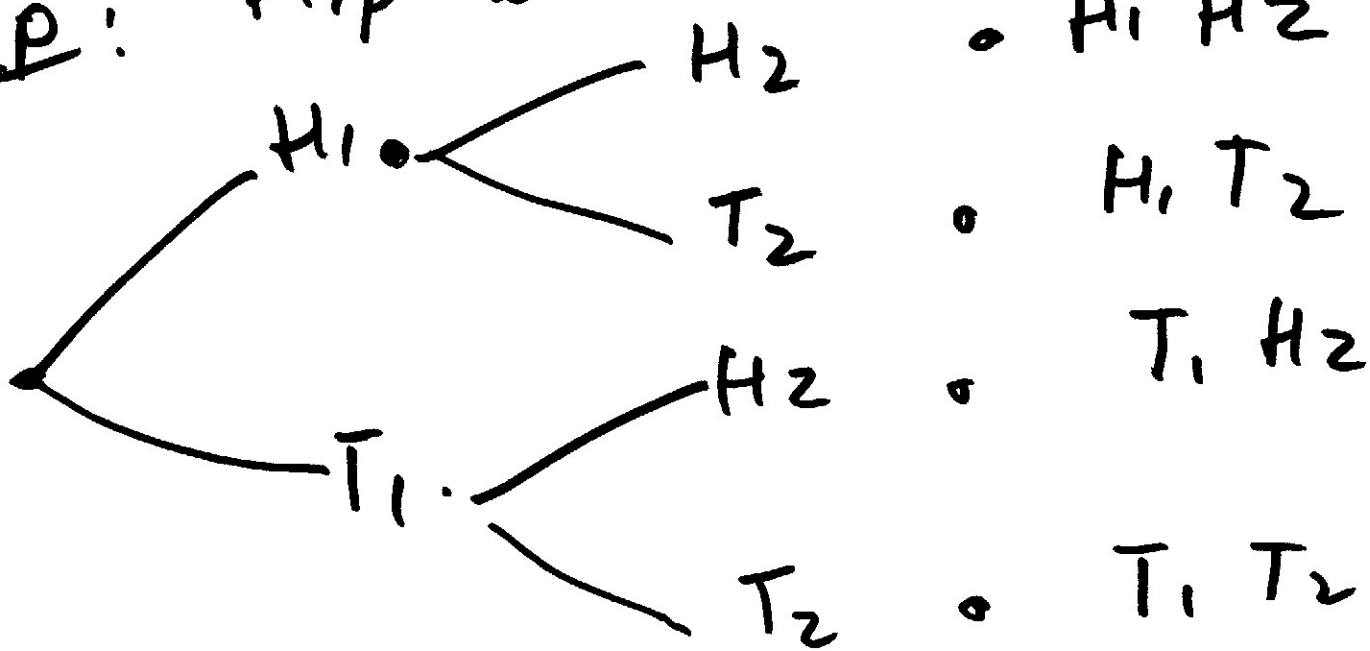
Notation Event $\begin{cases} H_n \\ T_n \end{cases}$ H_n → H on the nth Trial.

T_n → Tail on nth Trial

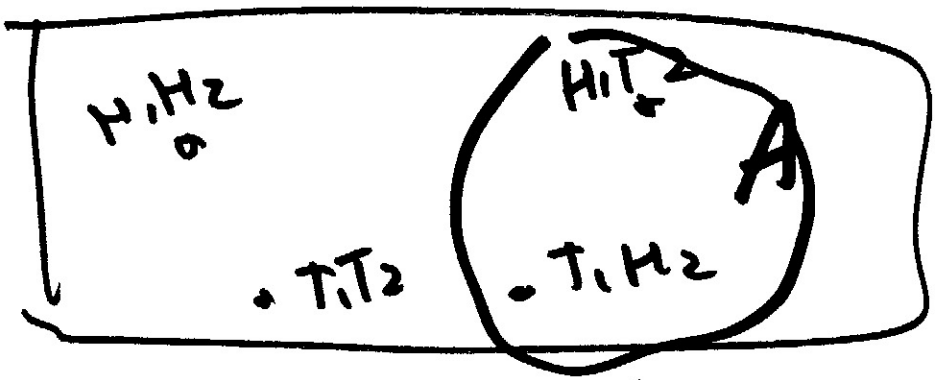
* Sequential sample space:

Rep.

Ex: Flip a coin twice.



Sample space. H_1H_2, H_1T_2, T_1H_2 T_1T_2.



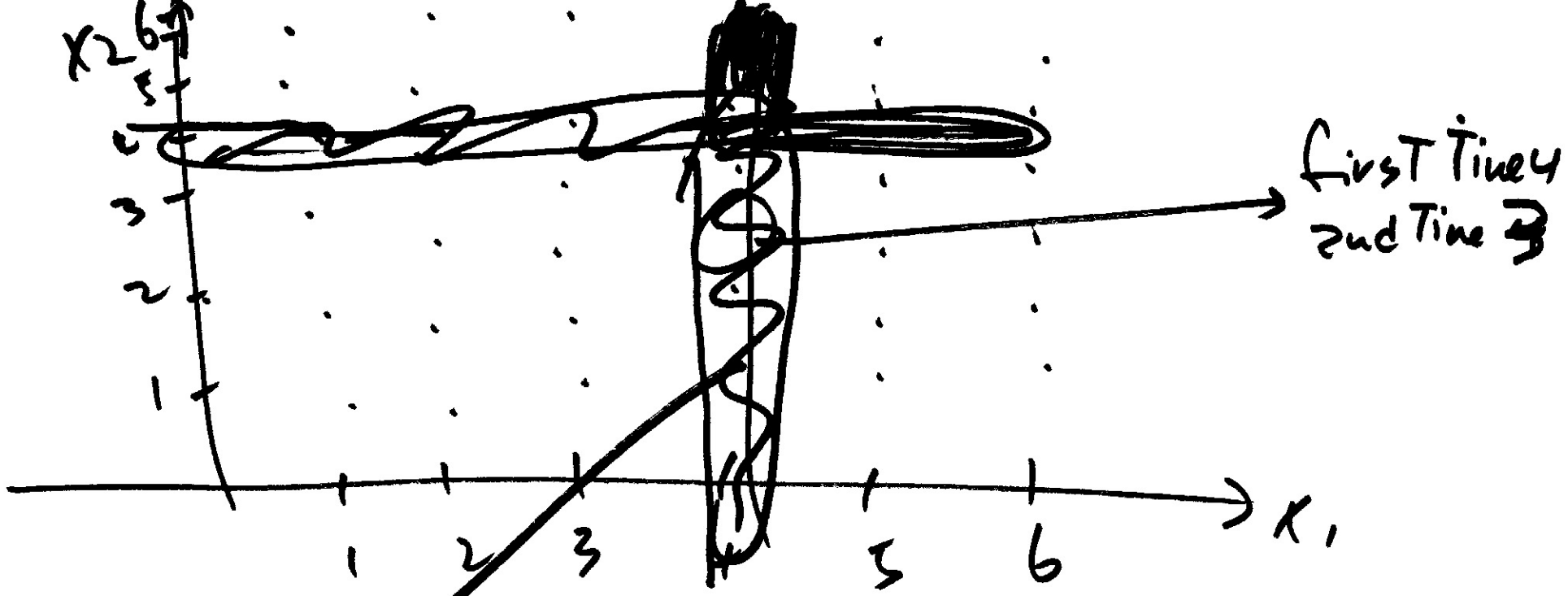
Event: exactly one head. = A

(2) If outcome of exp is expressed numerically, use coordinate sys.

Ex die. Toss it twice.

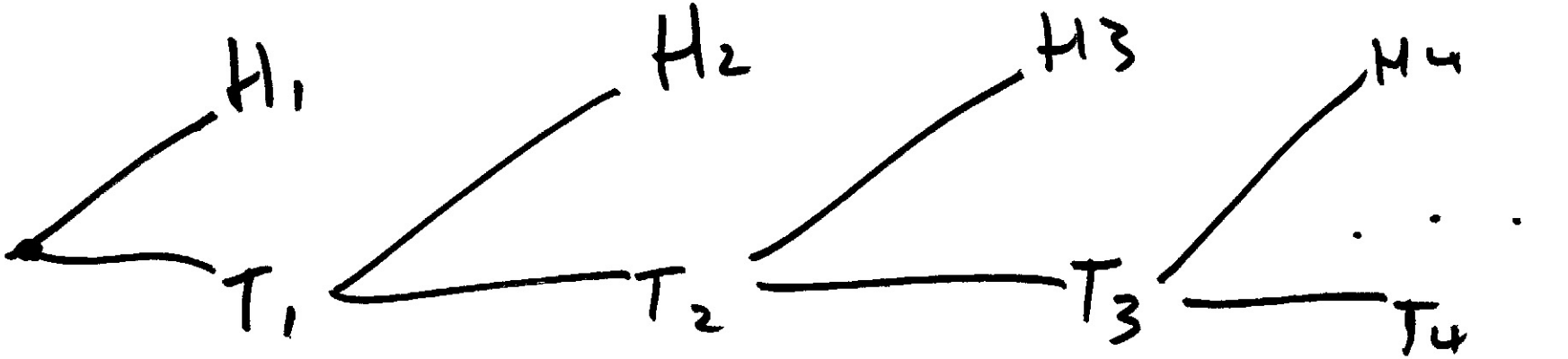
X_1 = # shows up in first try.

X_2 = # shows up in 2nd try.



at least one 4

EX Flip a coin until you get a first head.



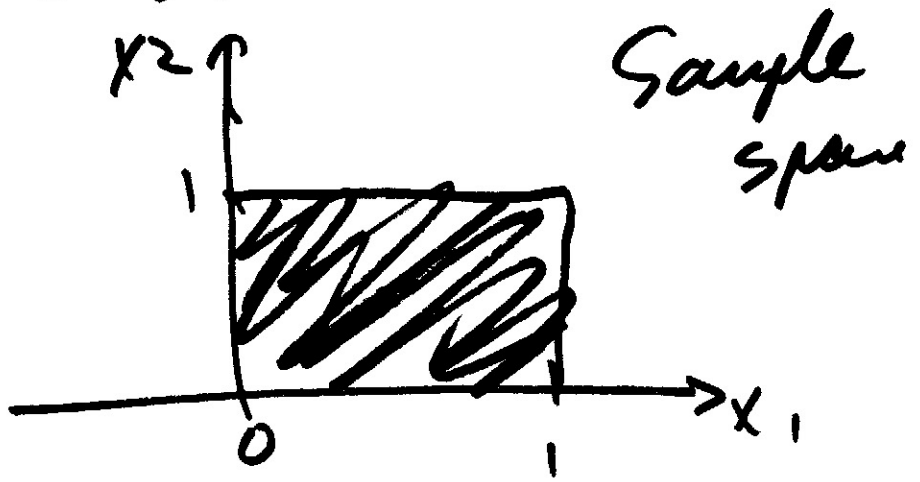
Sample space - $\{H_1, T_1H_2, T_1T_2H_3, T_1T_2T_3H_4, \ldots\}$

N = # of tosses. To do until getting a first head.

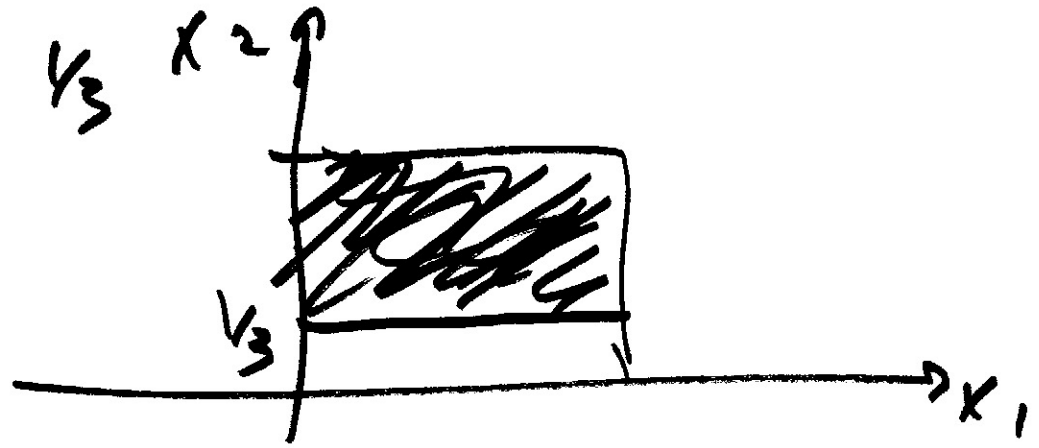
$N = 1, 2, 3, 4, \ldots$

Exp Spin a wheel of Fortune. Twice. each time a continuous value between $0 \to 1$

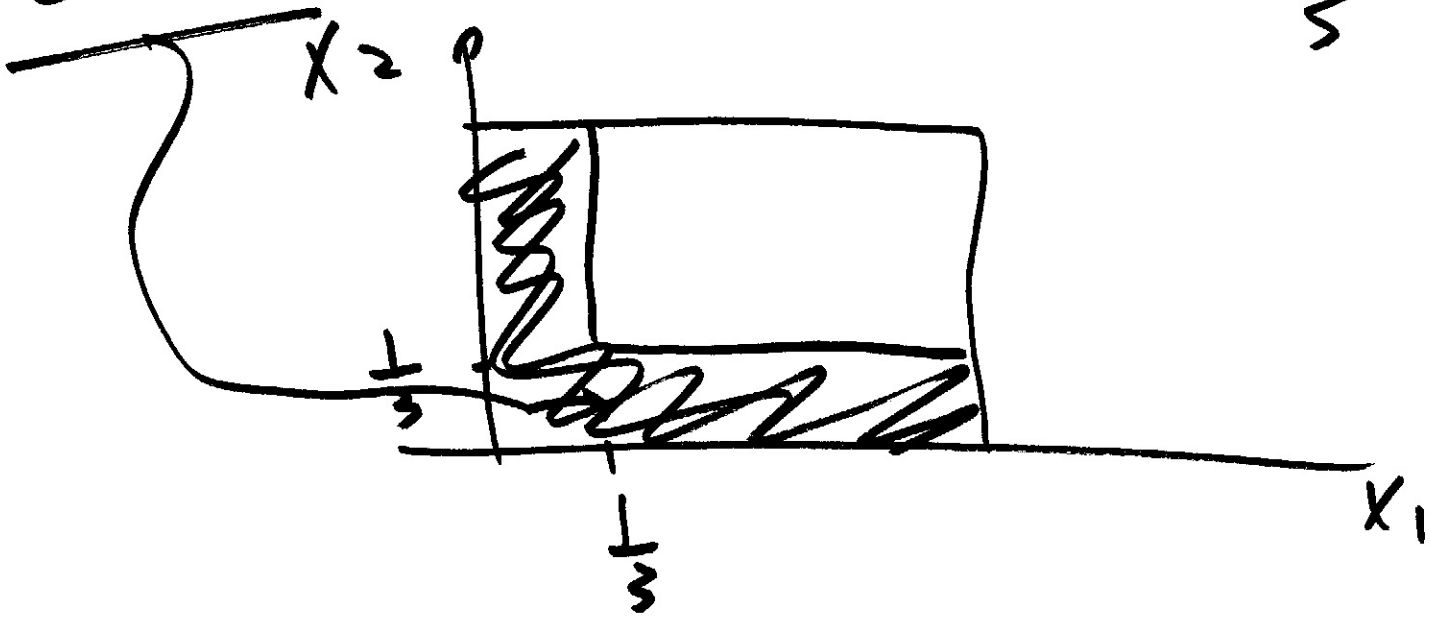
X_1 X_2



Event (A) $x_2 > \frac{1}{3}$

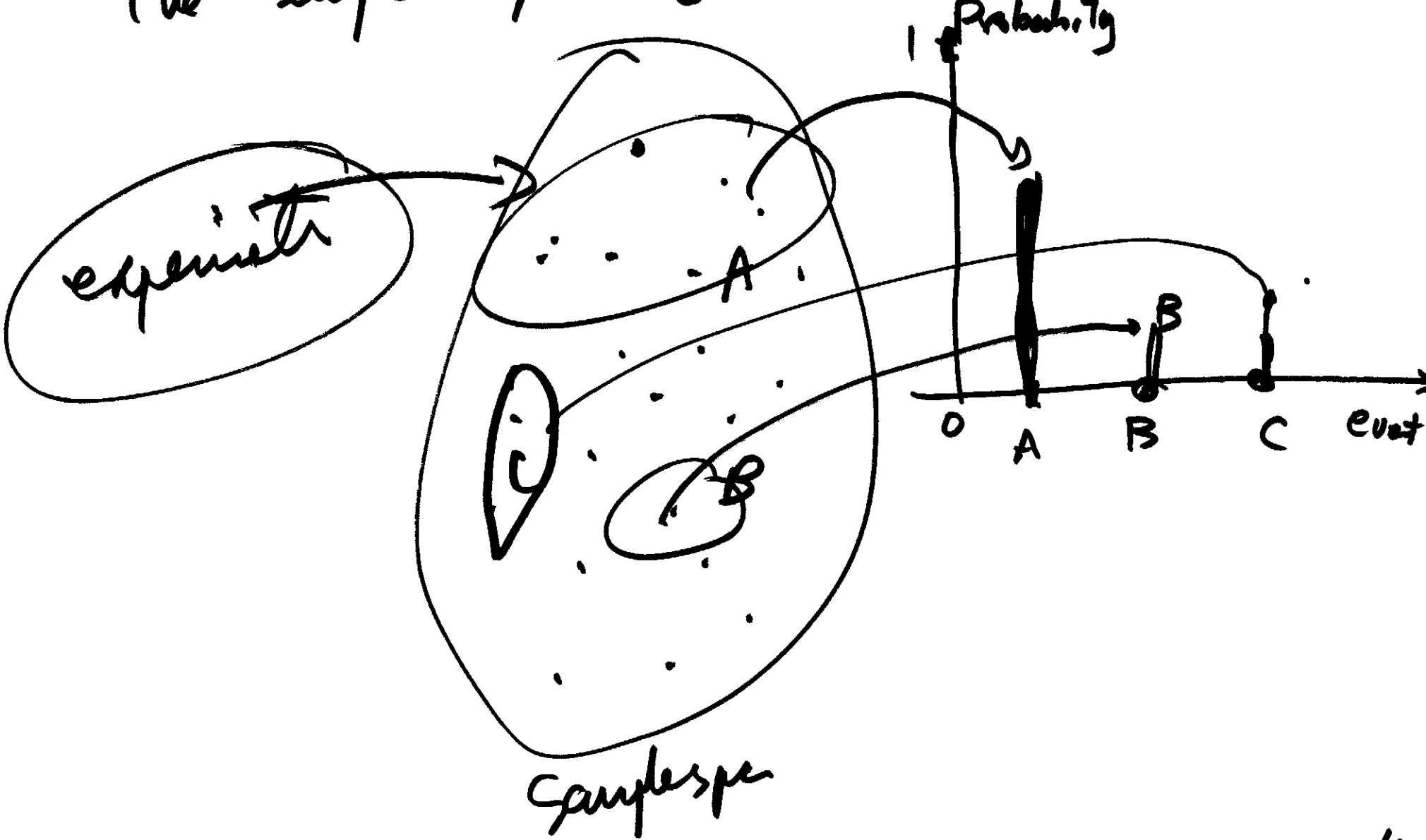


Event $\min(x_1, x_2) \leq \frac{1}{3}$



Probability means

Assign probability to the event in the sample space of an experiment.



probability

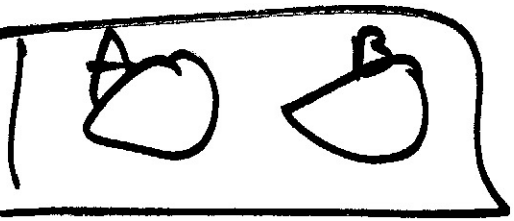
- non negative.
- between 0, 1
- chance or relative likelihood that performance of an experiment will result in occurance of that event.

Axioms of Probability Measure

(1) For any event A $P(A) \geq 0$

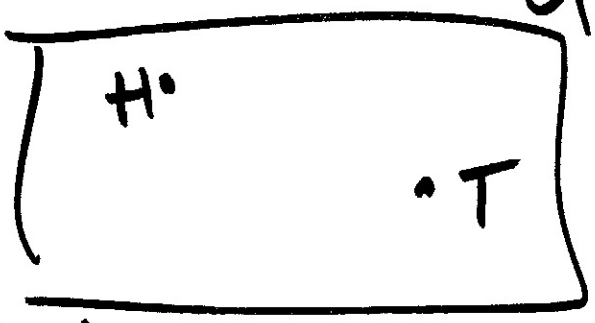
(2) $P(U) = 1$

(3) If $AB = \emptyset$ Then $P(A+B) = P(A) + P(B)$



Ex Single coin H, T • fair

$P(H) = \frac{1}{2}$ $P(T) = \frac{1}{2}$.



P(H or Tail)

$P(H + T) = P(H) + P(T)$

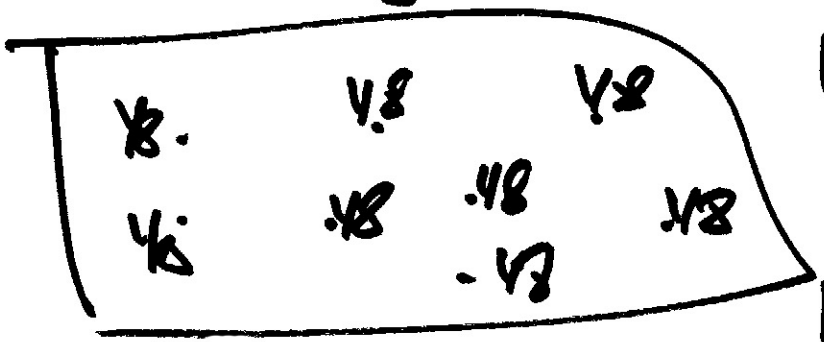
$= \frac{1}{2} + \frac{1}{2} = 1$

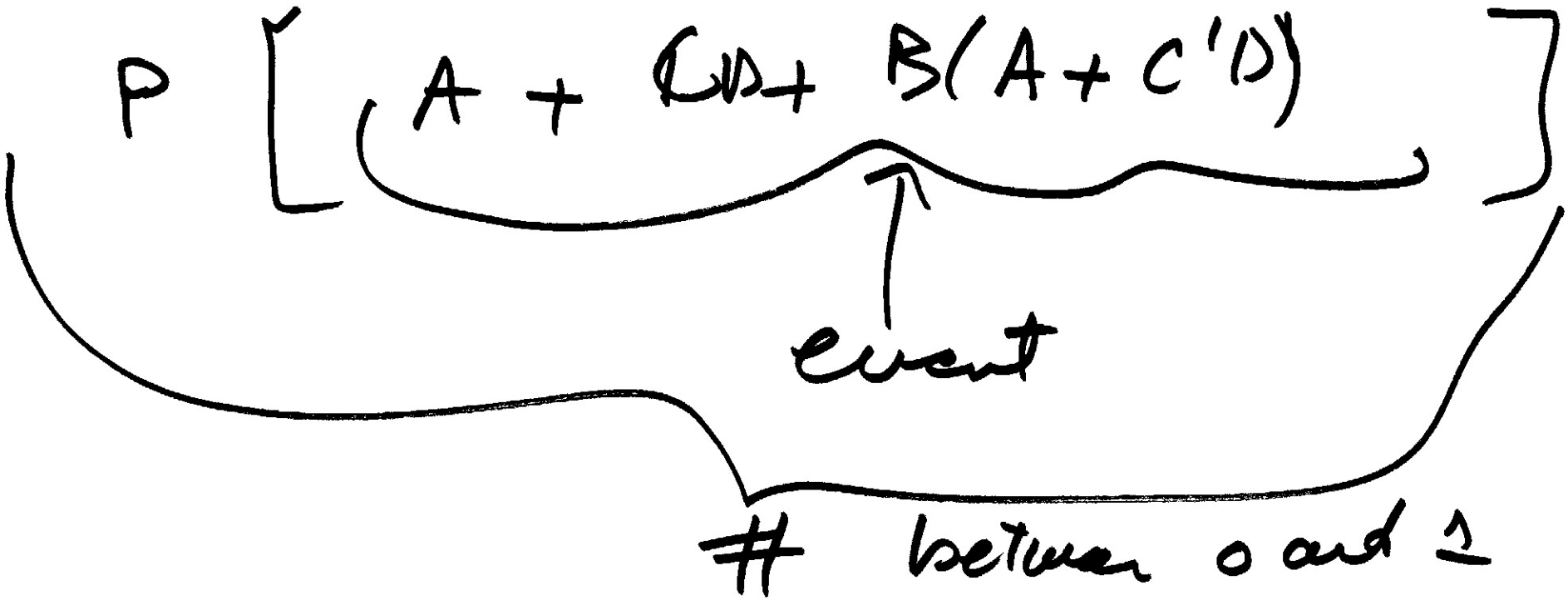
Ex 3 coins fair

Sample space = { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }

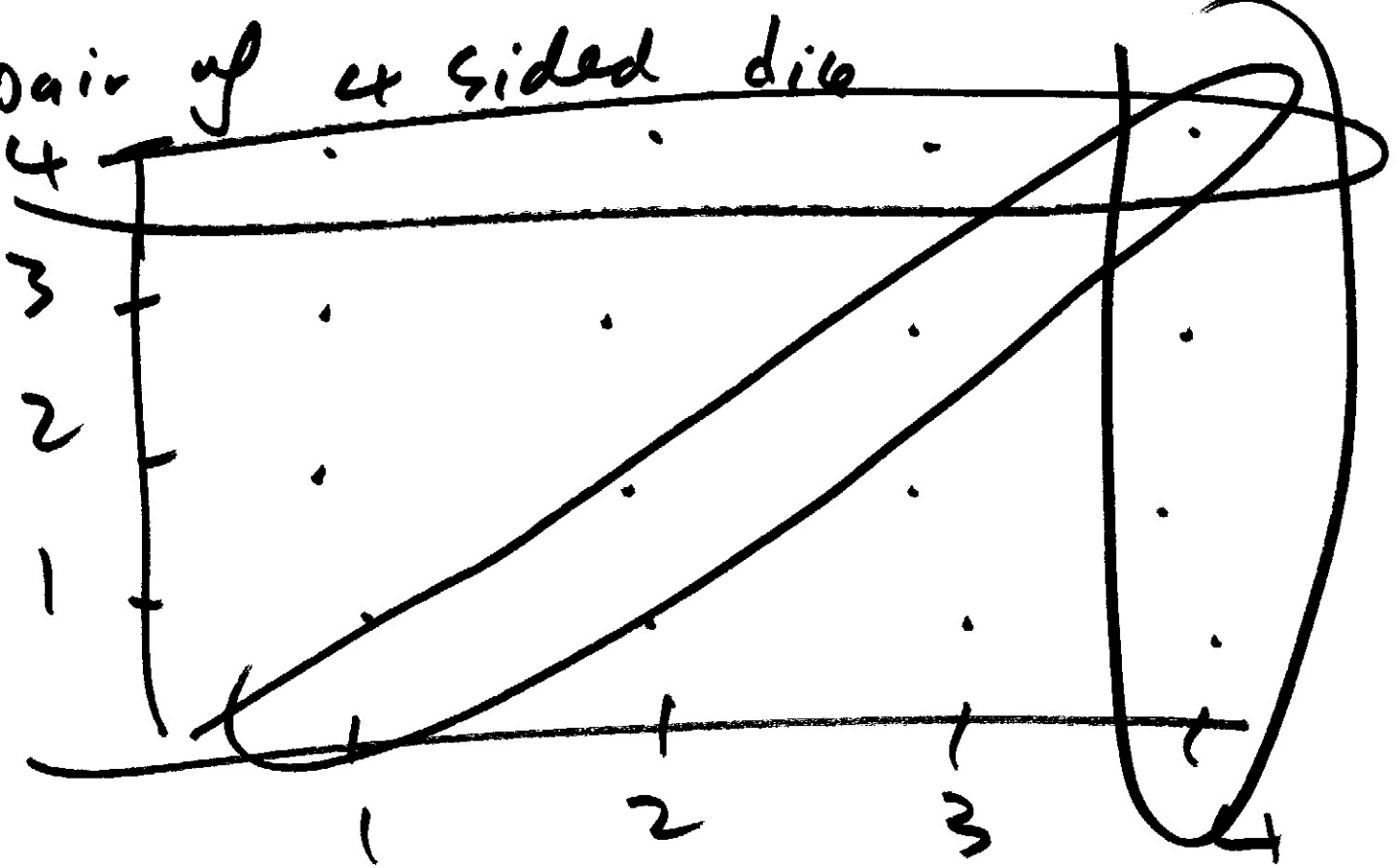
Event A = { exactly 2 heads } = { HHT, HTH, THH }

$P(A) = \frac{3}{8}$




P
A + CD + B(A + C'D)
event
between 0 and 1

a pair of 4 sided dice



Event = at least one 4 = A

$$P(A) = \frac{7}{16}$$

Event B: sum of rolls even

$$P(B) = \frac{1}{2}$$