UC Berkeley
Department of Electrical Engineering and Computer Sciences

ELECTRICAL ENGINEERING 126: PROBABILITY AND RANDOM PROCESSES

Problem Set 1
Fall 2017

Issued: August 31, 2017 **Due:** 9 AM, Thursday, September 7, 2017

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1. **Events**

   (a) Show that the probability that exactly one of the events A and B occurs is $\mathbb{P}(A) + \mathbb{P}(B) - 2\mathbb{P}(A \cap B)$.

   (b) If A is independent of itself, show that $\mathbb{P}(A) = 0$ or 1.

2. **Joint Occurrence**

   You know that, at least one of the events A_r (for $r \in \{1, \ldots, n\}$, $n \in \mathbb{Z}_{\geq 2}$) is certain to occur but certainly no more than two occur. Show that if the probability of occurrence of any single event is p, and the probability of joint occurrence of any two distinct events is q, we have $p \geq 1/n$ and $q \leq 2/n$.

3. **Coin Flipping & Symmetry**

   Alice and Bob have $2n + 1$ fair coins (where $n \in \mathbb{Z}_{>0}$), each coin with probability of heads equal to $1/2$. Bob tosses $n + 1$ coins, while Alice tosses the remaining n coins. Assuming independent coin tosses, show that the probability that, after all coins have been tossed, Bob will have gotten more heads than Alice is $1/2$.

   Hint: Use symmetry before diving into long calculations.

4. **Passengers on a Plane**

   There are N passengers in a plane with N assigned seats ($N \in \mathbb{Z}_{>0}$), but after boarding, the passengers take the seats randomly. Assuming all seating arrangements are equally likely, what is the probability that no passenger is in their assigned seat? Compute the probability when $N \to \infty$.

   Hint: Use the inclusion-exclusion principle.

5. **Variance**

   If $X_1, \ldots, X_n$, where $n \in \mathbb{Z}_{>0}$, are i.i.d. random variables with zero-mean and unit variance, compute the variance of $(X_1 + \cdots + X_n)^2$. You may leave your answer in terms of $\mathbb{E}[X_1^4]$, which is assumed to be finite.

6. **Poisson Practice**

   Suppose X is a Poisson random variable with parameter λ. Find:

   (a) $\mathbb{E}[X^2]$.

   (b) $\mathbb{P}(X \text{ is even})$. (*Hint*: Use the Taylor series expansion of e^x.)