UC Berkeley
Department of Electrical Engineering and Computer Sciences

Electrical Engineering 126: Probability and Random Processes

Discussion 13
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1. **Dynamic Programming**

   A decision problem is characterized by (state, action, noise) (x_k, u_k, w_k) for each positive integer k. The dynamics is governed by: $x_{k+1} = f_k(x_k, u_k, w_k)$, where f_k is some bounded function. Consider N (a positive integer) to be the horizon and the controller wants to minimize a cost function $g_k(x_k, u_k, w_k)$, which is additive over the discrete time step. The terminal cost $g_N(x_N)$ is given. Formulate the problem as a dynamic program and find the total expected cost. Define a policy sequence $\mu = (\mu_0, \dots, \mu_{N-1})$ as a mapping from state space to action space, i.e., $u_k = \mu_k(x_k)$. Find optimal policy as a function of total expected cost. Also state the dynamic programming update equations for the k-th iteration, using the principle of optimality.

2. **Infinite-Horizon Discounted Cost MDP**

   Consider a relay placement problem, where a deployment agent starts walking from state 0 on a line. He stops at regular intervals (consider the step length to be $\delta > 0$), and decides whether to place a relay there or not. At each step, he measures the power required to maintain a reasonable quality link. The goal of the agent is to minimize a linear combination of power cost and relay cost. Assume that the process restarts every time the agent deploys a relay. Also assume the length of the line is geometric with parameter θ. Formulate the problem as an infinite horizon MDP, with state space (r, γ) where r and γ are the respective distance and power from the previously placed relay. Using the Bellman equation, find the optimal policy structure. Mention a way to compute the optimal policy.